

Problem Set #21

Due: Thursday, 15 March 2012

1. Decide whether each of the following infinite series is convergent or divergent.

(a) $\sum_{i=1}^{\infty} \cos\left(\frac{1}{i^2}\right)$

(b) $\sum_{k=1}^{\infty} \left[\cos\left(\frac{1}{k^2}\right) - \cos\left(\frac{1}{(k+1)^2}\right) \right]$

2. Decide whether each of the following infinite series is convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{n-1}{n2^n}$

(b) $\sum_{k=1}^{\infty} \frac{\sqrt[k]{k}}{k}$

3. (a) Find all $p \geq 0$ such that the series $\sum_{n=1}^{\infty} \frac{1}{n[\ln(n+1)]^p}$ converges.

(b) If $a_k \geq 0$ is a bounded sequence, prove that $\sum_{k=0}^{\infty} \frac{a_k}{(k+1)^p}$ converges for all $p > 1$.