Problem Set #7

MATH 387 : 2015

Due: Thursday, 26 February 2015

1. An *affine plane* is a set of points and subsets called lines that satisfy the axioms:

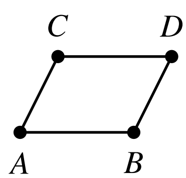
(A0) Any two distinct points lie on a unique line.
(A1) For any line ℓ and any point A, there exists a unique line m containing the point A and parallel to ℓ.
(A2) Every line contains at least two points.
(A3) There exist three non-collinear points.

(a) Show that any two lines in an affine plane have the same number of points (i.e. there exists a bijective correspondence between the points on two lines).

(b) If an affine plane has a line with exactly n points, then the total number of points in the plane is n^2.

(c) Show that there exist affine planes with 4 and 9 points.

2. In a Hilbert plane, suppose that we are given a quadrilateral $ABCD$ with $\overline{AB} \cong \overline{CD}$ and $\overline{AC} \cong \overline{BD}$. Prove that the line CD is parallel to the line AB (without using the Parallel Postulate or Hilb.P).



Hint. Join the midpoints of AB and CD, and use Eucl.I.27.

3. The circle–circle intersection property asserts:

(Hilb.E) Let Γ and Γ' be two circles. If Γ' contains at least one point inside Γ and Γ' contains at least one point outside Γ, then Γ and Γ' meet.

Use Hilb.E to justify Eucl.I.22. Specifically, given three line segments in a Hilbert plane satisfying Hilb.E such that sum of any two is greater than the third, construct a triangle with sides congruent to the three given segments.