Problem Set #8

MATH 387 : 2015

Due: Thursday, 5 March 2015

1. A field $\Bbbk$ together with an binary relation $<$ is an *ordered field* provided the following hold:

(O0) If $a \in \Bbbk$, then one and only one of the following holds: $0 < a$, $a = 0$, or $a < 0$.
(O1) If $a < b$ and $b < c$, then $a < c$.
(O2) If $a < b$, then $a + c < b + c$ for all $c \in \Bbbk$.
(O3) If $0 < a$ and $0 < b$, then $0 < a \cdot b$.

A *positive cone* in a field $\Bbbk$ is a subset $\Bbbk_+ \subset \Bbbk$ such that the following hold:

(P0) If $0 \neq a \in \Bbbk$, then either $a \in \Bbbk_+$ or $-a \in \Bbbk_+$.
(P1) For $a, b \in \Bbbk_+$, both $a + b \in \Bbbk_+$ and $a \cdot b \in \Bbbk_+$.
(P2) If $0 \neq a \in \Bbbk$, then $a^2 \in \Bbbk_+$.
(P3) The elements 0 and -1 is not in $\Bbbk_+$.

Given a field $\Bbbk$, show that there is a bijection between ordered fields structures on $\Bbbk$ and positive cones in $\Bbbk$.

2. Consider two triangles ABC and DEF. If $\angle BAC \cong \angle EDF$, and the sides $\overline{AB}$, $\overline{AC}$ are proportional to the sides $\overline{DE}$, $\overline{DF}$, then prove that the two triangles are similar.

3. Let ABC be any triangle. If $\overline{AD}$ is the angle bisector of $\angle BAC$ where D is between B and C, then prove that $\overline{AB}$ and $\overline{AC}$ are proportional to $\overline{BD}$ and $\overline{CD}$.