Problem Set #6

MATH 387 : 2015

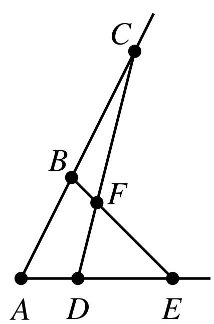
Due: Thursday, 12 February 2015

1. A *projective plane* is a set of points and subsets called lines that satisfy the axioms:

(P0) Any two distinct points lie on a unique line.
(P1) Any two lines meet in at least one point.
(P2) Every line contains at least three points.
(P3) There exist three non-collinear points.

(a) Show that every projective plane has at least seven points.
(b) Show that there exists a model of a projective plane having exactly seven points.
(c) Prove that the four axioms are independent.

2. Consider two distinct lines AC and AE that meet at the point A. Let B be a point between A and C and let D be a point between A and E. Using only the axioms of incidence, the axioms of betweenness, and the separation propositions, show that the line segment $\overline{BE}$ must meet the line segment $\overline{CD}$ at a point F.



3. Consider the real Cartesian plane $\mathbb{R}^2$ with the standard notions of lines and betweenness. Define a different notion of congruence of line segments using the distance function given by the sum of the absolute values:

$$d_0(A,B) := |a_1 - b_1| + |a_2 - b_2| ,$$

where $A = (a_1, a_2)$ and $B = (b_1, b_2)$. Specifically, we declare that $\overline{AB} \cong \overline{CD}$ if and only if $d_0(A,B) = d_0(C,D)$.

(a) Show that the axioms of congruence for line segments, namely Hilb.C1–C3, hold.

(Hilb.C1) Given a line segment $\overline{AB}$ and a ray r originating from the point C, there exists a unique point D on the ray r such that $\overline{AB} \cong \overline{CD}$.

(Hilb.C2) If $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \cong \overline{EF}$, then we have $\overline{CD} \cong \overline{EF}$. Every line segment is congruent to itself.

(Hilb.C3) Suppose that A, B, C are three points on a line such that B is between A and C, and suppose that D, E, F are three points on a line such that E is between D and F. If $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$, then we have $\overline{AC} \cong \overline{DF}$.

(b) What does the circle centred at $(0,0)$ and radius 1 look like in this model?