

# 7th Biannual Meeting on System and Control Theory

Queen's University, Kingston, ON, May 11-13, 2016

The titles and abstracts for the **Scheduled Talks** and **Posters** are included.

## Scheduled Talks:

### Wednesday May 11:

- **Reach Control Problem** (Mireille Broucke) 8:30-9:30
  - We discuss a class of control problems for continuous time dynamical systems featuring synthesis of controllers to meet certain logic specifications. Such problems fall in the area of hybrid systems. Hybrid systems have been studied for some time; unfortunately the area has not delivered all that it promised: a theory of control synthesis has remained elusive. Some work has been done at the high level on synthesis of controllers for logic specifications inspired by discrete event system theory. These approaches do not confront where the true challenge lies: a (hopefully structural) characterization of the intrinsic limits of a continuous time control system to achieve a non-equilibrium specification. We study affine systems and logic specifications encoded as inequality constraints. Mathematically, the model is an affine system defined on a polytopic state space, and control synthesis typically yields piecewise affine controllers. By studying this special model, synthesis tools have been recoverable. The core synthesis problem has been distilled in the so-called Reach Control Problem (RCP). Roughly speaking, the problem is for an affine system  $\dot{x} = A x + B u + a$  defined on a simplex to reach a pre-specified facet (boundary) of the simplex in finite time without first exiting the simplex. The significance of the problem stems from its capturing the essential requirements of logic specifications: state constraints and the notion of trajectories reaching a goal set of states in finite-time. In the talk I will give highlights of nearly 10 years of research on RCP: solvability by affine feedback, continuous state feedback, time-varying affine feedback, and piecewise affine feedback; an associated Lyapunov theory; a geometric structure theory; and emerging applications.
- **Mean field competitive Markov Decision Processes and Threshold Policies** (Minyi Huang) 9:30-10:30
  - This talk introduces a class of mean field games in a Markov decision process (MDP) framework as motivated by network security investment games and vaccination games. Each player has a continuum state, indicating its unfitness, and binary action, and is coupled with others through its cost function which is increasing in its own state. We show structural properties of the strategies in the mean field game in terms of threshold policies. Such strategies are appealing from the point of view of computation and implementation. Uniqueness of the solution of the stationary equation system of the mean field game is studied under a positive externality condition. This is based on a recent paper (M. Huang and Y. Ma, submitted, Feb. 2016)
- **Dynamic Collective Choice: A Mean Field Games Formulation** (Rabih Salhab) 11:00-11:30
  - Inspired by successful biological collective decision mechanisms such as honey bees searching for a new colony or the collective navigation of fish schools, we consider a Mean Field Games model of collective choice where a large number of agents choose between multiple alternatives while taking into account the group's behavior. For example, in elections, individual interests and collective opinion swings together contribute in the crystallization of final decisions. At first, the agents' decisions are determined by their initial states. Subsequently, the model is generalized to include a priori individual preferences towards the destination points. For example, personal preferences that transcend party lines in elections. We show that multiple approximate (epsilon) Nash equilibria may exist each characterized by a vector describing the way the population splits between the alternatives. These equilibria converge to an exact Nash equilibrium as the number of players converges to infinity.

- **$\epsilon$ -Nash Equilibria for Partially Observed Optimal Execution Problem: Mean Field Game Approach** (Dena Firoozi) 11:30-12:00

  - Partially observed mean field game (PO MFG) theory was introduced and developed in (Caines and Kizilkale, 2013, 2014, Ten and Caines 2014, 2015), where it is assumed the major agent’s state is partially observed by each minor agent, and the major agent completely observes its own state. Accordingly, each minor agent can recursively estimate the major agent’s state, compute the system’s mean field and thence generate the feedback control which yields the  $\epsilon$ -Nash property. This theory was further extended in our recent work (Firoozi and Caines, 2015) for major-minor LQG systems in which both the major agent and the minor agents partially observe the major agent’s state. The existence of  $\epsilon$ -Nash equilibria, together with the individual agents’ control laws yielding the equilibria, were established wherein each agent recursively generates estimates of the major agent’s state and hence generates a version of the system’s mean field. PO LQG MFG theory can be applied to the optimal execution problem in the financial sector where an institutional investor, interpreted as a major agent, has partial observations of its own inventories, and high frequency traders (HFTs), interpreted as minor agents, have partial observations of the major agent’s inventories. The objective for each agent is to maximize its own wealth. In this work we formulate the optimal execution problem with partial observations within the mean field game framework. Then, through utilizing the results of PO LQG MFG theory, the best rate of trading for each agent to maximize its profit is obtained.
  
- **Differential Private Filtering** (Jerome Le Ny) 1:30-2:30

  - Emerging systems such as smart grids or intelligent transportation systems often require end-user applications to continuously send information to external data aggregators performing monitoring or control tasks. This can result in an undesirable loss of privacy for the users in exchange of the benefits provided by the application. Motivated by this trend, we introduce privacy concerns in a system theoretic context, and address the problem of releasing filtered signals that respect the privacy of the users’ data streams. Our approach relies on a formal notion of privacy from the database literature, called differential privacy, which provides strong privacy guarantees against adversaries with arbitrary side information. Methods are developed to approximate a given filter by a differentially private version, so that the distortion introduced by the privacy mechanism is minimized. For model-based estimation, we discuss the design of a differentially private Kalman filter and, if time-permits, I will mention current extensions to simple nonlinear observers.
  
- **Mean Field Teams** (Jalal Arabneydi) 2:30-3:00

  - Teams refer to decentralized multi-agent systems in which all agents have a common objective. Such systems arise in many modern engineered systems including networked control systems, smart grids, and robotics. The optimal control of such systems is understood only for a few specific information structures. In this paper, we introduce a new class of information structures  $\epsilon$  called mean-field sharing  $\epsilon$  and develop a solution methodology to identify optimal decentralized control laws for such systems. In particular, we consider a population of agents that consists of multiple sub-populations of homogeneous agents. Mean-field refers to the empirical distribution of states in each sub-population. The dynamics of an agent are coupled to others through the mean-field. The system cost may be arbitrary. Using the common-information approach of Nayyar et. al. (TAC 2013) and by exploiting the symmetry between the agents, we identify a dynamic programming decomposition that determines the optimal control strategies of all agents. The key feature is that the complexity of the dynamic program is polynomial in the number of agents (but exponential in the number of sub-populations). We also consider the special case of linear dynamics and quadratic cost. For this model, we show that the optimal gains are determined by solving  $K + 1$  Riccati equations, where  $K$  is the number of sub-populations. The parameters of these Riccati equations do not depend on the number of agents; thus the solution complexity is independent of the number of agents and polynomial in the number of sub-populations. An interesting feature of our solution is that the optimal decentralized strategy has the same performance as the optimal centralized strategy. Applications to demand response in smart grids are also presented.
  
- **Distributed Nash Equilibrium Seeking By Gossip in Games on Graphs** (Farzad Salehisadaghiani) 3:00-3:30

  - We consider a gossip approach for finding a Nash equilibrium in a distributed multi-player network game. We extend previous results on Nash equilibrium seeking to the case when the players’ cost functions may be affected by the actions of any subset of players. An interference graph is employed

to illustrate the partially-coupled cost functions and the asymmetric information requirements. For a given interference graph, we consider network communication limitations between players. We design a generalized communication graph so that players exchange only their required information. The algorithm is designed in such a way that players, with possibly partially-coupled cost functions, make decisions based on the estimates of other players' actions obtained from local neighbors. We show that this choice of communication graph guarantees that all players' information is exchanged after sufficiently many iterations. Using a set of standard assumptions on the cost functions, interference and communication graphs, we prove almost sure convergence to a Nash equilibrium for diminishing step sizes. The efficacy of the proposed algorithm on a large-scale networked game is demonstrated via simulation.

- **Optimal Control of Hybrid Systems: Theory and Applications** (Ali Pakniyat) 4:00-4:30

  - Hybrid optimal control problems are studied for systems where autonomous and controlled state jumps are allowed at the switching instants and, in addition to running costs, switching between discrete states incurs costs. Features of special interest in this work are the possibility of state space dimension change, and existence of low dimensional switching manifolds. In other words, the hybrid state space is considered as the direct product of a set of discrete state components with finite cardinality and a set of Euclidean spaces whose dimensions depend upon the discrete state components and further, switching manifolds corresponding to autonomous switchings and jumps are allowed to be codimension  $k$  submanifolds in  $\mathbb{R}^{n_q}$  with  $1 \leq k \leq n_q$ . The statements of the Hybrid Minimum Principle and Hybrid Dynamic Programming are presented and it is shown that under certain assumptions the adjoint process in the Hybrid Minimum Principle and the gradient of the value function in Hybrid Dynamic Programming are identical to each other almost everywhere along optimal trajectories. In addition to analytical examples, an electric vehicle equipped with a dual-stage planetary transmission is modelled in this framework where, due to the special structure of the transmission, the mechanical degree of freedom changes during the transition period. Hybrid control problems for energy and time optimality of an electric vehicle acceleration task are studied which reveal unanticipated aspects of optimal energy saving strategies for the transmission control.
  
- **Generalized Scattering Transformation for Non-Planar Conic Systems** (Anastasiia Usova) 4:30-5:00

  - Scattering-based technique is presently among the most popular methods for stabilization of complex interconnections in the presence of communication delays; in particular, the technique has found prominent applications to bilateral teleoperation with communication constraints. Until recently, the scattering-based stabilization methods were limited to interconnections of passive systems. In this talk, an extension of the scattering-based techniques to stabilization of interconnections of nonlinear non-planar conic systems is discussed. The notion of non-planar conicity is a generalization of the classical notion of conic systems to the case where the dimension of the central subspace of the cone is greater than one. The extension of the scattering-based methods discussed in this talk is based on the representation of the scattering transformation as a rotation operator in the space of input-output signals. Constructive methods for scattering-based stabilization of interconnections of non-planar conic systems, with and without communication delays, will be discussed.
  
- **Extended Kalman Filtering Design for a Class of Quasi-Linear Distributed Parameter System** (Sepideh Afshar) 5:00-5:30

  - In many physical applications, such as diffusion, wave motion, and vibrations, the system's behavior is distributed in space. Such systems are called distributed parameter systems (DPSs). The mathematical models for these systems are partial differential equations. For many applications, the full state of these systems is required but cannot be measured. State estimation also called observer design is an important tool for reconstructing the state from available measurement. Observer design for linear DPSs has been well studied; see for instance [6], [3], [4], and also the book [2]. However, it has not been well explored for nonlinear DPSs. Some examples can be found in [7] and [8]. Most of the reported observers for nonlinear systems need detailed measurements or are based on an approximation of the system. Among different estimation techniques, Kalman filtering (KF) is well-known as an optimal way of approximating a system's states. In KF, a quadratic cost function is minimized; solving this minimization problem is equivalent to finding the solution of a Riccati equation. For general linear systems, the well-posedness of the Riccati equation is studied in [1]; however, the results are not valid

for nonlinear systems. The extension of the KF to nonlinear systems is developed in the case of finite-dimensional systems and is called extended Kalman filtering (EKF). In this approach, the nonlinear part of the system is linearized around the path of estimation vectors; then, the linearized operator is used in the Riccati equation whose solution provides a sub-optimal filtering. In this paper, EKF is extended to a class of quasi-linear infinite-dimensional systems. The form of Riccati equation employed here is a modified one like that of [5]; it is designed to provide exponential error convergence with arbitrary rate of convergence. Next, the well-posedness of the developed Riccati equation interconnected with the observer dynamics is proved. Finally, it is shown that the state of the developed observer locally exponentially converges to the that of the real system.

## Thursday May 12:

- **Finite Abstractions for Robust Control Synthesis** (Jun Liu)

9:30-10:30

- Abstraction-based control synthesis gained popularity in recent years for its ability to handle nontrivial dynamics and rich specifications at the same time. In practice, control systems are often affected by imperfections. Synthesis of robust control strategies thus becomes essential. In this talk, I will present a notion of finite abstractions that can be used for robust control synthesis in a hierarchical control design framework. I will focus on nonlinear dynamical systems described by ordinary differential equations, with specifications expressible in fragments of linear temporal logics. I will present computationally efficient procedures for abstracting nonlinear dynamics into finite-state transitions and discuss how analytical tools from dynamical system theory can provide correctness and robustness guarantees for these abstractions. Connections with reachability analysis will also be discussed, where I will focus on how computation of better over-approximations of local reachable sets can lead to less conservative abstractions. Finally, some preliminary results on feasibility guarantees for discrete synthesis resulting from finite abstractions will be discussed. The results will be illustrated with several applications in robot motion planning and automotive adaptive cruise control.

- **A Chronological Calculus and Chow-Rashevskii Theorem for Banach Manifolds** (Robert Kipka)

11:00-

11:30

- The classical Chow-Rashevskii theorem is a fundamental result of finite-dimensional geometric control theory, providing sufficient conditions for global controllability of affine systems in terms of Lie brackets of associated vector fields. We prove an infinite-dimensional variant of this theorem for smooth Banach manifolds using a recent extension of the Chronological Calculus. In this talk we introduce our extension of the Chronological Calculus for infinite-dimensional dynamical systems along with a corresponding calculus of remainder terms, prove a bracket formula for infinite-dimensional dynamical systems using the extended Chronological Calculus, and apply techniques of nonsmooth analysis to prove the variant of Chow-Rashevskii theorem for affine, infinite-dimensional control systems.

• **On Small-time Local Controllability** (Saber Jafarpour)

11:30-12:00

- In geometric control theory, a real analytic control system on  $\mathbb{R}^n$  is considered as a family of real analytic vector fields and the evolution of the system is defined using the concatenations of trajectories of these vector fields. A control system is called small-time locally controllable (STLC for short) from a point  $x_0$  if, for every small enough time  $t$ , one can reach a neighbourhood of  $x_0$  in  $\mathbb{R}^n$  by starting from  $x_0$  and traveling along trajectories of the system for times less than  $t$ . Small-time local controllability is one of the fundamental properties of a control system. While STLC is defined using trajectories of the system, it is usually difficult, if not impossible, to solve for the trajectories of a control system. Thus, it is reasonable to study STLC of a system using the vector fields of the system and their derivatives. One of the interesting questions about STLC of a real analytic control system is whether it can be determined using only finite number of differentiation of vector fields of the system at point  $x_0$ . This question has been formulated by Andrei Agrachev as follows: Conjecture 1. Given a real analytic control system  $\Sigma$  which is STLC from  $x_0$ , does there exist  $N \in \mathbb{N}$  such that every other control system whose vector fields have the same Taylor polynomial of order  $N$  at point  $x_0$  as vector fields of  $\Sigma$  is STLC from  $x_0$ ? Some partial answers for this conjecture exist in the literature. However, for the general case where  $n \geq 3$ , this question is still open. Another important question about STLC is the rate of change of the reachable sets of a system with respect to time. It would be interesting to know if the reachable sets of a STLC control system shrink to  $x_0$  with a polynomial rate with respect to time. More precisely: Conjecture 2. Let  $\Sigma$  be a real analytic control system which is STLC from  $x_0$ , does there exist  $N \in \mathbb{N}$  such that, for small enough  $t$ , the reachable set of the system from  $x_0$  for times less than  $t$  contains a ball of radius  $t^N$  with center at  $x_0$ ? In this talk, we study these two conjectures and the relation between them. We show that, for a real analytic control system whose reachable sets satisfy a polynomial growth condition with respect to time, STLC can be recognized using finite number of differentiations of vector fields of the system. In particular, this shows that if the answer to conjecture 2 is positive, then we have an affirmative answer for conjecture 1.

• **Observer Design on  $SO(3)$  - Fast Convergence and Practical Considerations** (James Forbes)

1:30-2:00

- Robotic vehicles require accurate knowledge of their position and orientation to execute autonomous maneuvers. However, orientation, also referred to as attitude, cannot be directly measured. Instead, the attitude of a vehicle must be ascertained from a variety of available measurements, many of which may be of poor quality. Complicating matters is the fact that direction cosine matrices (DCMs), which are a global and unique representation of attitude, are elements of the special orthogonal group,  $SO(3)$ . As such, linear estimation methods are not directly applicable to estimation of position and orientation. This talk will discuss observer design on  $SO(3)$  within a deterministic framework. In particular, an alternate attitude error function to what is typically used in the literature is presented and used in Lyapunov analysis. The alternate attitude error function leads to a different innovation term that in turn results in estimator dynamics that have superior convergence properties for large initial estimation error compared to existing schemes in the literature. Estimator design with bias compensation is also considered. Motivated by the fact that there are no "attitude sensors" the use of vector measurements directly within the estimator is also considered.

• **Control of Sampled-Data Systems on Matrix Lie Groups** (Philip James McCarthy)

2:00-2:30

- Systems on matrix Lie groups are common in engineering applications. Rotational and translational dynamics, oscillators, and even quantum systems can be modelled on matrix Lie groups. Systems on matrix Lie groups, unlike most nonlinear systems, admit exact closed-form solutions for piecewise constant inputs. This allows for sampled-data control using exact discretization. We present two of our results pertaining to sampled-data control of systems on matrix Lie groups: 1) passivity-based stabilization and reference tracking; 2) phase synchronization in networks of oscillators. Given a fixed storage function, passivity is typically lost under sampling, even with LTI systems. By redefining the output of the discretized dynamics, passivity is preserved. Given a passive continuous-time plant that satisfies a standard observability assumption, we asymptotically stabilize the closed-loop sampled-data system. A common control objective for a network of oscillators is to achieve some sense of synchronization. This can mean, for example, driving all agents to the same phase, or to the same frequency. Using a simple control law, we achieve phase synchronization for a network of oscillators on  $SO(2)$  and, in the case of two agents,  $SO(3)$ .

- **Integrated Guidance and Control to a Desired Pose in SE(3) with Four Control Inputs** (Sasi P. Viswanathan) 2:30-3:00

  - An integrated guidance and feedback control scheme for steering an underactuated unmanned vehicle to a desired pose, i.e., combined position and orientation, is developed in this paper. The unmanned vehicle platform considered here is modeled as a rigid body with four control inputs. The control inputs actuate the three degrees of rotational motion and one degree of translational motion in a vehicle body-fixed coordinate frame. This actuation model is appropriate for a wide range of vehicles including fixed-wing and multirotor unmanned aerial vehicles (UAVs), highly maneuverable robotic vehicles, and spacecraft. Here, the guidance problem is developed on the special Euclidean group SE(3) in the framework of geometric mechanics, which represents the vehicle dynamics globally on this configuration manifold. The integrated guidance and control algorithm selects desired translational and angular velocities at the desired final pose, based on knowledge of the initial pose and initial velocities. A feedback control law is then obtained to steer the underactuated vehicle towards the desired final pose with selected final velocities. This integrated scheme takes into account known bounds on control inputs and generates a trajectory that is continuous and at least twice differentiable, which can be implemented with continuous control inputs that satisfy the bounds. The scheme can be extended to generate a continuous trajectory through a series of given waypoints in SE(3). A discretization of this scheme is also obtained as a Lie group variational integrator, which is used to numerically simulate the scheme for with the dynamics model on a quadcopter UAV.
  
- **Virtual Holonomic Constraints for Euler-Lagrange Systems** (Alireza Mohammadi) 3:00-3:30

  - Abstract: The theory of virtual holonomic constraints (VHCs) pioneered by J. Grizzle and collaborators is a promising approach for designing agile and energy-efficient locomotion control algorithms for robotic systems. The idea in Grizzle's work is to design feedback controllers that enforce constraints on the configuration variables of the robot, thereby inducing a stable walking gait. In this talk we present two results concerning underactuated mechanical control systems with  $n$  degrees-of-freedom and  $n-1$  controls when  $n-1$  VHCs have been enforced via feedback. In this context the constrained dynamics are described by a two-dimensional unforced differential equation. The first result are necessary and sufficient conditions for the constrained dynamics to be Lagrangian. In this context, we highlight the sometimes counter-intuitive discrepancies with results of classical mechanics. The second result is concerned with stabilizing energy level sets associated with Lagrangian constrained dynamics. The challenge in this context is that the constrained dynamics have no control inputs. The stabilization mechanism we propose relies on dynamically changing the geometry of the VHCs.
  
- **Global Exponential Stabilization on SO(3)** (Soulaïmane Berkane) 4:00-4:30

  - The attitude control on the Special Orthogonal Group SO(3) is a long lasting problem that has received a broad interest during the last decades. Recent investigations focus on using differential geometric tools to design attitude control systems directly on the attitude configuration space SO(3); where the orientation of the rigid body is uniquely and globally represented by a rotation matrix in SO(3). The available results obtained via smooth feedback are limited to almost global asymptotic stability, with slow convergence rates near some critical points. These limitations are mainly due to the topological nature of the closed compact manifold SO(3). An interesting hybrid framework involving synergistic potential functions has been developed by A. Teel and his collaborators, overcoming the topological obstruction to global stabilization on SO(3) and leading to global asymptotic stability results. Recently, we extended the synergistic hybrid control design approach and proposed a comprehensive framework for the design of hybrid control and estimation schemes providing stronger global exponential stability results. Our framework encompasses the use of either differentiable or non-differentiable attitude potential functions. A hybrid switching mechanism is employed to avoid the critical points or the singular points of each individual control law, when using smooth or non-smooth attitude potential functions, respectively. We provide new results on the design of the hysteresis gap that is necessary for the implementation the switching mechanism. We show that these control strategies guarantee global exponential stability results on SO(3); a result that was considered as the holy grail for many years.
  
- **Patterned Systems: A New Framework for Distributed Control** (Adam Sniderman) 4:30-5:00

  - We have developed a way to solve geometric control problems on distributed systems while simultaneously preserving those systems' interconnection structures. Most distributed control frameworks to

date have taken one of two forms: some allow for distributed systems with arbitrary interconnections, but only provide limited results on the solvability of control problems; while others provide those solvability results, but restrict the allowable interconnections in the system parts (often requiring them to form a symmetry). Our framework combines these two methods: specifically, by encoding distributed system structures with commuting relationships ? called patterns in linear systems ? we can carry those structures through the fundamental control procedures of Stabilization (by state or measurement feedback), Output Stabilization, and Disturbance Decoupling. This work can be considered a sequel to Sniderman's presentation at the previous Meeting on Systems and Control, on the control of block circulant systems. In that presentation, we showed that the block circulant structure could be preserved through all the aforementioned control procedures by keeping track of a certain inherent commuting relationship. Now, by zeroing in on commuting relationships in other distributed systems, this framework has been expanded in a number of notable directions. We can effect control syntheses while preserving any inherent symmetries in a patterned system ? thus encompassing all systems allowed by prior frameworks ? and we can further preserve patterns that cannot be characterized by symmetries, most notably for the distributed structure of a unidirectional chain. Our framework is the first to move beyond symmetries, and is also the first to admit all patterns, without restriction. In addition to solving control problems on patterned systems, we have also determined how to find patterns in distributed systems. This line of research is completely new in the literature ? all prior work assumes that a distributed structure has already been turned into a pattern (or other such encoding) a priori. Our new method removes this deficiency, detailing not only how to beget a pattern from a distributed structure when such a thing is achievable, but also how to get as close as possible otherwise. Overall, these results on patterns present a promising new approach to the control of distributed systems.

## Friday May 13:

- **Quadratic Performance of Distributed Optimization Algorithms** (John W. Simpson-Porco) 8:30-9:30
  - Primal-dual gradient algorithms have recently attracted interest as a set of systematic techniques for distributed optimization. One of the proposed applications has been optimal frequency regulation in smart grids, where the algorithm is implemented for online optimization as a distributed controller, minimizing the total cost of power generation. In this context however, disturbances arise from fluctuating load and renewable generation, and input/output performance becomes important to understand. This talk presents preliminary results on how effectively these algorithms reject disturbances. For the linear primal-dual controllers which arise from quadratic programs, we present simple expressions for the  $H_2$  norm under several input/output choices, and examine the performance gain achieved by augmenting the Lagrangian. Our results suggest that the conventional algorithm may perform poorly when applied to large-scale systems, and that augmentation can partially (or completely) alleviate these scaling issues. Time permitting, we compare theoretically the performance of primal-dual methods with that of recently proposed optimal frequency controllers based on distributed averaging.
- **An Extremum-Seeking Control Approach for Observer Design** (Martin Guay) 9:30-10:30
  - In this study, we propose an extremum seeking control (ESC) approach for the design of nonlinear observers. ESC is a real-time optimization control algorithm that is used to steer an unknown dynamical control system to the unknown optimum of a measured, but unknown, objective function. In this work, ESC is used to circumvent the difficulties associated with the computation of optimal observer gains for nonlinear systems. The main challenge associated with nonlinear observer design is the unknown nature of the relation between the measured outputs and the states of the system. For linear system, this relation is easily derived and exploited in the design of reliable observers. Observability of the system leads to existence of a solution to a Riccati equation. The resulting solution can be used directly to design a suitable observer gain. In nonlinear systems, this task is considerably more difficult since the analogue of the Riccati equation typically takes the form on a nonlinear partial differential equation. Although the structure of the nonlinear PDE can be exploited, the solutions are usually approximated to arrive at a useable observer formulation. In this study, we attempt to circumvent the difficulties associated with the computation of the observer gain for nonlinear systems by using an extremum-seeking control approach to generate optimal observer gains for a general class of nonlinear systems. This presentation proposes a proportional-integral extremum-seeking control (PIESC) technique for the

design of nonlinear observers. The PIESC technique is a generalization of the standard perturbation based techniques that was initially developed to improve the transient performance of ESC control systems. In this application of PIESC, the gain of the nonlinear observer is computed in real-time to minimize the least-squares output prediction error. The resulting observer provides a very effective design technique for nonlinear systems that is reminiscent of classical Chandrasekhar-type algorithms introduced by Kailath and Lindquist. Finally, it is shown that the optimal observer gain computed using the PI-ESC technique can also be used to solve a dual nonlinear stabilization problem.

- **A Smooth Distributed Control Law for Global Rendezvous of Unicycles** (Ashton Roza) 11:00-11:30

- This talk presents a solution to the rendezvous control problem for a network of unicycles on the plane using continuous feedbacks that are local and distributed. While a time-varying feedback solving this problem exists in the literature, we present the first time-invariant solution. Each unicycle is equipped with an onboard camera and can measure its relative displacement to its neighbors in body frame. The unicycles' control inputs depend only on onboard measurements and no global positioning system is required. The objective of the rendezvous control problem is to design the control inputs for each unicycle so as to drive the group to a common position from arbitrary initial conditions. The solution we present relies on a control scheme made of two nested loops. An outer loop treats the unicycles as fully-actuated single integrators and computes a reference velocity signal for the inner loop by leveraging existing consensus algorithms from the literature. The inner loop processes the reference velocity to assign local and distributed feedbacks that solve the rendezvous control problem. We conclude the talk by extending the result to networks of dynamic, underactuated rigid bodies in three dimensions.

- **The Strategic Formation of Multi-Layer Networks** (Ebrahim Moradi) 11:30-12:00

- Many real-world networks consist of multiple layers of relationships between a common set of nodes. Examples include friendship and professional relationships in social networks, different transportation networks between a set of cities, and coupled communication and energy infrastructure networks. We describe a model to capture the strategic formation of multi-layer networks, where each layer is designed to maximize some utility that depends on the topology of that layer and those of the other layers. We start by generalizing distance-based network formation to the two-layer setting, where edges are constructed in one layer (with fixed cost per edge) to minimize distances between nodes that are neighbors in another layer. We show that designing an optimal network in this setting is NP-hard. Despite the underlying complexity of the problem, we characterize certain properties of the optimal networks. We then formulate a multi-layer network formation game where each layer corresponds to a player that is optimally choosing its edge set in response to the edge sets of the other players. For utility functions that view the different layers as strategic substitutes, we show that players with low edge-costs drive players with high edge-costs out of the game, and that hub-and-spoke networks that are commonly observed in transportation systems arise as Nash equilibria in this game.

## Posters:

- **Termination Time of the One-Sided Asymmetric Hegselmann-Krause Dynamics** (Jeremy Coulson and Drew Steeves)
  - We provide a novel upper bound for the termination time of the one-dimensional asymmetric Hegselmann-Krause dynamics, when the asymmetry is one-sided. In addition to the number of agents, our upper bound depends on the ratio of asymmetry and the confidence range in the opinions of agents, and recovers the known  $O(n^3)$  results of the symmetric case. Our proof technique relies on a novel Lyapunov-like function, which measures the spread of the opinion profile. As a by-product, we fully characterize the switching pattern in the opinions of the agents.
- **Fundamental limits of remote estimation of Markov processes under communication constraints** (Jhelum Chakravorty)
  - The remote estimation system consists of a sensor and an estimator. The sensor observes a discrete-time Markov process driven by a symmetric and unimodal innovations process. At each time, the sensor either transmits the current state of the Markov process or does not transmit at all. The estimator estimates the Markov process based on the transmitted observations. In such a system, there is a trade-off between communication cost and estimation accuracy. Two fundamental limits of this trade-off are characterized for infinite horizon discounted cost and average cost setups. First, when each transmission is costly, we characterize the minimum achievable cost of communication plus estimation error. Second, when there is a constraint on the average number of transmissions, we characterize the minimum achievable estimation error. Transmission and estimation strategies that achieve these fundamental limits are also identified.
- **Robust Controller Design Using the Large Gain Theorem** (Ryan James Caverly)
  - In this talk, controller synthesis methods are presented that invoke the Large Gain Theorem and the concept of minimum gain to guarantee robust closed-loop input-output stability. The Large Gain Theorem provides a guarantee of robust stability in the proposed controller synthesis methods, which parallels the use of the Small Gain Theorem to guarantee robust stability in H-infinity controller design. A significant benefit of the Large Gain Theorem includes the ability to accommodate unstable uncertainties, which cannot be directly accounted for with the Small Gain Theorem. The proposed controllers are synthesized to either maximize robustness or maximize performance while satisfying a linear matrix inequality that enforces the stipulations of the Large Gain Theorem. Numerical examples are presented to illustrate the effectiveness of the proposed controller synthesis methods.
- **Constrained Control of the Synchronesh Operating State** (Hossein Vahid Alizadeh)
  - This work considers the constrained control problem of the friction regimes in sliding lubricated surfaces with the purpose of speed synchronization, wear reduction and increasing the lifetime of the friction lining material. For the purpose of controlling such systems, the control objectives and the constraints are defined, and a controller design method is proposed. The controller design method is based on solving a set of linear programming (LP) problems in the offline phase, which results in a piecewise affine (PWA) feedback law that can be easily applied on the system in the real-time closed-loop configuration. The case study here is the engagement process of the synchronesh cone clutch system. Such a system performs the clutchless gear shifting in a 2-speed automated manual transmission (AMT) of an electric vehicle. In this study, the frictional behavior of the cone clutch system is investigated by considering the involved lubricated friction regimes. By knowing the lubricated sliding friction regimes, the dynamic model of the system is derived according to the variable coefficient of friction. Moreover, the primary sources of the uncertainty and disturbance are recognized and considered in the dynamic model of the system. The performance of the proposed control approach is assessed by presenting the closed-loop control results for the ideal situation as well as the perturbed systems in the presence of the disturbance. Finally, the controller is implemented on a real-time embedded industrial controller and the performance of the closed-loop control system is verified by conducting a set of experiment on a test rig developed at the McGill Centre for Intelligent Machines (CIM).

- **Optimized Decentralized Control of Large Scale Systems** (Xiaoqi Shi)
  - This paper presents a new optimal decentralized controller design method for solving the tracking and disturbance rejection problems for the class of large scale LTI systems, using only low order decentralized controllers. In particular, the class of large scale systems considered in this paper is described by the LTI model  $\dot{x} = Ax + Bw + Dd$ , where  $x$  is the output,  $w$  is the input,  $d$  is an unknown constant disturbance,  $e$  is the tracking error in the system and  $u$  is a constant given signal. To illustrate the type of results which can be obtained using the new optimal decentralized control design method, the control of a large flexible space structure is studied and compared with the standard centralized LQR-Observer controller. In this case it is shown that the new resultant decentralized controller obtained is orders of magnitude smaller than the standard centralized LQR-Observer controller. The proposed controller also has the property that if a sensor actuator failure occurs, the resultant failed system has certain fail-safe properties. The proposed optimal controller also has the property that it is strongly robust as compared to the standard centralized LQR-Observer controller. In particular, the proposed controller can have the property of being some 5 orders of magnitude more robust than the standard LQR-Observer controller. An illustration of the new decentralized controller design method is applied to a large flexible space structure (LFSS) system with 5 inputs and 5 outputs and of order 24.
- **Synchronization of diffusively coupled LTI systems** (Tian Xia)
  - In this talk we investigate the output-feedback synchronization problem for a network of identical, linear time-invariant systems. In the first part of the talk we introduce and study the notion of output-feedback synchronizability and we present a sufficient and necessary condition for the systems to be synchronizable. A consequence of this condition is that output-feedback stabilizability is sufficient but not necessary for synchronizability. In the second part of the talk we focus on special classes of synchronizable systems and we use our formalism to show that a network of passive systems asymptotically synchronize if and only if the interconnection graph has at least one globally reachable node. This extends a classical result for consensus in networks of single integrators to networks of passive systems. We conclude the talk by considering the special case of SISO systems and by illustrating the theory with some examples.
- **Exact observability problem in infinite dimensional dynamical systems** (Mohammad sajjad Edalatzadeh)
  - The concept of observability for infinite dimensional dynamical systems has received considerable attention in recent years. The survey paper of Lagnese (1997) and the general exposition of Bensoussan (1990) provide a comprehensive study in this field. Dolecki and Russell (1977) showed that the concept of exact observability for an infinite dimensional dynamical system is dual to that of exact controllability. Furthermore, the duality between admissible observation operator and admissible control operator in this framework is introduced by Salamon (1987). A general necessary condition for the exact observability of the systems governed by partial differential equations (PDEs) is then attained by Russell and Weiss (1994). To tackle an observability problem, most researchers adopt a time domain approach. This means that the governing PDE or its dual counterpart is manipulated in various ways to meet the necessary conditions for observability or controllability; this includes: multiplier method (Komornik and Gattulli, 1997), microlocal analysis technique (Bardos et al., 1992), or nonharmonic Fourier series (Avdonin and Ivanov, 1995). The introduction of Hautus test for the infinite dimensional dynamical systems established a basis for frequency domain approach, which is rarely adopted by researcher. Liu et al (2001) obtained a Hautus-type criterion test for exact controllability of systems with bounded input operator; they then applied the criterion for several elastic systems. Recently, a criterion was established for unbounded observation operators with application to Schrödinger equation (Miller, 2005). In addition, the Hautus-type test is developed to characterize exact observability only in terms of the observation operator and the spectral elements of the state operator (Ramdani et al., 2005). In my research, the exact observability of micro-scale beam model under various observation operators is studied using both time and frequency domain approaches. Based on the only observation operator that is proved to give an exactly observable system, an infinite dimensional observer is then designed. Moreover, the input-output stability of the system, the semigroup generation property of the observer state operator, the admissibility of the observer control operators, the exponential stability of error dynamic and of closed loop system are all shown.