A Model for Correlated Rician Fading Channels Based on a Finite Queue

Libo Zhong, Fady Alajaji and Glen Takahara

Department of Mathematics and Statistics

Queen's University, Kingston, Ontario, Canada, K7L 3N6

Abstract

We study the problem of approximating the family of hard-decision frequency-shift keying demodulated correlated flat Rician fading channels via a recently introduced queue-based channel (QBC) model for binary communication channels with memory. For a given "discretized" fading channel, we construct a QBC whose noise process is statistically "close" in the Kullback-Leibler sense to the error or noise process generated by the fading channel and the modeling accuracy is evaluated in terms of noise autocorrelation function (ACF) and channel capacity. Numerical results indicate that the QBC provides a good approximation of the fading channels for a wide range of channel conditions. Furthermore, it estimates the noise ACF more accurately than the finite-state Markov models recently studied by Pimentel *et. al.* while at the same time remaining mathematically tractable.

Keywords: Modeling of communication channels with memory; correlated Rician fading; Kullback-Leibler divergence rate; autocorrelation function; error statistics; capacity.

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I. INTRODUCTION

In recent years, there has been an increasing interest in transmitting voice, data, image and video signals over wireless communication channels. However, wireless channels undergo a variety of time-varying signal impairments caused by propagation loss, shadowing, multipath fading, and thermal noise. In particular, it is important to understand the deleterious effects of fading on wireless transmission. A common feature of many fading channels is that they cause symbol errors to occur in clusters or bursts [2].

In the presence of error bursts, interleaving is usually applied to destroy or mitigate the memory because most coding systems and protocols are designed under the assumption of memoryless error processes. With perfect or ideal interleaving, it is possible to model the fading channels as memoryless channels. However, the use of interleaving introduces extra delay and complexity, and perfect interleavers do not exist in any practical system. In real-time personal communication systems, data is transmitted in short blocks and fairly strict delay constraints must be obeyed (e.g., see [3]). Non-interleaved or finite-interleaved packet transmission over fading channels has received significant attention recently [4], [5].

Therefore, in this work we start with the premise that the inherent memory of fading communication channels cannot be neglected. Actually, an advantageous feature of memory is that the channel quality in the near future can be forecast based on the knowledge of previous channel conditions due to the statistical dependence of errors. In order to obtain highly reliable data transmission over channels with memory, we should take advantage of the channel memory by constructing effective error control coding strategies. For this reason, it is critical to fully understand the error structure of such channels. This is achieved via channel modeling, where the main objective is to provide a model whose properties are both complex enough to closely capture the real channel statistical characteristics, and simple enough to allow mathematically tractable system analysis. In this work, we employ a binary additive channel model with memory based on a finite queue that reliably and tractably describes a family of correlated fading channels.

During the past several decades, a variety of channel models have been proposed and studied for the modeling of wireless fading channels (e.g., see [6], [7], [8] and the models therein). A finite-state Markov channel (FSMC) is a discrete valued channel with a finite set of possible states whose transition is governed by an underlying Markov chain and with a probability assignment that is independent of time [9], [2], [10]. FSMCs have been widely adopted for the description of the correlation structures and success/failure processes of wireless channels with bursty behavior [11] because they are efficient in quick simulations, system performance evaluations and protocol investigations. Two of the earliest FSMC models for representing the "discretized" version (under hard-decision demodulation) of binary-input fading channels with memory are the Gilbert-Elliott channel (GEC) [12], [13] and the Fritchman channel (FC) [14]. They were for example employed to model high-frequency channels [15], mobile radio channels [16], [17], [18], low earth orbit satellite channels [19] and magnetic tape recorders [20]. The GEC model also has been used to evaluate the performance of coding and decoding schemes over bursty channels [21], [22], [3].

Many FSMC models, including the above mentioned works, have been proposed to fit realistic wireless channels. In [23], Wang and Moayeri proposed an FSMC based on the partitioning of the received signal-to-noise ratio (SNR) into a finite number of states to model Rayleigh fading channels. The same approach was also presented independently in [4] and used in [24], [25]. The model proposed in [23] attracted much attention because it has a good balance between accuracy and complexity. It was applied to the evaluation of system-related channel properties (such as the correlation properties of the fading mobile radio channel) in [26], [27] by modeling the channel as a first-order Markov process whose transition probabilities are a function of the channel characteristics. In [28], an analytical model was used to evaluate the effect of mobile velocity on the performance of a communication system operating in a multi-path fading channel.

FSMCs are often generated via finite-state hidden Markov models (HMMs).[1] General HMMs were studied in [29] to model flat fading channels. Due to their HMM structure, such channels can be difficult to mathematically analyze (e.g., they do not admit an exact closed-form expression for their capacity and/or their block transition distribution is not transparently expressed in terms of the channel parameters), particularly when incorporated within an overall source and/or channel coded system. This may partly explain why to date, few coding techniques that effectively exploit the noise memory have been successfully constructed for HMM-based channel models and for channels with memory in general. Instead, most current wireless communication systems are designed for memoryless channels and employ channel interleaving in an attempt to disperse the channel memory and make the channel appear memoryless (even burst-error correcting codes such as Reed Solomon codes operating on a

[1]A description of a large class of finite or infinite state HMM based channel models is provided in [2].

HMM-based channel perform best when interleaving is used, e.g., see [5]). However, in addition to the increased complexity/delay caused by interleaving, the failure to exploit the channel's memory at the encoder and/or decoder leads to a waste of channel capacity since it is well known that *memory increases capacity*[2] for a wide class of channels (the class of information stable channels [31], [32]). It is therefore vital to construct channel models which can well represent the behavior of real-world channels while remaining analytically tractable for design purposes.

In [32, Section VI], Alajaji and Fuja proposed a simple binary additive noise channel with memory, referred to as the finite memory contagion channel (FMCC), where the noise process is generated via a finite-memory version of Polya's urn scheme for the spread of a contagious disease through a population [33]. In such a channel, every error (or "infection", if we use the contagion interpretation) effectively increases the probability of future errors ([33]), and hence may lead to a clustering or burst of errors (i.e., an "epidemic" in the population). The resulting channel, which is fully described by only three parameters, has a stationary ergodic Mth order Markov noise source and admits single-letter analytical expressions for its block transition distribution and capacity. This model was adopted in several source-channel and channel coding studies (e.g., [34]-[42]) where the channel statistics are incorporated into the system design in order to exploit the noise memory.

The queue-based channel (QBC), recently introduced in [43], [44], [45], is a binary additive noise channel with memory based on a finite queue. The QBC is a more general model than the FMCC as it subsumes the later as a special case. It also features a stationary ergodic Mth order Markov noise source and it is fully characterized by four parameters (ϵ, α, p and M), thus having one more degree of freedom than the FMCC (by setting $\alpha = 1$, the QBC reduces to the FMCC for the same bit error rate, correlation coefficient and memory order). It is important to point out that Pimentel, Falk and Lisbôa recently showed in a numerical study [46] that the class of binary channel models with additive Kth order Markov noise (to which the QBC belongs) is a good approximation, in terms of the autocorrelation function (ACF) and variational distance, to the family of hard-decision frequency-shift keying demodulated time-correlated flat Rayleigh and Rician fading channels for a good range of fading environments, particularly for medium and fast fading rates. Note however, that the general Kth order Markov noise channels considered in [46] have a complexity (number of parameters) that grows exponentially with K, rendering it impractical for the modeling of channels with large memory such as very slow Rayleigh fading channels (e.g.,

[2]In other words, the capacity of the "equivalent" memoryless channel achieved by ideal interleaving (with infinite interleaving span) is smaller than the capacity of the original channel (e.g., see [30]).

see Fig. 2 or [46, Fig. 11]). The QBC model, on the other hand, does not suffer from this limitation as it has a fixed number of parameters (four parameters) and it can accommodate very large values of the memory M. Like the FMMC, it enjoys a transparent formula for its n-fold statistics and a closed form formula for its capacity, which are appealing features as they provide powerful analytical tools for code design and system analysis. In a recent related work [47], the problem of modeling the GEC using the QBC was investigated, and it was shown (numerically) that the QBC provides a good approximation of the GEC for various channel conditions.

In this work, we investigate the problem of approximating the same class of Rician fading channels studied in [46] via the QBC. Specifically, for a given hard-decision demodulated fading channel, we construct a QBC whose error (i.e., noise) process is statistically as close as possible to the error process generated by the fading channel. This is achieved by selecting the QBC parameters that minimize the Kullback Leibler divergence rate between both noise processes for identical bit error rates and correlation coefficients. Since the QBC model has a simple Markovian structure and low complexity as it is fully described by only four parameters (while still allowing for large memory values), the optimization problem involves only two parameters and can be efficiently solved numerically. Modeling results indicate that the QBC is a good fit for fading channels as it accurately models (in terms of autocorrelation function and capacity) their burst-error behaviour for a wide range of channel environments, including slow fading. The QBC is thus an interesting alternative to existing models for channels with memory (such as HMMs) which tend to be either too complex for tractable analysis and code design that exploits the channel's memory, or too limiting for realistic modeling.

The rest of this paper is organized as follows. Preliminaries on the GEC and QBC channel models are presented in Section II. In Section III, we investigate the modeling of the Rician fading channels via the QBC. In Section IV, we provide the numerical fitting results. For the sake of comparison, we also model the fading channels via the GEC (which has the same number of parameters as the QBC) using the parameterization method of Pimentel *et. al.* in [46]. The accuracy of both methods is evaluated in terms of ACF and capacity. In Section V, we conclude with a summary along with several directions for future work.

II. The GEC and QBC Binary Channel Models

Hereafter, a discrete-time binary additive noise communication channel refers to a channel with common input, noise and output alphabet $\mathcal{X} = \mathcal{Z} = \mathcal{Y} = \{0, 1\}$, described by $Y_n = X_n \oplus Z_n$, for $n = 1, 2, 3, \cdots$, where $\oplus$ denotes addition modulo 2, and where X_n, Z_n, and Y_n denote, respectively, the channel's input, noise, and output at time n. Hence a transmission error occurs at time n whenever $Z_n = 1$. It is assumed that the input and noise sequences are independent of each other. In this work, a given noise process $\{Z_n\}_{n=1}^{\infty}$ will be generated according to one of the GEC, the QBC and the discretized Rician fading channel.

A. Gilbert-Elliott Channel

The GEC model [12], [13], [30] is driven by an underlying stationary ergodic Markov chain $\{S_k\}$ with two states: a good state and a bad state, denoted by G (or state 0) and B (or state 1). In a fixed state, the channel behaves like a binary symmetric channel (BSC). The GEC is thus a time-varying BSC, where p_G and p_B are the crossover probabilities in the good and bad states, respectively (the Gilbert channel (GC) [12] is obtained when $p_G = 0$; i.e., it behaves like a noiseless BSC in the good state). After every channel transmission, the chain makes a state transition according to the transition probability matrix

$$\boldsymbol{P} \triangleq \begin{bmatrix} Pr\{S_k = 0 | S_{k-1} = 0\} & Pr\{S_k = 1 | S_{k-1} = 0\} \\ Pr\{S_k = 0 | S_{k-1} = 1\} & Pr\{S_k = 1 | S_{k-1} = 1\} \end{bmatrix} = \begin{bmatrix} 1-b & b \\ g & 1-g \end{bmatrix},$$

where $0 < b < 1$ and $0 < g < 1$. A useful approach for calculating the probability of an error or noise sequence for the GEC is discussed in [6]. The probability of a noise sequence of length n, $z^n = (z_1, z_2, \cdots, z_n)$, can be expressed as

$$\mathrm{P}_{\text{GEC}}(z^n) \triangleq \mathrm{P}_{\text{GEC}}\{Z^n = z^n\} = \boldsymbol{\pi}^T \left(\prod_{k=1}^{n} \boldsymbol{P}(z_k) \right) \mathbf{1}, \tag{1}$$

where $\cdot^T$ denotes transposition, $\mathbf{P}(z_k)$ is a 2×2 matrix whose (i, j)th entry is the probability that the output symbol is z_k when the chain makes a transition from state $S_{k-1} = i$ to $S_k = j$, i.e.,

$$\mathbf{P}(0) = \begin{bmatrix} (1-b)(1-p_G) & b(1-p_B) \\ g(1-p_G) & (1-g)(1-p_B) \end{bmatrix}, \quad \mathbf{P}(1) = \begin{bmatrix} (1-b)p_G & bp_B \\ gp_G & (1-g)p_B \end{bmatrix}, \tag{2}$$

1 is the 2-dimensional vector with all ones and the vector $\boldsymbol{\pi}$ indicates the stationary distribution vector of the underlying Markov chain

$$\boldsymbol{\pi} = \begin{bmatrix} \pi_0 \triangleq Pr\{S_k = 0\} \\ \pi_1 \triangleq Pr\{S_k = 1\} \end{bmatrix} = \begin{bmatrix} \frac{g}{b+g} \\ \frac{b}{b+g} \end{bmatrix}. \tag{3}$$

B. *Queue-Based Channel with Memory*

The additive noise process of the queue-based binary channel with memory [43], [44], [45] is generated according to a sampling mechanism involving the following two parcels.

- **Parcel 1** is a queue of length M that contains initially M balls, either red or black.
- **Parcel 2** is an urn that contains a very large number of balls where the proportion of black balls is $1-p$ and the proportion of red balls is p, where $p \in (0,1)$, $p \ll 1/2$.

We assume that the probability of selecting parcel 1 (the queue) is ε, while the probability of selecting parcel 2 (the urn) is $1-\varepsilon$ and $\varepsilon \in [0,1)$. Notice that the channel is actually a BSC with crossover probability p when $\varepsilon = 0$; in this case we experiment on the urn only.

The noise process $\{Z_n\}_{n=1}^{\infty}$ is generated according to the following procedure. By flipping a biased coin (with $\Pr$(Head)=ε), we select one of the two parcels (select the queue if Heads and the urn if Tails). If parcel 2 (the urn) is selected, a pointer randomly points at a ball, and identifies its color. If parcel 1 (the queue) is selected, the procedure is determined by the length of the queue. If $M \geq 2$, a pointer points at the ball in cell k with probability $1/(M-1+\alpha)$, for $k = 1, 2, \cdots, M-1$ and $\alpha \geq 0$, and points at the ball in cell M with probability $\alpha/(M-1+\alpha)$, and identifies its color. If $M = 1$, a pointer points at the ball in the only cell of the queue with probability 1; in this case we set $\alpha = 1$. If the selected ball from either parcel is red (respectively black), we introduce a red (respectively black) ball in cell 1 of the queue, pushing the last ball in cell M out. The noise process $\{Z_n\}_{n=1}^{\infty}$ is then modeled as follows:

$$Z_n = \begin{cases} 1, & \text{if the } n\text{th experiment points at a red ball,} \\ 0, & \text{if the } n\text{th experiment points at a black ball.} \end{cases}$$

It can be shown that the resulting channel noise process $\{Z_n\}_{n=1}^{\infty}$ is a stationary ergodic (irreducible) Mth order Markov process. Moreover, various statistical and information theoretic quantities of the QBC, such as the channel block transition probability, capacity and ACF, can be determined (in closed-form) in terms of M, p, ε, and α [43], [44], [45]. The expressions for these quantities are herein summarized.

Block Transition Probability: For a given input block $X^n = (X_1, \cdots, X_n)$ and a given output block $Y^n = (Y_1, \cdots, Y_n)$, where n is the blocklength, the channel block transition probability is

$$\mathrm{P}_{\text{QBC}}^{(M)}\{Y^n = y^n | X^n = x^n\} = \mathrm{P}_{\text{QBC}}^{(M)}\{Z^n = z^n\} \stackrel{\triangle}{=} \mathrm{P}_{\text{QBC}}^{(M)}(z^n),$$

where $z_i = x_i \oplus y_i$, for $i = 1, 2, \cdots, n$, and the noise n-fold distribution is as follows.

- For blocklength $n \leq M$,

$$\mathrm{P}_{\text{QBC}}^{(M)}(z^n) = \frac{\prod_{j=0}^{n-d_1^n-1}\left[j\frac{\varepsilon}{M-1+\alpha} + (1-\varepsilon)(1-p)\right]\prod_{j=0}^{d_1^n-1}\left[j\frac{\varepsilon}{M-1+\alpha} + (1-\varepsilon)p\right]}{\prod_{j=M-n}^{M-1}\left[1-(\alpha+j)\frac{\varepsilon}{M-1+\alpha}\right]}, \tag{4}$$

  where $d_a^b = z_b + z_{b-1} + \cdots + z_a$ ($d_a^b = 0$ if $a > b$), and $\prod_{j=0}^{a}(\cdot) \triangleq 1$ if $a < 0$.

- For blocklength $n \geq M+1$,

$$\begin{aligned}\mathrm{P}_{\text{QBC}}^{(M)}(z^n) &= L^{(M)}\prod_{i=M+1}^{n}\left[\left(d_{i-M+1}^{i-1} + \alpha z_{i-M}\right)\frac{\varepsilon}{M-1+\alpha} + (1-\varepsilon)p\right]^{z_i} \\ &\quad \left\{\left[\left(M-1-d_{i-M+1}^{i-1}\right) + \alpha(1-z_{i-M})\right]\frac{\varepsilon}{M-1+\alpha} + (1-\varepsilon)(1-p)\right\}^{1-z_i},\end{aligned} \tag{5}$$

  where

$$L^{(M)} = \frac{\prod_{j=0}^{M-1-d_1^M}\left[j\frac{\varepsilon}{M-1+\alpha} + (1-\varepsilon)(1-p)\right]\prod_{j=0}^{d_1^M-1}\left[j\frac{\varepsilon}{M-1+\alpha} + (1-\varepsilon)p\right]}{\prod_{j=0}^{M-1}\left[1-(\alpha+j)\frac{\varepsilon}{M-1+\alpha}\right]}.$$

Note that the channel's bit error rate (BER) and noise correlation coefficient are given by

$$\text{BER}_{\text{QBC}} = \mathrm{P}_{\text{QBC}}^{(M)}\{Z_i = 1\} = \mathrm{P}_{\text{QBC}}^{(M)}\{Z_1 = 1\} = p, \tag{6}$$

and

$$\text{Cor}_{\text{QBC}} = \frac{E[Z_2 Z_1] - E[Z_2]E[Z_1]}{\text{Var}[Z_1]} = \frac{\frac{\varepsilon}{M-1+\alpha}}{1 - \frac{M-2+\alpha}{M-1+\alpha}\varepsilon}, \tag{7}$$

respectively, where $E[\cdot]$ denotes expectation, and $\text{Var}[\cdot]$ is the variance.

Autocorrelation Function (ACF): The ACF of a binary stationary process $\{Z_n\}_{n=1}^{\infty}$ is defined by $R[m] = E[Z_i Z_{i+m}] = \mathrm{P}_{\text{QBC}}^{(M)}\{Z_i = 1, Z_{i+m} = 1\}$. It can be shown that the ACF of the QBC satisfies the following.

$$R_{\text{QBC}}[m] = \begin{cases} p & \text{if } m = 0; \\ \frac{\frac{\varepsilon}{M-1+\alpha} + (1-\varepsilon)p}{1 - \frac{M-2+\alpha}{M-1+\alpha}\varepsilon}p & \text{if } 1 \leq m \leq M-1; \\ (1-\varepsilon)p^2 + \frac{\varepsilon}{M-1+\alpha}\left(\sum_{i=m-M+1}^{m-1} R_{\text{QBC}}[i] + \alpha R_{\text{QBC}}[m-M]\right) & \text{if } m \geq M. \end{cases} \tag{8}$$

Capacity: Since the QBC is a channel with stationary ergodic additive noise, it is information stable, and its (operational) capacity, $C_{\text{QBC}}^{(M)}$, is given by the familiar mutual information rate formula (e.g., [48]):

$$C_{\text{QBC}}^{(M)} = \lim_{n\to\infty}\sup_{X^n}\frac{1}{n}I(X^n; Y^n), \tag{9}$$

where $I(\cdot;\cdot)$ denotes mutual information [49]. It can be shown that input n-tuples X^n that are uniformly distributed over $\{0,1\}^n$ maximize $I(X^n;Y^n)$; this yields the following expression for $C_{\text{QBC}}^{(M)}$.

$$\begin{aligned} C_{\text{QBC}}^{(M)} &= 1-\sum_{\omega=0}^{M-1}\binom{M-1}{\omega}L_{\omega}^{(M)}h_b\Big[\omega\frac{\varepsilon}{M-1+\alpha}+(1-\varepsilon)p\Big] \\ &\quad -\sum_{\omega=1}^{M}\binom{M-1}{\omega-1}L_{\omega}^{(M)}h_b\Big[(\omega+\alpha-1)\frac{\varepsilon}{M-1+\alpha}+(1-\varepsilon)p\Big], \end{aligned} \tag{10}$$

where

$$L_{\omega}^{(M)}=\frac{\prod_{j=0}^{M-1-\omega}\left[j\frac{\varepsilon}{M-1+\alpha}+(1-\varepsilon)(1-p)\right]\prod_{j=0}^{\omega-1}\left[j\frac{\varepsilon}{M-1+\alpha}+(1-\varepsilon)p\right]}{\prod_{j=0}^{M-1}\left[1-(\alpha+j)\frac{\varepsilon}{M-1+\alpha}\right]},$$

$h_b(x)\stackrel{\triangle}{=}-x\log_2 x-(1-x)\log_2(1-x)$ is the binary entropy function and $\prod_{j=0}^{a}(\cdot)\stackrel{\triangle}{=}1$ for $a<0$. Finally, it should be noted that the FMMC channel of [32] is a special case of the QBC: by setting $\alpha=1$, the QBC reduces to the FMMC (for identical BER, Cor and memory). Hence, the QBC is a more flexible channel model than the FMMC due to an additional degree of freedom.

III. Fitting Rician Fading Channels via the QBC

We next consider the problem of fitting discretized Rayleigh and Rician fading channels via the QBC model. For the sake of comparison, we also model the fading channels via the GEC (which has the same number of parameters as the QBC) according to the parameterization method of Pimentel *et. al.* in [46]. The accuracy of both methods is evaluated in terms of ACF and capacity.

A. Fading Channel Model

We consider a discrete (binary-input, binary-output) communication system, referred to as the discrete channel with Clarke's autocorrelation (DCCA) model, that employs binary frequency-shift keying (FSK) modulation, a time-correlated Rician flat-fading channel, and a hard quantized noncoherent demodulation [46]. As in [6], [46], [50], we consider non-coherent FSK modulation; however any other modulation/demodulation scheme, for which the expression of the probability of length-n error sequences is available (as in (11) below), can also be studied by our model. The complex envelope of the received signal at the input to the demodulator is corrupted by a multiplicative Rician fading and by an additive white Gaussian noise, i.e.,

$$\tilde{R}(t)=\sqrt{2E_s}\tilde{G}(t)\tilde{S}(t)+\tilde{N}(t),$$

where E_s is the symbol energy. $\tilde{S}(t)$ is the complex envelope of the symbol which can be expressed as $\tilde{S}(t) = \sum_{k=1}^{\infty} p_{a_k}(t-kT)$, where the binary information bearing symbols a_k are embedded in the signals $p_i(t)$, $i=0,1$, which are equally probable orthogonal signals with unit energy. T is the symbol interval and $\tilde{N}(t)$ is the complex envelope of the white Gaussian noise with autocorrelation function given by $\frac{1}{2}E[\tilde{N}(t+\tau)\tilde{N}^*(t)] = N_0\delta(\tau)$, where N_0 is the variance of $\tilde{N}(t)$ [7]. The complex envelope of the fading process $\tilde{G}(t) = \tilde{G}_I(t) + j\tilde{G}_Q(t)$ is a complex, wide sense stationary, Gaussian process with mean η, $j=\sqrt{-1}$, and the quadrature components $\tilde{G}_I(t)$ and $\tilde{G}_Q(t)$ are mutually independent Gaussian processes with the same covariance function Cov(τ) which, adopting Clarke's fading model [51], [52], can be expressed as

$$\text{Cov}(\tau) = \frac{1}{2}E\left[(\tilde{G}(t+\tau)-\eta)(\tilde{G}^*(t)-\eta)\right] = \sigma_g^2 J_0(2\pi f_D \tau),$$

where

$$J_0(x) = \sum_{k=0}^{\infty}(-1)^k(\frac{x^k}{2^k k!})^2$$

is the zero-order Bessel function of the first kind, f_D is the maximum Doppler frequency experienced by the moving vehicle, and σ_g^2 is the variance of $\tilde{G}(t)$. At each signaling interval of length T, the demodulator forms two decision variables $\{0,1\}$ and decides which signal was more likely to have been transmitted. A general block diagram for visualizing the behavior of such systems is given in Fig. 1.

The combination of digital modulator, fading channel, and digital demodulator yields the equivalent DCCA model. The study and analysis of the statistical behavior of the DCCA model is important since the design and construction of effective error control schemes for this simplified (binary-input, binary-output) model helps us better exploit the system memory and achieve reliable communication over the underlying correlated fading channel.

The DCCA is represented as an additive noise channel with binary error process $\{Z_n\}_{n=1}^{\infty}$, where

$$Z_n = \begin{cases} 0 & \text{if the } n\text{th transmitted symbol is correctly received,} \\ 1 & \text{if the } n\text{th transmitted symbol is incorrectly received.} \end{cases}$$

The probability of an error sequence of length n, $z^n = (z_1, z_2, \cdots, z_n)$, can be obtained directly from [46, Eq. (3) with $\Omega = 1$]:

$$\begin{aligned} \text{P}_{\text{DCCA}}(z^n) &\triangleq \Pr\{Z^n = z^n\} \\ &= \sum_{l_1=z_1}^{1}\cdots\sum_{l_n=z_n}^{1}\left(\prod_{k=1}^{n}\frac{(-1)^{l_k+z_k}}{l_k+1}\right) \times \frac{\exp\{-\frac{E_s}{N_0}K_R\mathbf{1}^T\boldsymbol{F}\big((K_R+1)\boldsymbol{I}+\frac{E_s}{N_0}\bar{\boldsymbol{C}}\boldsymbol{F}\big)^{-1}\mathbf{1}\}}{\det(\boldsymbol{I}+\frac{E_s}{N_0}(1+K_R)^{-1}\bar{\boldsymbol{C}}\boldsymbol{F})}, \end{aligned} \quad (11)$$

where $\boldsymbol{F}$ is a diagonal matrix defined as

$$\boldsymbol{F} = \text{diag}\left(\frac{l_1}{l_1+1}, \cdots, \frac{l_n}{l_n+1}\right),$$

$\boldsymbol{I}$ is the identity matrix, $K_R = \eta/2\sigma_g^2$ is the Rician factor, and $\bar{\boldsymbol{C}}$ is the normalized covariance matrix with entries

$$\bar{C}_{ij} = (1/\sigma_g^2)\text{Cov}(|i-j|) = J_0(2\pi f_D T|i-j|), \qquad 1 \le i, j \le n.$$

The QBC is next used to model the equivalent binary error sequence of the DCCA, which represents the successes and failures that result from the transmission of symbols over the above fading channel.

B. Estimation of Channel Parameters

1) QBC Parameter Estimation: For a given DCCA, we construct a QBC whose noise or error process is statistically "close" in the Kullback-Leibler sense to the noise process generated by the DCCA. The Kullback-Leibler distance or divergence is an approximation quality measure widely used to determine the statistical closeness between two sources (e.g., see [49], [6], [11]). Specifically, given a DCCA with fixed average signal-to-noise ratio (SNR) E_s/N_0, normalized Doppler frequency $f_D T$ and Rician factor K_R resulting in bit error rate BER_{DCCA} and correlation coefficient Cor_{DCCA}, we estimate the QBC parameters M, p, ε, and α that minimize the Kullback-Leibler divergence rate (KLDR),

$$\lim_{n\to\infty} \frac{1}{n} D_n(\text{P}_{\text{DCCA}} \parallel \text{P}_{\text{QBC}}^{(M)}),$$

subject to the constraints

$$\text{BER}_{\text{QBC}} = \text{BER}_{\text{DCCA}} \ \text{ and } \ \text{Cor}_{\text{QBC}} = \text{Cor}_{\text{DCCA}},$$

where $D_n(\text{P}_{\text{DCCA}} \parallel \text{P}_{\text{QBC}}^{(M)})$ is the Kullback-Leibler divergence between the n-fold DCCA and QBC noise distributions, P_{DCCA} and $\text{P}_{\text{QBC}}^{(M)}$, respectively:

$$D_n(\text{P}_{\text{DCCA}} \parallel \text{P}_{\text{QBC}}^{(M)}) = \sum_{z^n \in \{0,1\}^n} \text{P}_{\text{DCCA}}(z^n) \log_2 \frac{\text{P}_{\text{DCCA}}(z^n)}{\text{P}_{\text{QBC}}^{(M)}(z^n)},$$

where $\text{P}_{\text{QBC}}^{(M)}$ is given in closed form by (4) and (5) and P_{DCCA} is given by (11). Note that we focus on minimizing the KLDR, which is an asymptotic quantity (as opposed to minimizing the normalized divergence $(1/n)D_n(\text{P}_{\text{DCCA}} \parallel \text{P}_{\text{QBC}}^{(M)})$ for finite n), since it is vital to have identical statistical behavior on both channels for large blocklengths (as large blocklengths are required to achieve reliable communication by the channel coding theorem [49]).

It can be shown (e.g., see [53]) that the KLDR between the DCCA noise process (which is stationary) and the QBC noise process (which is Markovian) does exist and can be expressed as

$$\lim_{n\to\infty}\frac{1}{n}D_n(\mathrm{P}_{\text{DCCA}} \parallel \mathrm{P}_{\text{QBC}}^{(M)}) = -\mathcal{H}_{\text{DCCA}}(Z) - E_{\mathrm{P}_{\text{DCCA}}}[\log_2 \mathrm{P}_{\text{QBC}}^{(M)}(Z_{M+1}|Z^M)], \tag{12}$$

where $\mathcal{H}(Z)\stackrel{\triangle}{=}\lim_{n\to\infty}(1/n)H(Z^n)$ denotes the entropy rate [49],

$$E_{\mathrm{P}_{\text{DCCA}}}[\log_2 \mathrm{P}_{\text{QBC}}^{(M)}(Z_{M+1}|Z^M)]\stackrel{\triangle}{=}\sum_{z^{M+1}} \mathrm{P}_{\text{DCCA}}(z^{M+1})[\log_2 \mathrm{P}_{\text{QBC}}^{(M)}(z_{M+1}|z^M)],$$

and $\mathrm{P}_{\text{QBC}}^{(M)}(z_{M+1}|z^M)$ is the QBC conditional error probability of symbol $M+1$ given the previous M symbols. Then the above minimization reduces to maximizing the second term in (12) (which is independent of n) over the QBC parameters. Note that in our approximation, we match BER and Cor of both channels to guarantee identical noise marginal distributions and identical probabilities of two consecutive errors (ones). Hence, given these constraints, the above optimization problem reduces to an optimization over only two QBC parameters: M and ε. This is achieved numerically by sequentially incrementing $M \geq 1$ and varying $0.0001 \leq \varepsilon \leq 0.9999$ for each given M.

2) GEC Parameter Estimation: We next briefly describe the method of modeling the DCCA via the GEC introduced by Pimentel *et. al.* in [46]. For a given DCCA, the parameterization of the GEC is based on the following lemma.

Lemma 1: **[46]** The probability of any observed sequence z^n generated by the GEC satisfies the following recurrence equation:

$$\mathrm{P}_{\text{GEC}}(z^n\varsigma\kappa) = c(\varsigma,\kappa)\mathrm{P}_{\text{GEC}}(z^n\varsigma) + d(\varsigma,\kappa)\mathrm{P}_{\text{GEC}}(z^n), \tag{13}$$

where $\mathrm{P}_{\text{GEC}}(z^n\varsigma\kappa)\stackrel{\triangle}{=}\Pr\{Z_1 = z_1,\cdots,Z_n = z_n, Z_{n+1} = \varsigma, Z_{n+2} = \kappa\}$, ς and κ are binary symbols,

$$c(0,0) = (1-p_G)(1-b) + (1-p_B)(1-g),\;\; c(1,1) = p_G(1-b) + p_B(1-g), \tag{14}$$

$$d(0,0) = -(1-g-b)(1-p_G)(1-p_B),\;\; d(1,1) = -(1-g-b)p_G)p_B, \tag{15}$$

$c(1,0) = 1 - c(1,1)$, $c(0,1) = 1 - c(0,0)$, $d(0,1) = -d(0,0)$, and $d(1,0) = -d(1,1)$.

Lemma 1 shows that the parameters $c(\varsigma,\kappa)$ and $d(\varsigma,\kappa)$ can be calculated via a linear system of equations. For example, setting $z^n = \phi$, where ϕ is an empty sequence, $\mathrm{P}_{\text{GEC}}(\phi) = 1$, and $z^n = \varsigma$ in (13), $c(\varsigma,\kappa)$ and $d(\varsigma,\kappa)$ can be determined by the probabilities of error sequences of length at most 3:

$$c(\varsigma,\kappa) = \frac{\mathrm{P}_{\text{GEC}}(\varsigma\varsigma\kappa) - \mathrm{P}_{\text{GEC}}(\varsigma\kappa)\mathrm{P}_{\text{GEC}}(\varsigma)}{\mathrm{P}_{\text{GEC}}(\varsigma\varsigma) - \mathrm{P}_{\text{GEC}}^2(\varsigma)}, \tag{16}$$

and

$$d(\varsigma,\kappa)=\frac{\mathrm{P}_{\text{GEC}}(\varsigma\kappa)\mathrm{P}_{\text{GEC}}(\varsigma\varsigma)-\mathrm{P}_{\text{GEC}}(\varsigma\varsigma\kappa)\mathrm{P}_{\text{GEC}}(\varsigma)}{\mathrm{P}_{\text{GEC}}(\varsigma\varsigma)-\mathrm{P}^2_{\text{GEC}}(\varsigma)}. \tag{17}$$

The GEC parameters follow by solving the nonlinear equations in (14)-(15) as follows.

Proposition 1: **[46]** If $\mathrm{P}_{\text{GEC}}(01)\neq\mathrm{P}_{\text{GEC}}(0)\mathrm{P}_{\text{GEC}}(1)$, the parameters of the GEC are uniquely determined by the four probabilities $\mathrm{P}_{\text{GEC}}(0)$, $\mathrm{P}_{\text{GEC}}(00)$, $\mathrm{P}_{\text{GEC}}(000)$ and $\mathrm{P}_{\text{GEC}}(111)$. The four parameters b, g, p_G, and p_B are given by the following: p_G and p_B are the roots of the quadratic equation

$$[-1+c(1,1)+c(0,0)]x^2+[1-c(1,1)-c(0,0)+d(1,1)-d(0,0)]x-d(1,1)=0,$$

and

$$b=\frac{c(0,0)p_B-c(1,1)(1-p_B)+(p_G-p_B)}{p_G-p_B},$$

$$g=\frac{c(0,0)p_G-c(1,1)(1-p_G)+(p_B-p_G)}{p_B-p_G},$$

Hence, if $\mathrm{P}_{\text{DCCA}}(0)$, $\mathrm{P}_{\text{DCCA}}(00)$, $\mathrm{P}_{\text{DCCA}}(000)$ and $\mathrm{P}_{\text{DCCA}}(111)$ are known, where $\mathrm{P}_{\text{DCCA}}(z^n)$ is the probability of error sequences generated by the DCCA (see (11)), the parameters of the GEC can be obtained by (16), (17) and Proposition 1 by setting $\mathrm{P}_{\text{GEC}}(z^n)=\mathrm{P}_{\text{DCCA}}(z^n)$, $n=1,2,3$.

IV. Modeling Results and Discussions

We evaluate how well the QBC model fits or approximates the DCCA according to two criteria: ACF and channel capacity. The QBC ACF and capacity expressions are provided in Section II-B. The ACF of the DCCA can be obtained directly from (11):

$$R_{\text{DCCA}}[m]=\frac{(1+K_R)^2}{\left(2+2K_R+\frac{E_s}{N_0}\right)^2-\left(\frac{E_s}{N_0}\rho(m)\right)^2}\times\exp\left\{-\frac{2K_R\frac{E_s}{N_0}}{2+2K_R+\frac{E_S}{N_0}\left(\rho(m)+1\right)}\right\},$$

where $\rho(m)=J_0(2\pi m f_D T)$.

As in (9), the capacity of the DCCA is given by

$$C_{\text{DCCA}}=\lim_{n\to\infty}\sup_{X^n}\frac{1}{n}I(X^n;Y^n)=1-\mathcal{H}_{\text{DCCA}}(Z).$$

The entropy rate $\mathcal{H}_{\text{DCCA}}(Z)$ of the (stationary ergodic) DCCA error process is not known in closed form. However, we can approximate it by calculating the normalized block noise entropy $(1/n)H(Z^n)$ for large values of n and thus obtain a lower bound on C_{DCCA}, given by:

$$C_{\text{DCCA}}\geq C_{\text{DCCA, n}}\stackrel{\triangle}{=}1-\frac{1}{n}H_{\text{DCCA}}(Z^n).$$

In our calculations, we used values of n as large as 21.

For the sake of comparison, we also present modeling results via the GEC using the method of Pimentel *et. al.* in [46] (which we briefly described in Section III-B.2). Note that in [46], the authors also employ arbitrary Kth order Markov noise models to approximate the fading channels. However, unlike our QBC model which has only four parameters (as the GEC) and allows large values for its memory order M (since its noise is a specially structured Mth order Markov process generated by our queue scheme), the Kth order Markov models of [46] are unstructured and hence suffer from the limitation of having a number of parameters that grows exponentially[3] with K. Therefore, with the exception of a brief comparison with the Markov model of [46] (see Fig. 2), we herein mainly compare our QBC-based modeling method with the GEC-based modeling method of [46] since both channels have identical number of parameters, hence identical degrees of freedom and complexity.

The capacity of the GEC is obtained via the algorithm in [30]. The ACF of the GEC can also be obtained directly from (1):

$$R_{\text{GEC}}[m] = \boldsymbol{\pi}^T \boldsymbol{P}(1) \left(\prod_{k=1}^{m-1} \boldsymbol{P} \right) \boldsymbol{P}(1)\mathbf{1}, \tag{18}$$

where $\boldsymbol{\pi}$, $\boldsymbol{P}(1)$, and $\boldsymbol{P}$ are defined in Section II-A.

A wide range of DCCA channel parameters is investigated with SNR = 15 dB and 25 dB, $f_D T = 0.001$, 0.005, 0.01 and 0.1 for Rayleigh fading ($K_R = -\infty$ dB), and SNR = 15 dB and $f_D T = 0.001$, 0.005, 0.01 and 0.05 for Rician fading ($K_R = 5$ dB). The SNR, $f_D T$ and K_R values (except for $f_D T = 0.005$) were chosen to match the conditions of the correlated Rician and Rayleigh fading channels studied in [46, Fig. 6, Fig. 7, Fig. 9 and Fig. 11]. The QBC and GEC parameters, obtained as explained in Sections III-B.1 and III-B.2, respectively, are provided in Tables I-III.

A subset of the modeling results in terms of the ACF for the DCCA, its QBC approximation and its GEC approximation is shown in Figs. 2-5 (the complete results are available in [45]). We observe a strong ACF agreement between the QBC and the DCCA in these figures.[4] This behavior is indeed observed for all computations, especially for $f_D T = 0.1$, where the ACF curves of the DCCA and its QBC approximation are identical [45]. For slow and

[3]As a result, only models with memory order up to 6 are studied in [46]. Such models are shown to approximate well channels with fast and medium fading rates ($f_D T > 0.02$); but they are inadequate for slow fading rates. As we later show in this section, the QBC model can accommodate large values of the memory order; thus, it can provide a good approximation of channels with slow fading ($f_D T < 0.02$) in addition to medium and fast fading.

[4]Note that the ACF of the QBC $R_{\text{QBC}}[m]$ is equal to a constant for $m \leq M - 1$ as indicated by (8).

medium fading (e.g., see Fig. 2, and [45]), the ACF curve for the GEC takes a longer span of m before eventually converging, which indicates that the GEC (as fitted in [46]) might not be adequate for modeling very slow Rayleigh fading ($f_D T = 0.001$) and very slow to medium Rician fading ($f_D T = 0.001$, 0.005 and 0.01). We observe that the QBC has a better performance than the Markov models in [46] (see Fig. 2), but with significantly smaller complexity since it is fully described by four parameters and allows us closed-form expressions for various fading characteristics. Compared with [46, Fig. 7.(a)], the QBC has similar performance as the Markov models of [46] with order 4 or 5, but with smaller complexity.

Note that since the QBC noise is a homogeneous Markov process, the KLDR between the DCCA and the QBC error processes exists and admits a simple expression given by (12). Hence, it is practical to minimize this KLDR by maximizing the expected value in (12) over the QBC parameters which is independent of n (see Section III-B.1). However, this approach is not easily applicable to the GEC since the KLDR between the DCCA and the GEC noise processes does not admit a simple expression in general (as the GEC noise is hidden Markovian). The method of parameterization of the GEC of Section III-B.2 is simple, but it only takes into account error sequences no longer than 3, which implies that this method is not appropriate for approximating slow fading.

Our results show that the largest Markovian memory M for the QBC model that best fits the DCCA is 20, while the largest Markovian memory K for the (unstructured) Markov noise channel models considered in [46] is 6 (higher order unstructured models could not be obtained in [46] due to their prohibitive exponential complexity). This explains why the QBC is more suitable for fitting slow fading with large memory than the Markov noise model considered in [46].

Modeling results in terms of capacity are shown in Figs. 6-7, where the lower bound for the capacity of the DCCA and the capacities of the QBC approximation and the GEC approximation are shown for different SNR values and $f_D T$ values. We clearly observe from the figures that the capacity curves of the QBC and the lower bound curves for the capacity of the DCCA match quite well, and the capacities for $f_D T = 0.1$ (fast Rayleigh fading) are almost identical. The last observation can be explained by the fact that the DCCA has low memory at $f_D T = 0.1$ (fast fading); hence the lower bound for its capacity is tight (since $(1/n)H(Z^n) = H(Z_1)$ if Z^n is memoryless). Overall, we observe a strong match in capacity between the DCCA and its QBC approximation. In terms of capacity, the GEC has nearly as good a performance as the QBC in fitting the DCCA.

V. Summary

In this work, we approximate hard-decision demodulated correlated Rician fading channels (represented by the DCCA model) via the QBC model. Numerical results show a strong agreement between the ACF and capacity curves of the QBC and the DCCA. This leads us to conclude that the QBC provides a very good approximation of the DCCA under a variety of channel conditions. The QBC provides a much better performance in terms of ACF for fitting the DCCA than the GEC and the Markov models of [46] for the range of slow and very slow fading. An important feature of this queue-based channel model is that it is valuable for characterizing a wide class of communication channels with memory, while remaining mathematically simple and flexible.

One possible direction for future work is the modeling and analysis of wired/wireless Internet traffic and channel coding, as an extension and application of this work. Sanneck and Carle [54] used an Mth order Markov chain process to describe the dependencies between packet losses. However, their models have a complexity (number of parameters) that grows exponentially with M, rendering it impractical for the modeling of packet loss processes with large memory. The QBC model, on the other hand, does not suffer from this limitation as it is fully described by only four parameters and allows single-letter analysis. The QBC can hence be employed to characterize the packet-loss patterns introduced by the Internet, especially to capture loss burstiness and distances between loss bursts. Another topic of future interest is the design, construction and analysis of channel codes for the QBC. One important objective in this problem is the judicious design of the powerful channel codes in order to fully exploit the channel memory. Some results in this direction involving low density parity check (LDPC) codes are reported in [55].

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Model	$f_D T$			
	0.001	0.005	0.01	0.1
QBC	$M = 20$ $\varepsilon = 0.8593$ $p = 0.0297$ $\alpha = 0.8959$	$M = 11$ $\varepsilon = 0.7602$ $p = 0.0297$ $\alpha = 0.3828$	$M = 7$ $\varepsilon = 0.6556$ $p = 0.0297$ $\alpha = 0.3387$	$M = 2$ $\varepsilon = 0.0893$ $p = 0.0297$ $\alpha = 0.131$
GEC	$b = 0.0000339$ $g = 0.000479$ $p_B = 0.3393$ $p_G = 0.00783$	$b = 0.000841$ $g = 0.0118$ $p_B = 0.3393$ $p_G = 0.00766$	$b = 0.00329$ $g = 0.045$ $p_B = 0.3395$ $p_G = 0.00713$	$b = 0.0324$ $g = 0.7466$ $p_B = 0.5199$ $p_G = 0.00849$

TABLE I

QBC AND GEC MODELING PARAMETERS FOR $K_R = -\infty$ dB (RAYLEIGH) AND SNR = 15 dB.

Model	$f_D T$			
	0.001	0.005	0.01	0.1
QBC	$M = 18$ $\varepsilon = 0.8506$ $p = 0.00314$ $\alpha = 0.2607$	$M = 6$ $\varepsilon = 0.6226$ $p = 0.00314$ $\alpha = 0.2525$	$M = 4$ $\varepsilon = 0.4666$ $p = 0.00314$ $\alpha = 0.2019$	$M = 2$ $\varepsilon = 0.0145$ $p = 0.00314$ $\alpha = 0.1054$
GEC	$b = 0.0000333$ $g = 0.00466$ $p_B = 0.3339$ $p_G = 0.000783$	$b = 0.000773$ $g = 0.1014$ $p_B = 0.334$ $p_G = 0.000622$	$b = 0.0025$ $g = 0.2887$ $p_B = 0.3343$ $p_G = 0.000279$	$b = 0.00103$ $g = 0.8338$ $p_B = 0.4523$ $p_G = 0.00259$

TABLE II

QBC AND GEC MODELING PARAMETERS FOR $K_R = -\infty$ dB (RAYLEIGH) AND SNR = 25 dB.

Model	$f_D T$			
	0.001	0.005	0.01	0.05
QBC	$M = 18$ $\varepsilon = 0.8195$ $p = 0.00853$ $\alpha = 0.9619$	$M = 17$ $\varepsilon = 0.8054$ $p = 0.00853$ $\alpha = 0.3971$	$M = 11$ $\varepsilon = 0.7219$ $p = 0.00853$ $\alpha = 0.3299$	$M = 3$ $\varepsilon = 0.3426$ $p = 0.00853$ $\alpha = 0.3726$
GEC	$b = 0.00000259$ $g = 0.000139$ $p_B = 0.3112$ $p_G = 0.00289$	$b = 0.0000646$ $g = 0.00347$ $p_B = 0.3113$ $p_G = 0.00288$	$b = 0.000257$ $g = 0.0137$ $p_B = 0.3115$ $p_G = 0.00284$	$b = 0.00542$ $g = 0.2533$ $p_B = 0.3185$ $p_G = 0.0019$

TABLE III

QBC AND GEC MODELING PARAMETERS FOR $K_R = 5$ dB (RICIAN) AND SNR = 15 dB.

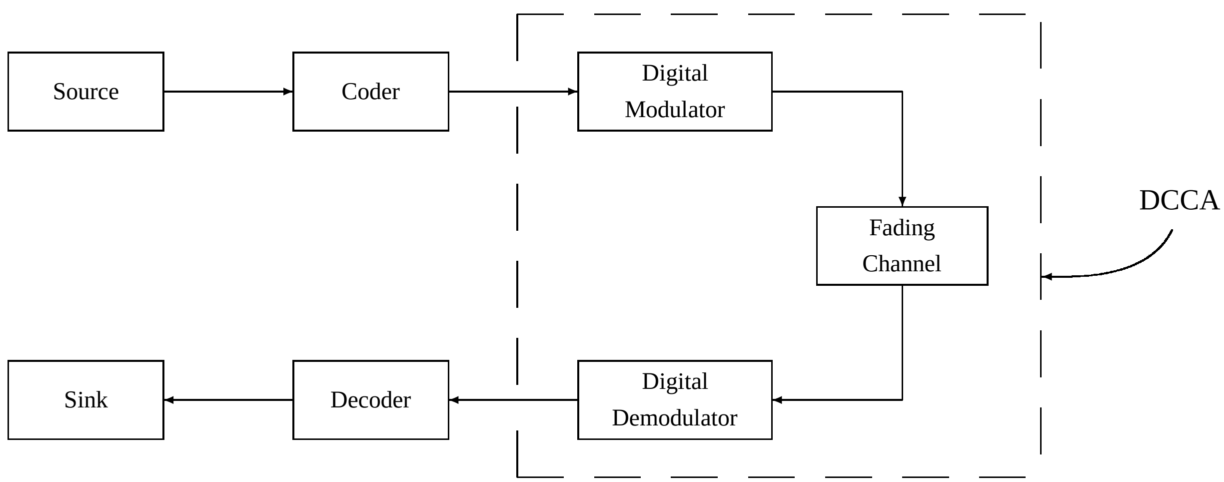

Fig. 1. Overall system and the equivalent DCCA model.

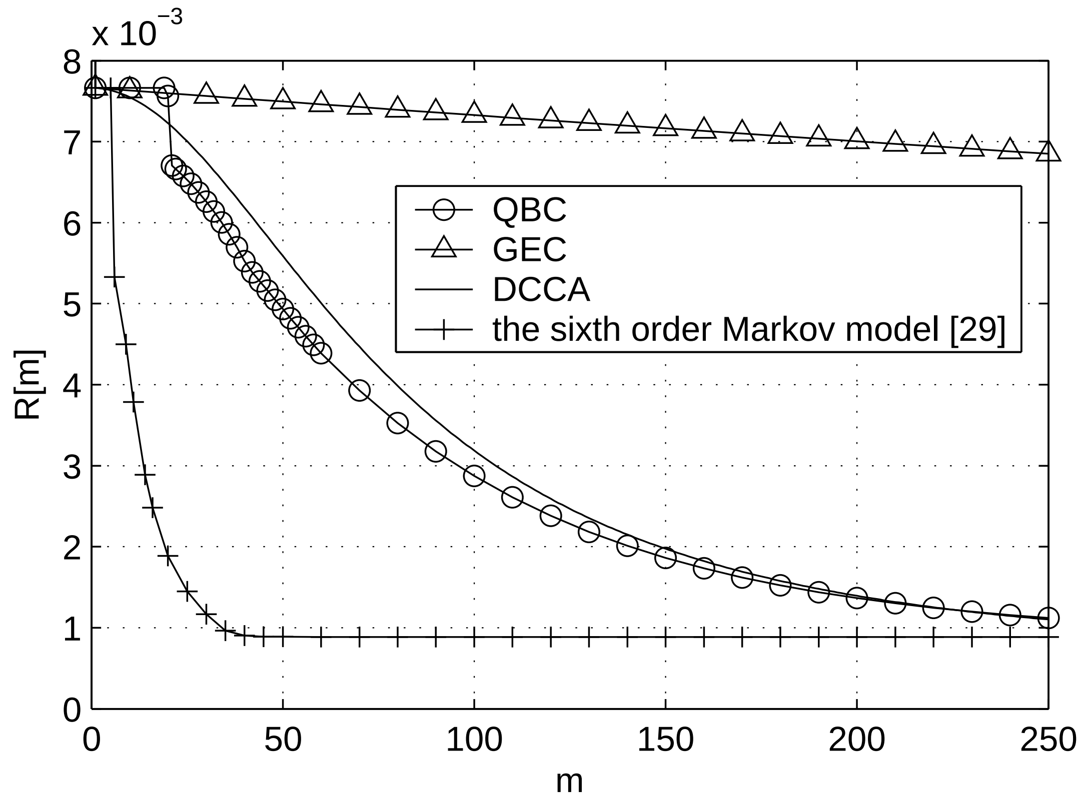

Fig. 2. DCCA fitting via the QBC: ACF vs m for $f_D T = 0.001$, SNR = 15 dB and $K_R = -\infty$ dB (Rayleigh).

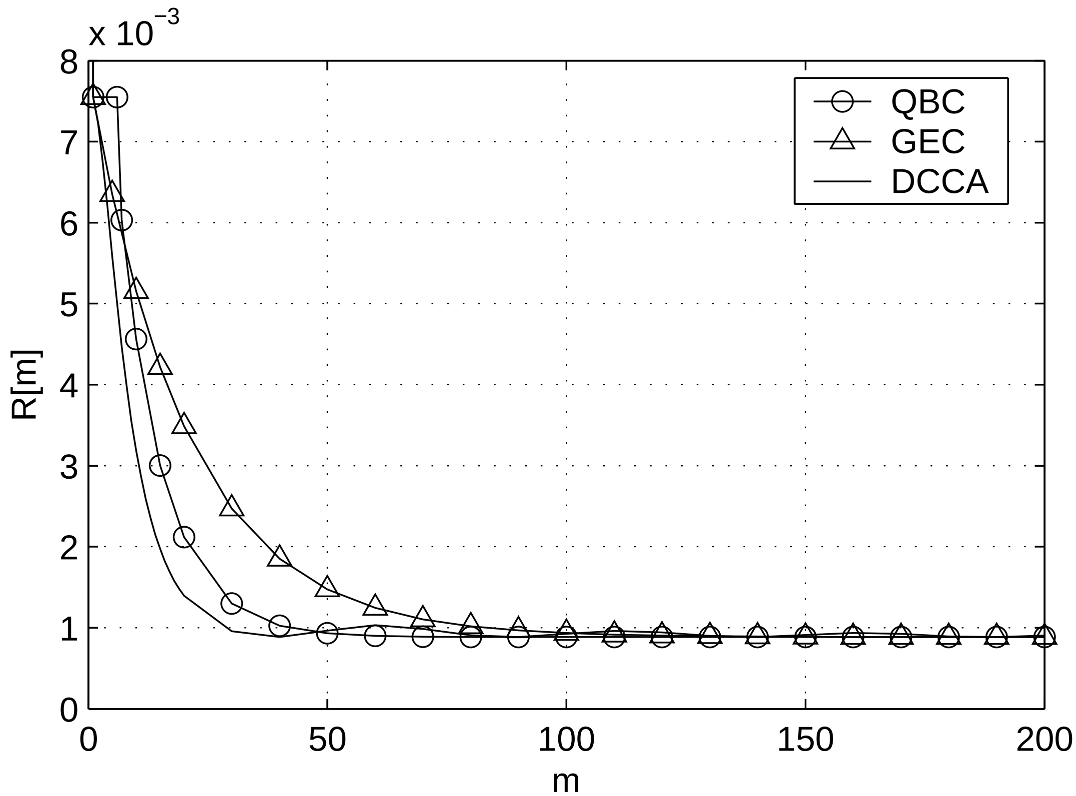

Fig. 3. DCCA fitting via the QBC: ACF vs m for $f_D T = 0.01$, SNR = 15 dB and $K_R = -\infty$ dB (Rayleigh).

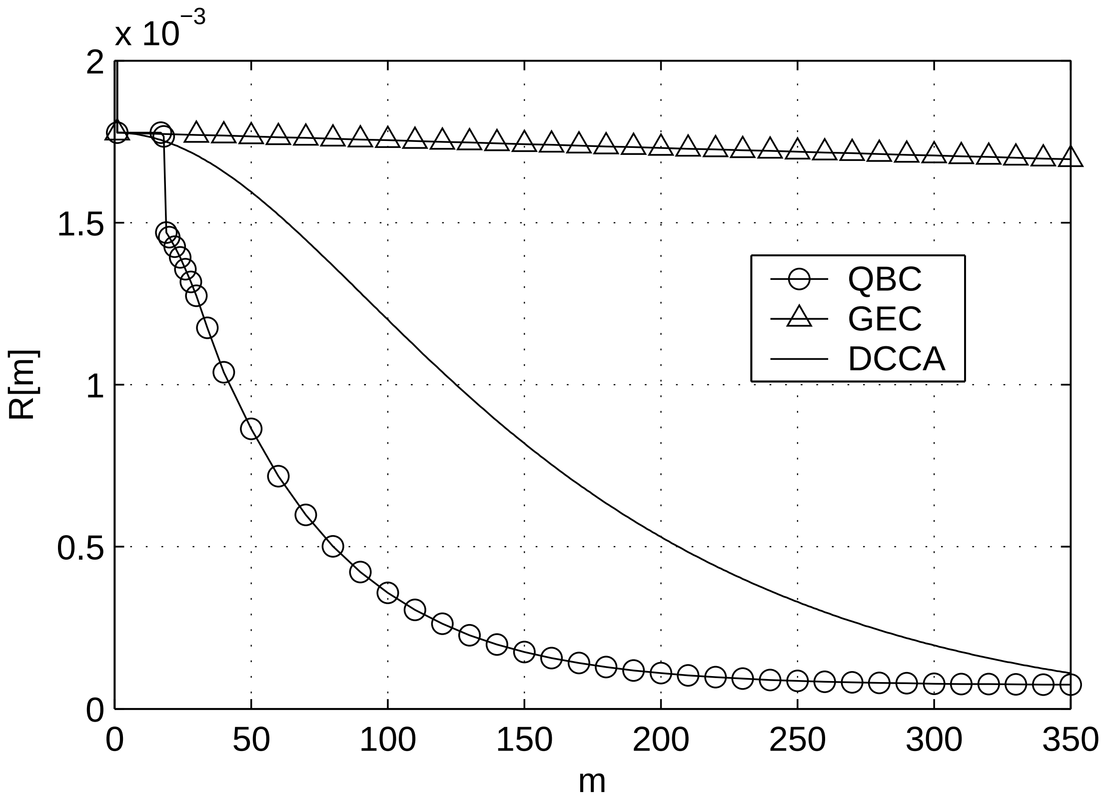

Fig. 4. DCCA fitting via the QBC: ACF vs m for $f_D T = 0.001$, SNR = 15 dB and $K_R = 5$ dB (Rician).

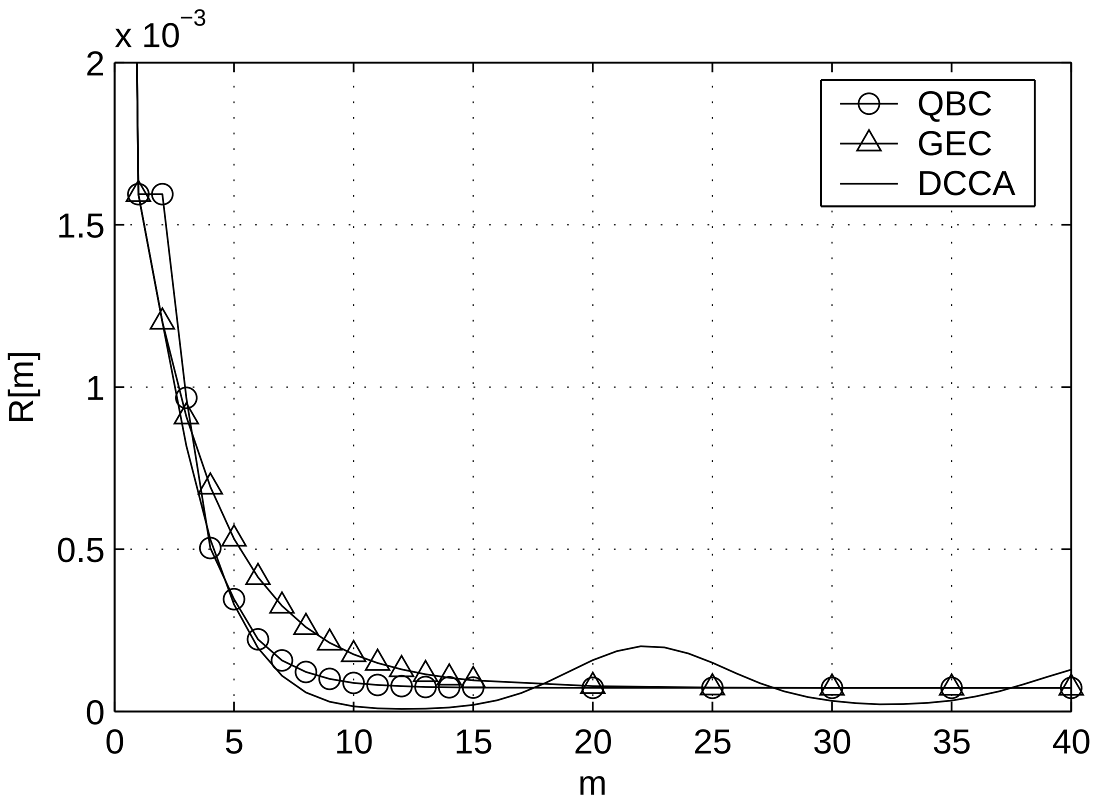

Fig. 5. DCCA fitting via the QBC: ACF vs m for $f_D T = 0.05$, SNR = 15 dB and $K_R = 5$ dB (Rician).

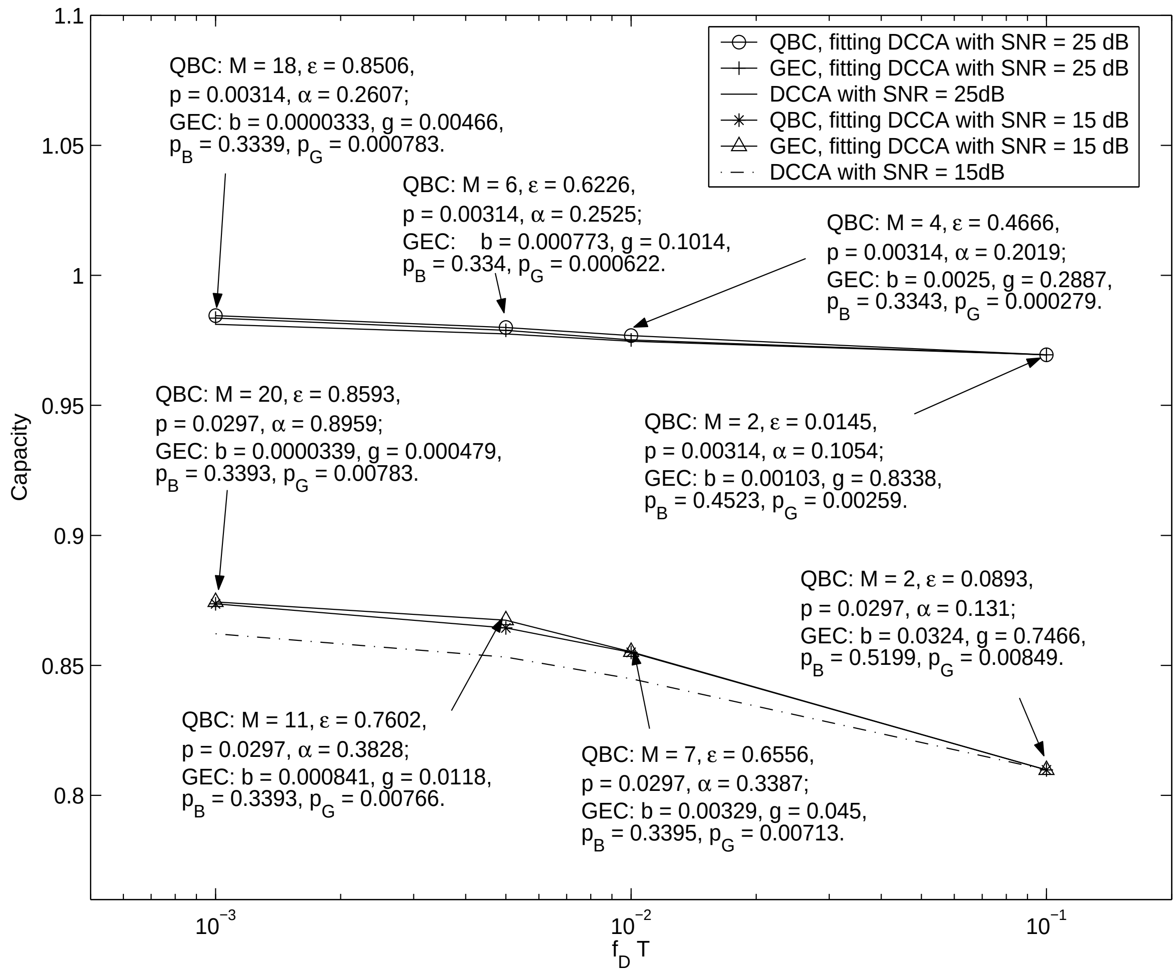

Fig. 6. DCCA fitting via the QBC: Capacity (in bits/channel use) vs normalized Doppler frequency $f_D T$ for Rayleigh fading.

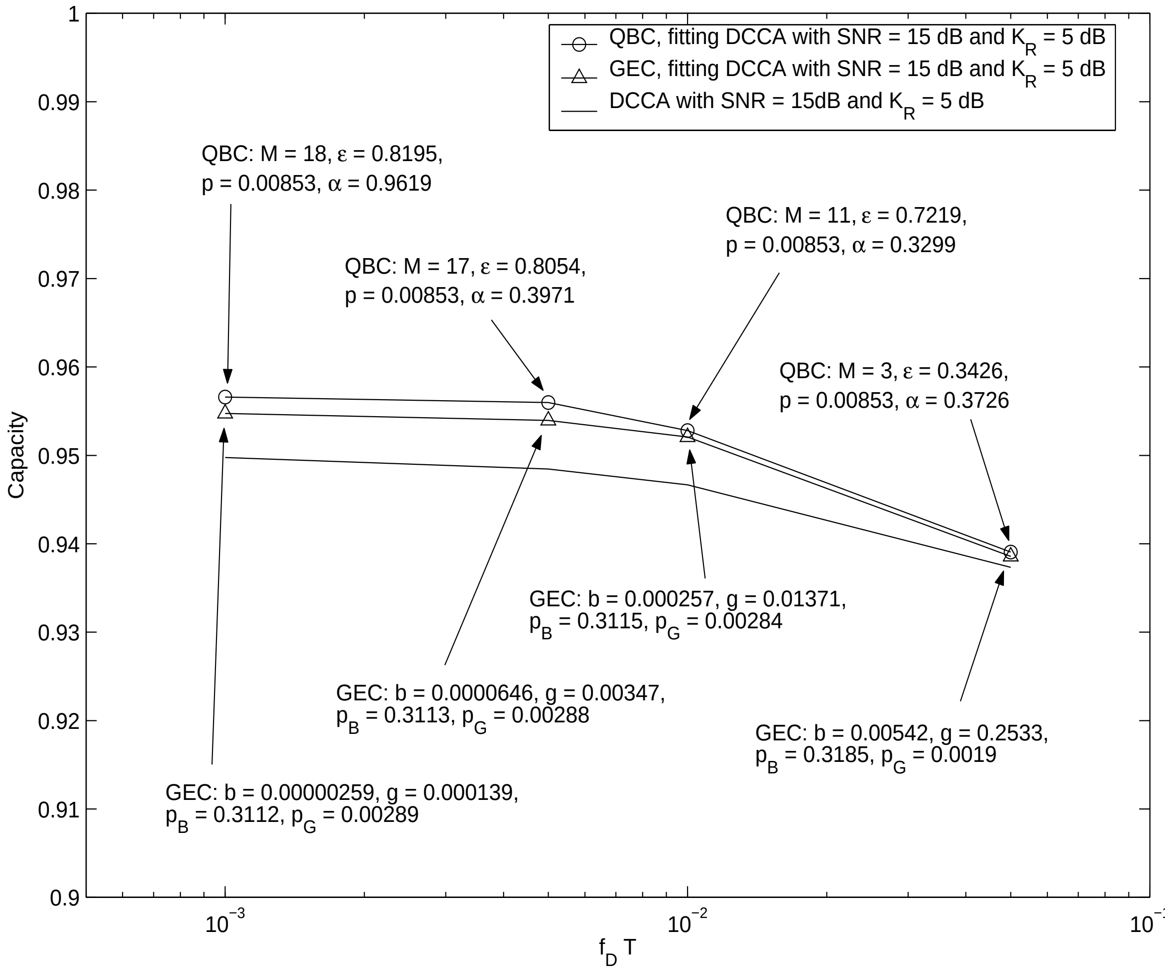

Fig. 7. DCCA fitting via the QBC: Capacity (in bits/channel use) vs normalized Doppler frequency $f_D T$ for Rician fading.