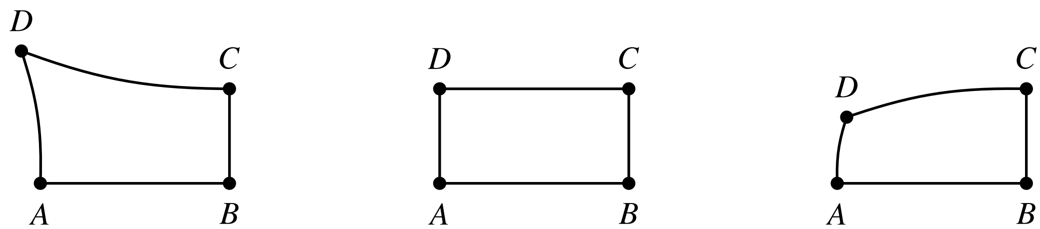
Problem Set #10

MATH 387 : 2015

Due: Thursday, 19 March 2015

1. A *Lambert quadrilateral* is a quadrilateral $ABCD$ with right angles at $\angle DAB$, $\angle ABC$, and $\angle BCD$. Show that the fourth angle $\angle CDA$ is acute, right, or obtuse if and only if the geometry is semi-hyperbolic, semi-Euclidean, or semi-elliptic respectively.



2. In a semi-hyperbolic or semi-elliptic plane, prove the Angle–Angle–Angle Congruence Theorem for triangles:

> If two triangles ABC and $A'B'C'$ satisfy $\angle ABC \cong \angle A'B'C'$, $\angle BCA \cong \angle B'C'A'$, and $\angle CAB \cong \angle C'A'B'$, then the two triangles are congruent.

3. The field of real rational functions $\frac{f(t)}{g(t)}$, where $f(t)$ and $0 \neq g(t)$ are univariate polynomials with real coefficients, can be made into an ordered field by defining $\frac{f(t)}{g(t)} > 0$ whenever $\frac{a_n}{b_m} > 0$ and $f(t) = a_n t^n + a_{n-1} t^{n-1} + \cdots + a_0$ and $g(t) = b_m t^m + a_{m-1} t^{m-1} + \cdots + b_0$. Arrange the following elements in increasing order:

$$0, \quad 1, \quad 5, \quad t, \quad \frac{1}{t}, \quad t+1, \quad \frac{1}{t+1}, \quad t-1, \quad \frac{t^2}{2}, \quad t^2-t, \quad t^2-1, \quad t+\frac{1}{t}, \quad \frac{t-1}{t+1}.$$