# Problem Set #13

Due: Thursday, 12 January 2012

**1.** Suppose that $h$ is a continuous function, $f$ and $g$ are differentiable functions, and

$$F(x) := \int_{f(x)}^{g(x)} h(t)\,dt\,.$$

Prove that $F'(x) = h\big(g(x)\big) \cdot g'(x) - h\big(f(x)\big) \cdot f'(x)$.

**2.** A function $f$ is ***periodic*** with ***period*** $a$, if $f(x) = f(x+a)$ for all $x$.

**(a)** If $f$ is continuous and periodic with period $a$, then show that

$$\int_0^a f(t)\,dt = \int_b^{b+a} f(t)\,dt \qquad \text{for all } b \in \mathbb{R}.$$

**(b)** Find a function $g$ such that $g$ is not periodic, but $g'$ is.

**(c)** Suppose that $f'$ is continuous and periodic with period $a$. Prove that $f$ is periodic with period $a$ if and only if $f(a) = f(0)$.

**3.** Compute the following integral: $\displaystyle\int_0^1 \left(\sqrt{2-x^2} - \sqrt{2x-x^2}\right) dx$.

**Hint.** Interpret the definite integral as the area bounded by appropriate curves.