

On the Transmission of a Memoryless Gaussian Source over a Memoryless Fading Channel

Hamid Behroozi[†], Javad Haghghat^{††}, Fady Alajaji[†] and Tamas Linder[†]

[†]Department of Mathematics and Statistics, Queen's University, Kingston, ON, Canada

^{††}Department of Electrical & Computer Engineering, McGill University, Montreal, QC, Canada

Email: behroozi@mast.queensu.ca, javad.haghghat@mcgill.ca, {fady, linder}@mast.queensu.ca

Abstract—We consider the transmission of a discrete memoryless Gaussian source over a discrete memoryless fading channel with additive white Gaussian noise (AWGN) where the decoder has perfect channel state information (CSI). Our goal is to characterize the optimal tradeoff between the average transmission power constraint, P and the average estimation distortion, D . It is well known that for point-to-point transmission of a single Gaussian source over an AWGN channel, if the channel bandwidth is equal to the source bandwidth, linear (scalar) joint source-channel coding, i.e., uncoded transmission, achieves the optimal power-distortion tradeoff (Shannon's limit). But this result does not hold in the presence of fading [1]. In this work, we show that a relatively simple joint source-channel coding scheme, proposed by Lapidath *et al.*, which is based on the transmission of scaled versions of vector-quantized source sequences, can approach the optimal power-distortion tradeoff. This coding scheme is still optimal when the CSI is available at both the encoder and the decoder.

I. INTRODUCTION

In this paper, we consider the transmission of a discrete memoryless Gaussian source, X , over a discrete memoryless fading channel with AWGN. The encoder is subject to a transmission cost constraint which comes from the restrictions on the power resources that are available at the transmitter. The decoder aims to reconstruct the source, X , with an average mean squared-error distortion D , at the smallest cost in the communication link. Assuming that the decoder has perfect channel state (fading gain) information while the encoder has only statistical information about the channel state, the goal is to provide a joint source-channel coding scheme that can achieve the optimal power-distortion tradeoff.

Shannon proved that the separate design of source and channel coding is an optimal strategy for ergodic point-to-point communication system (asymptotically as coding/decoding delay and complexity become unconstrained) [2]. This scheme is referred to as the conventional tandem source-channel coding scheme. It is known that this conceptually simple coding scheme does not lead to the optimal performance theoretically attainable (OPTA) in networks, see e.g. [3, p. 449], [4]. Joint source and channel coding (JSCC) has attracted a lot of interest not only because of its role in reducing the complexity of the overall system, but also because of its optimality in network communication systems. In particular, JSCC can significantly outperform separation based coding schemes in multi-user systems. For the point-to-point

transmission of a single Gaussian source through an AWGN channel it is known (e.g., see [5], [4]) that if the channel bandwidth is equal to the source bandwidth, a simple uncoded transmission, which can be regarded as a special case of JSCC, achieves OPTA. The optimality of uncoded transmission in some multi-user communication systems was recently shown in [6], [7], [8]. Furthermore in [1], it is shown that uncoded transmission cannot achieve OPTA in the presence of fading.

In this work, similarly to [1] we study the reliable transmission of a memoryless Gaussian source over a memoryless fading channel. Specifically, we consider another relatively simple JSCC scheme, proposed in [6] for the AWGN channel, where the encoder is a rate- ρ Gaussian vector quantizer that scales the quantized (reproduction) sequence to meet the channel input power constraint. This scheme lies between the tandem source-channel coding scheme and the uncoded transmission scheme, i.e., there is no explicit (digital) channel coding and the scaled source codewords act as the joint source-channel codewords. (Note that in a tandem transmission scheme, a conventional source code is in tandem with a channel code, and the index of the source codeword is channel coded and transmitted over the channel.) We show that this simple JSCC scheme, which we refer to as the *VQ-based JSCC* scheme, can achieve OPTA for two cases; 1) when CSI is available at the decoder only (DCSI); 2) when the CSI is available at both the encoder and the decoder (full CSI or FCSI).

The remainder of this paper is organized as follows. In Section II, we formally define the problem. Section III provides review of the system's OPTA (Shannon's limit) and the performance of the uncoded transmission method, presented in [1]. In Section IV, we analyze the performance of the VQ-based JSCC scheme and show that it can (asymptotically) achieve OPTA. Section V concludes the paper.

II. PROBLEM STATEMENT

We consider the transmission of a memoryless Gaussian source $\{X(t) : t = 1, 2, \dots\}$ over a memoryless fading channel with AWGN, i.e., $X(t) \sim \mathcal{N}(0, \sigma_X^2)$ is independent and identically distributed (i.i.d.) over t . We represent the first n instances of $\{X(t)\}_{t=1}^{\infty}$ by the data sequence $\mathbf{X} = \{X(1), X(2), \dots, X(n)\}$. The sequence \mathbf{X} is encoded to $\mathbf{S} = f_n(\mathbf{X})$ where the encoder

function is given by

$$f_n : \mathbb{R}^n \rightarrow \mathbb{R}^n. \quad (1)$$

The transmitted sequence \mathbf{S} is average-power limited to P , i.e.,

$$\frac{1}{n} \sum_{t=1}^n E[|S(t)|^2] \leq P. \quad (2)$$

The encoder communicates the coded sequence to the decoder through a discrete memoryless fading channel with AWGN. Thus each component of the transmitted sequence is multiplied by a real-valued random channel gain $b(t)$ that is not known by the encoder but is available to the decoder. The fading coefficients are i.i.d. and independent of the source and of the channel noise. The time- t output of the channel is given by

$$Y(t) = b(t)X(t) + Z(t), \quad (3)$$

where $Z(t) \sim \mathcal{N}(0, \sigma_Z^2)$ is i.i.d. over t , and is independent of $X(t)$. Based on the channel output $\mathbf{Y} \in \mathbb{R}^n$, the decoder makes an estimate $\hat{\mathbf{X}}$ of the source \mathbf{X} . The measure of the fidelity between \mathbf{X} and $\hat{\mathbf{X}}$ is the average distortion $\Delta = \frac{1}{n} E \left[\sum_{j=1}^n d(X(j), \hat{X}(j)) \right]$ where $d(X(j), \hat{X}(j))$ is the mean squared-error (MSE) distortion measure. If we represent the fading sequence by $\mathbf{b} = \{b(1), b(2), \dots, b(n)\}$, the reconstructed signal can be described by $\hat{\mathbf{X}} = g_n(\mathbf{Y}, \mathbf{b})$, where the decoder is a mapping

$$g_n : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n. \quad (4)$$

Let $\mathcal{F}^{(n)}(P)$ denote all encoder and decoder functions (f_n, g_n) that satisfy (1)-(4). For a particular coding scheme (f_n, g_n) , the performance is determined by the required power P and the incurred distortion Δ . The power-distortion region \mathcal{P} is defined as the closure of the set of all achievable power-distortion pairs (P, D) , where a power-distortion pair (P, D) is achievable if for any $\delta > 0$ and any $n \geq n_0(\delta)$, there exists $(f_n, g_n) \in \mathcal{F}^{(n)}(P)$ such that $\Delta \leq D + \delta$. In the next section, we first review the OPTA, which is defined for a given $P > 0$, as

$$D_{min}(P) = \inf \{D \mid (P, D) \in \mathcal{P}\}, \quad (5)$$

for the transmission of Gaussian source over a fading channel. We also review the results of [1] which imply that uncoded transmission cannot achieve OPTA for this system (this also holds when the CSI is available at both the encoder and the decoder). Our main goal, presented in Section IV, is to show that OPTA can be achieved using the simple VQ-based JSCC scheme.

III. PRELIMINARIES: SHANNON'S BOUND AND PERFORMANCE OF UNCODED TRANSMISSION

A. Theoretical Limit (Shannon's Bound)

Shannon's separation theorem states that to obtain the optimal performance in a point-to-point communication system, data compression and error correction can be optimized separately and performed sequentially (in tandem). This scheme achieves the optimal performance asymptotically as the delay and the complexity become unconstrained [2, Thm. 21]. As a result, all

achievable pairs of (P, D) can be obtained by combining the rate-distortion region and the channel capacity region. The OPTA can be derived by equating the rate-distortion function of the source to the capacity of the channel. For a memoryless Gaussian source X with variance σ_X^2 and an MSE distortion, the rate-distortion function is known to be

$$R_X(D) = \max \left\{ 0, \frac{1}{2} \log \left(\frac{\sigma_X^2}{D} \right) \right\}. \quad (6)$$

The capacity of a fading channel depends on the availability of CSI at the encoder/decoder.

1) *OPTA with Decoder CSI (DCSI)*: The capacity of a fading channel, when the CSI is only available at the decoder and the average power constraint on the channel input symbols is P , is given by [9]

$$C^{DCSI}(P) = E_b \left[\frac{1}{2} \log \left(1 + \frac{P|b|^2}{\sigma_Z^2} \right) \right]. \quad (7)$$

Combining (6) with (7) leads to the following OPTA:

$$D_{min}^{DCSI}(P) = \sigma_X^2 \sigma_Z^2 \exp \left(E_b \left[\log \frac{1}{P|b|^2 + \sigma_Z^2} \right] \right), \quad (8)$$

where $D_{min}^{DCSI}(P) \leq \sigma_X^2$. Note that this OPTA is achievable in the limit of large blocklengths where the channel code transmits the source coded bits with vanishing error.

2) *OPTA with Full CSI (FCSI)*: When the CSI is available at both the encoder and the decoder, the capacity of the fading channel is given by [9]

$$C^{FCSI}(P) = E_b \left[\frac{1}{2} \log \left(1 + \frac{P(b)|b|^2}{\sigma_Z^2} \right) \right], \quad (9)$$

where $P(b)$ waterfills over the fading states

$$P(b) = \left(\frac{1}{\lambda} - \frac{\sigma_Z^2}{|b|^2} \right)^+, \quad (10)$$

and λ satisfies $E_b[P(b)] = P$. Again combining (6) and (9), the system's OPTA can be obtained:

$$D_{min}^{FCSI}(P) = \sigma_X^2 \sigma_Z^2 \exp \left(E_b \left[\log \frac{1}{P(b)|b|^2 + \sigma_Z^2} \right] \right). \quad (11)$$

Next we review from [1] the performance of an uncoded transmission scheme for sending a discrete memoryless Gaussian source over a discrete memoryless fading channel with AWGN. We consider the performance of both CSI availability cases.

B. Uncoded Transmission with DCSI

In this coding scheme, the encoder transmits the scaled version of its observation, scaled to its power constraint, i.e., $S(t) = \sqrt{\frac{P}{\sigma_X^2}} X(t)$ [5], [4]. This scheme is also referred to as linear JSCC with block length $n = 1$ [1]. The received signal at the decoder is then given by

$$Y(t) = \sqrt{\frac{P}{\sigma_X^2}} b(t)X(t) + Z(t). \quad (12)$$

Since the encoding is memoryless, the optimal estimator is the minimum mean squared error (MMSE) estimator of $X(t)$ from

received signal, $Y(t)$, which is given by

$$\hat{X}(t) = E[X(t)|Y(t), b(t)] = \frac{b(t)\sqrt{P\sigma_X^2}}{P|b(t)|^2 + \sigma_Z^2} Y(t). \quad (13)$$

Evaluating the average MSE distortion of this estimator, D_{unc} , gives the following distortion-power tradeoff

$$D_{unc}^{DCSI}(P) = \sigma_X^2 \sigma_Z^2 E_b \left[\frac{1}{P|b|^2 + \sigma_Z^2} \right]. \quad (14)$$

Rewriting (14) as

$$D_{unc}^{DCSI}(P) = \sigma_X^2 \sigma_Z^2 \exp \left(\log \left(E_b \left[\frac{1}{P|b|^2 + \sigma_Z^2} \right] \right) \right),$$

and then using Jensen's inequality [3] and the concavity of the logarithm confirms that $D_{unc}^{DCSI}(P) \geq D_{min}^{DCSI}(P)$. Thus uncoded transmission achieves the best possible distortion if and only if

$$E_b \left[\log \frac{1}{P|b|^2 + \sigma_Z^2} \right] = \log \left(E_b \left[\frac{1}{P|b|^2 + \sigma_Z^2} \right] \right),$$

i.e., when $|b|^2$ is a constant. In all other cases uncoded transmission cannot achieve OPTA [1].

So far we have assumed that the CSI is only available at the decoder. Since the average power constraint was the only constraint on the channel input symbols, the optimal power allocation along the time index t was a uniform power allocation [1]. However, when the fading realization $b(t)$ is also known by the encoder, the uniform power allocation is not optimal.

C. Uncoded Transmission with FCSI

When there is CSI at the encoder and the decoder, the transmitted signal is given by $S(m) = \sqrt{\frac{P_{unc}(b_m)}{\sigma_X^2}} X(m)$ where b_m denotes the channel realization (state) at $t = m$ and $P_{unc}(b_m)$ denotes the transmit power at $t = m$ which depends on the channel state. The MMSE estimator as well as the instantaneous MSE can be obtained as follows:

$$\hat{X}(m) = \frac{b_m \sqrt{\sigma_X^2 P_{unc}(b_m)}}{P_{unc}(b_m) |b_m|^2 + \sigma_Z^2} Y(m), \quad (15)$$

$$E(|X(m) - \hat{X}(m)|^2) = \sigma_X^2 \sigma_Z^2 E_{b_m} \left[\frac{1}{P_{unc}(b_m) |b_m|^2 + \sigma_Z^2} \right].$$

The optimal power allocation can be obtained by solving

$$\begin{aligned} \min \quad & E_b \left[\frac{1}{P_{unc}(b) |b|^2 + \sigma_Z^2} \right] \\ \text{s.t.} \quad & E_b [P_{unc}(b)] = P, P_{unc}(b) \geq 0 \end{aligned} \quad (16)$$

It can be shown that the optimal power allocation in terms of the channel state b is [1]

$$P_{unc}^{opt}(b) = \frac{1}{|b|} \left(\frac{1}{\mu} - \frac{\sigma_Z^2}{|b|} \right)^+, \quad (17)$$

where μ is a common threshold for all channel states and can be calculated by solving $E_b [P_{unc}^{opt}(b)] = P$, i.e.,

$$\int_{\mu}^{\infty} \frac{1}{|b|} \left(\frac{1}{\mu} - \frac{\sigma_Z^2}{|b|} \right) f(|b|) d|b| = P. \quad (18)$$

With this power allocation, which is a water-filling allocation over time, the minimum achievable distortion by uncoded transmission can be expressed as

$$D_{unc}^{FCSI}(P) = \sigma_X^2 \sigma_Z^2 E_b \left[\frac{1}{P_{unc}^{opt}(b) |b|^2 + \sigma_Z^2} \right]. \quad (19)$$

IV. VQ-BASED JSCC SCHEME

In this section, we consider the transmission of vector-quantized source sequences, as proposed in [6] for the AWGN case. In this VQ-based JSCC scheme, the encoder is a rate- ρ Gaussian vector quantizer that scales the quantized sequence to meet the channel input power constraint.

A. VQ-Based JSCC with DCSI

First we generate the quantization codebook $\mathcal{C}^{(n)}$ and then describe the encoding part. Let ρ denote the rate of the Gaussian vector quantizer. There are $2^{n\rho}$ codewords that lie on the sphere of approximate radius $\sqrt{n\sigma_X^2(1-2^{-2\rho})}$. Therefore, the approximate normalized squared-norm of each codeword in $\mathcal{C}^{(n)}$ is

$$\frac{1}{n} \|\mathbf{U}\|^2 = \sigma_X^2 (1 - 2^{-2\rho}), \quad (20)$$

where $\|\mathbf{U}\|^2$ denotes the sum of the squares of the components in \mathbf{U} .

The encoder uses nearest neighbor encoding to choose the codeword \mathbf{U}^* in the codebook $\mathcal{C}^{(n)}$ that is closest to the source sequence and scales the resulting vector \mathbf{U}^* so that the power constraint P is satisfied. Therefore, the transmitted sequence can be expressed as

$$\mathbf{S} = \beta \cdot \arg \min_{\mathbf{U} \in \mathcal{C}^{(n)}} \|\mathbf{X} - \mathbf{U}\|^2 = \beta \cdot \arg \max_{\mathbf{U} \in \mathcal{C}^{(n)}} \langle \mathbf{X}, \mathbf{U} \rangle, \quad (21)$$

where

$$\beta = \sqrt{\frac{P}{\sigma_X^2 (1 - 2^{-2\rho})}}, \quad (22)$$

and $\langle \cdot, \cdot \rangle$ denotes the standard inner product in \mathbb{R}^n . It can be shown that the closest codeword \mathbf{U}^* in $\mathcal{C}^{(n)}$ to the source sequence \mathbf{X} would be *nearly orthogonal* [6], [10] to $(\mathbf{X} - \mathbf{U}^*)$ as the blocklength n tends to infinity. Thus, for the quantization error, $(\mathbf{X} - \mathbf{U}^*)$, we approximately have for large n

$$\begin{aligned} \frac{1}{n} \|\mathbf{X} - \mathbf{U}^*\|^2 &\stackrel{(a)}{=} \frac{1}{n} \|\mathbf{X}\|^2 - \frac{1}{n} \|\mathbf{U}^*\|^2 \\ &\stackrel{(b)}{=} \sigma_X^2 2^{-2\rho} \end{aligned} \quad (23)$$

where (a) follows from the near orthogonality between \mathbf{U}^* and $(\mathbf{X} - \mathbf{U}^*)$ and (b) follows from the law of large numbers for $\frac{1}{n} \|\mathbf{X}\|^2$ and from (20).

The decoder reconstructs the source sequence \mathbf{X} as $\hat{\mathbf{X}} = \hat{\mathbf{U}}^* \in \mathcal{C}^{(n)}$ using the fading sequence $\mathbf{b} = \{b(1), b(2), \dots, b(n)\}$, where the decoding rule is given as

$$\hat{\mathbf{U}}^* = \arg \max_{\mathbf{U} \in \mathcal{C}^{(n)}} \langle \mathbf{Y}, \mathbf{b} \circ \mathbf{U} \rangle, \quad (24)$$

and \circ denotes the Schur product (coordinate-wise product) between \mathbf{b} and \mathbf{U} . This decoding method, which has been referred to by the minimum angle decoding in [10], [6], and the nearest

neighbor decoding in [11], is in fact the minimum Euclidean distance decoding. When there is no fading, it is shown in [10] that the decoder will succeed with high probability if

$$\rho < \frac{1}{2} \log \left(1 + \frac{P}{\sigma_Z^2} \right). \quad (25)$$

By combining this result with the previous results of Lapidth [11] for fading channels with Gaussian codebooks, we can show that the decoder will succeed with high probability if

$$\rho < E_b \left[\frac{1}{2} \log \left(1 + \frac{P|b|^2}{\sigma_Z^2} \right) \right]. \quad (26)$$

Replacing this inequality with equality and calculating the MSE of the proposed estimator, we obtain

$$D_{VQ}^{DCSI}(P) = \sigma_X^2 \sigma_Z^2 \exp \left(E_b \left[\log \frac{1}{P|b|^2 + \sigma_Z^2} \right] \right). \quad (27)$$

Comparing (27) with (8) reveals that this VQ-based JSCC scheme is optimal in the sense of achieving OPTA. In other words, as the quantization dimension tends to infinity, this JSCC scheme is optimal if we choose the rate of the vector quantizer arbitrarily close to the capacity of the fading channel.

B. VQ-based JSCC with FCSI

In the presence of the CSI at the encoder and the decoder, the codebook $\mathcal{C}^{(n)}$ generation is the same as before (Section IV-A), i.e., there are $2^{n\rho}$ codewords that lie on the sphere of approximate radius $\sqrt{n\sigma_X^2(1-2^{-2\rho})}$. The encoder uses nearest neighbor encoding to choose the codeword \mathbf{U}^* in the codebook $\mathcal{C}^{(n)}$ that is closest to the source sequence and scales each symbol U_m^* ($m = 1, 2, \dots, n$) in the resulting vector $\mathbf{U}^* = [U_1^* U_2^* \dots U_n^*]$ (based on the available CSI at the encoder) so that the average power constraint P is satisfied and the achievable MSE distortion is minimized. Using the optimal power allocation scheme for the memoryless fading channels [9], it can be shown that the m th component of the transmitted sequence $\mathbf{S} = [S_1 S_2 \dots S_n]$ is

$$S_m = \beta_m \cdot U_m^* = \sqrt{\frac{P_m^{opt}}{\sigma_X^2(1-2^{-2\rho})}} \cdot U_m^*, \quad (28)$$

for $m = 1, 2, \dots, n$ where $P_m^{opt} = \left(\frac{1}{\lambda} - \frac{\sigma_Z^2}{|b_m|^2} \right)^+$ and λ is chosen such that $E_{b_m}[P_m^{opt}] = P$. The decoder reconstructs the transmitted codeword \mathbf{U}^* as $\hat{\mathbf{U}}^* \in \mathcal{C}^{(n)}$ with the same decoding rule as (24). Again, it can be shown that the decoder will succeed with high probability if

$$\rho < E_b \left[\frac{1}{2} \log \left(1 + \frac{P(b)|b|^2}{\sigma_Z^2} \right) \right], \quad (29)$$

where $P(b)$ is given in (10). Replacing this inequality with equality and calculating the MSE of the estimator, we obtain

$$D_{VQ}^{FCSI}(P) = \sigma_X^2 \sigma_Z^2 \exp \left(E_b \left[\log \frac{1}{P(b)|b|^2 + \sigma_Z^2} \right] \right). \quad (30)$$

Comparing (30) with (11) shows that this VQ-based JSCC scheme still achieves OPTA for the FCSI case. Therefore, regarding the transmission of a memoryless Gaussian source over a memoryless fading channel with FCSI, the VQ-based JSCC

is optimal while the uncoded transmission shows a performance loss.

V. CONCLUSION AND FUTURE WORK

In this paper, we showed that to transmit a memoryless Gaussian source over a memoryless fading channel with AWGN, the transmission of scaled versions of vector-quantized source sequences is optimal in that it (asymptotically) achieves the OPTA distortion. Since we considered a point-to-point communication system, Shannon's separation theorem was used as our benchmark. However, the separation theorem does not hold in general in multi-user systems. As one of the future directions, we plan to apply this scheme to the two-user sensor network where two sensors transmit independent noisy versions of a Gaussian source to a central decoder over a multi-access channel with fading. For this network in general, only separate necessary and sufficient conditions for achievability of all power-distortion tuples exist [12]. In the symmetric case with no fading, where the encoders are subject to the same average power constraint and the observations of sensors have the same variance, it is known that uncoded transmission is optimal [7]. But determining the optimal power-distortion tradeoff for the asymmetric case and in the presence of fading is still an open problem.

REFERENCES

- [1] J. Xiao, Z. Q. Luo, and N. Jindal, "Linear joint source-channel coding for Gaussian sources through fading channels," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM)*, San Francisco, CA, Nov. 2006.
- [2] C. E. Shannon, "A mathematical theory of communication," *Bell Systems Technical Journal*, vol. 27, pp. 379–423, pp. 623–656, 1948.
- [3] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: Wiley, 1991.
- [4] M. Gastpar, B. Rimoldi, and M. Vetterli, "To code, or not to code: Lossy source channel communication revisited," *IEEE Trans. Inf. Theory*, vol. 49, no. 5, pp. 1147–1158, May 2003.
- [5] T. J. Goblick, "Theoretical limitations on the transmission of data from analog sources," *IEEE Trans. Inf. Theory*, vol. 11, pp. 558–567, Oct. 1965.
- [6] A. Lapidth and S. Tinguely, "Sending a bivariate Gaussian source over a Gaussian MAC," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Seattle, Washington, Jul. 2006, pp. 2124–2128.
- [7] M. Gastpar, "Uncoded transmission is exactly optimal for a simple Gaussian sensor network," in *Proc. 2007 Information Theory and Applications Workshop*, San Diego, CA, USA, Jan. 2007.
- [8] S. Bross, A. Lapidth, and S. Tinguely, "Broadcasting correlated Gaussians," arXiv:0710.5376v1[cs.IT], Oct. 2007.
- [9] D. N. C. Tse and P. Viswanath, *Fundamentals of Wireless Communications*. Cambridge, U.K.: Cambridge University Press, 2005.
- [10] S. Bross, A. Lapidth, and S. Tinguely, "Superimposed coded and uncoded transmissions of a Gaussian source over the Gaussian channel," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Seattle, Washington, Jul. 2006, pp. 2153–2155.
- [11] A. Lapidth, "Nearest neighbor decoding for additive non-Gaussian noise channels," *IEEE Trans. Inf. Theory*, vol. 42, no. 5, pp. 1520–1529, Sep. 1996.
- [12] M. Gastpar and M. Vetterli, "Power, spatio-temporal bandwidth, and distortion in large sensor networks," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 4, pp. 745–754, Apr. 2005.