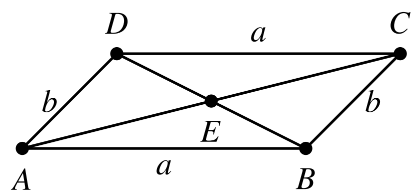
Problems #2

MATH 387 : 2015

Due: Thursday, 15 January 2015

1. Prove that the diagonals of a parallelogram bisect each other and that, in addition, the diagonals of a rhombus are perpendicular to each other.



2. Two triangles are **congruent** if their corresponding sides are equal in length and their corresponding angles are equal in size. Prove the following proposition (which does not appear explicitly in Euclid's *Elements*).

> If two right triangles have two sides equal, not containing the right angle, then they are congruent.

3. Explain what is wrong with the following argument which purports to prove that every triangle is isosceles.

> Let ABC be any triangle. Let D be the midpoint of BC. The perpendicular to BC at D meet the angle bisector at A at the point E. Drop perpendiculars EF and EG to the sides of the triangle. Draw BE and CE. The triangles AEF and AEG have the side common and two angles equal, so they are congruent (Eucl.I.26). Hence, we have $AF = AG$ and $EF = EG$. The triangles BDE and CDE have DE common, two other sides equal, and the included right angles equal. Thus, they are congruent (Eucl.I.4) and $BE = CE$. It follows that the triangles BEF and CEG are right triangles with two sides equal, so they are congruent (by Problem 2.2) and $BF = CG$. Therefore, we see that $AB = AF + FB$ is equal to $AC = AG + GC$, so the triangle ABC is isosceles.