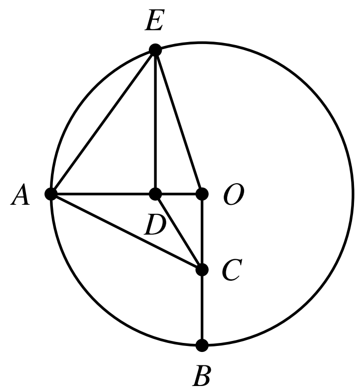
Problem Set #9

MATH 387 : 2015

Due: Thursday, 12 March 2015

1. Consider two perpendicular radii $\overline{OA}$ and $\overline{OB}$ in a circle Γ centred at the point O. Let C be the midpoint of $\overline{OB}$, and let $\overline{CD}$ be the angle bisector of $\angle ACO$ where D is between O and A. If $\overline{DE}$ is perpendicular to the line OA and E lies on the circle Γ, then prove that $\overline{AE}$ is the side of a regular pentagon inscribed in Γ.



Hint. Use Problem 8.3, to find the length of $\overline{AE}$.

2. In Cartesian plane over the ordered field $\Bbbk$, consider an angle α formed by two rays lying on lines of slope m and m'. The *tangent* of α is defined to be

$$\tan(\alpha) = \pm\left|\frac{m'-m}{1+m\cdot m'}\right|,$$

where we take the positive sign if the angle is acute and the negative sign if the angle is obtuse. Using this definition, verify that for any two acute angles α and β, we have

$$\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\cdot\tan(\beta)}.$$

3. In the Cartesian plane over an ordered field $\Bbbk$, consider a right triangle ABC where $\angle ABC$ is a right angle. Let D and E be the midpoints of the segments $\overline{AB}$ and $\overline{AC}$ respectively. Show that there exists a line segment $\overline{FG}$ such that F is between B and D, G is between C and E, $\overline{FG}$ is parallel to $\overline{BC}$, and $\overline{EF}$ is parallel to $\overline{BG}$ if and only if $\sqrt{2} \in \Bbbk$.