STAT 269 – Winter 2009

Homework Assignment 2

Assignment 2 — due Friday, January 23 (in the class or in the mail-box for office 511 during the day)

1. Suppose that X follows a geometric distribution,

$$P(X = x) = p(1-p)^{x-1}, \quad x = 1, 2, \ldots ,$$

and assume an i.i.d. sample of size n.

a) Find the method of moments estimate of p.

b) Find the maximum likelihood estimate of p.

2. Consider an i.i.d. sample of random variables with the Laplace density

$$f(x|\theta) = \frac{1}{2\theta} e^{-|x|/\theta}, \quad (\theta > 0).$$

a) Can a method of moments estimator of θ be based on the first moment $E(X)$?

b) Find the method of moments estimator of θ using second moment $E(X^2)$.

c) Find the method of moments estimator of θ using first absolute moment $E(|X|)$.

d) Find the maximum likelihood estimator $\hat{\theta}$ of θ.

e) Is the maximum likelihood estimator $\hat{\theta}$ unbiased?

f) What is the variance of the maximum likelihood estimator $\hat{\theta}$?

g) Is the maximum likelihood estimator $\hat{\theta}$ the best unbiased estimator of θ?

3. Suppose a random sample of size n from a Poisson population is given.

a) Find the maximum likelihood estimator $\hat{\lambda}$ for the parameter λ.

b) Is $\hat{\lambda}$ the best unbiased estimator of λ?

4. Let $x_1, ..., x_n$ be a random sample from a *gamma distribution*, with the probability density

$$f(x|\alpha, \beta) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, \quad x > 0.$$

Assume that the parameter α is given.

a) Find the method of moments estimator of β.

b) Find the maximum likelihood estimator $\hat{\beta}$ of β.

c) Find the *mean squared error* of $\hat{\beta}$.

d) Is your estimator the best unbiased estimator of β? Explain.

5. The family of *beta densities* on the interval $[0, 1]$ has the following form

$$f(x|\alpha, \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 < x < 1, \quad (\alpha, \beta > 0).$$

On a single printout, produce 16 different plots of beta densities corresponding to the following combinations of parameters: 1) $\alpha = \beta = 1/2$; 2) $\alpha = \beta = 1$; 2) $\alpha = \beta = 2$; 3) $\alpha = \beta = 3$; 4) any combination of the above for which $\alpha < \beta$ and $\alpha > \beta$. Provide corresponding titles. **Hint:**

```
par(mfrow=c(4,4))
a=1/2
b=1/2
m=100
x=(0:m)/m
plot(x,dbeta(x,a,b),type="l", ylab="f(x)")
title(paste("a=",a,", b=",b))
```

6. **(Optional)** Let $x_1, ..., x_n$ be a random sample from the beta distribution.

a) Find the *method of moments* estimates of the parameters α and β.

b) Are your estimators *consistent?* Explain.

c) Generate a random sample of size $n = 100$ from the beta distribution with $\alpha = 2$ and $\beta = 3$. Make the histogram of the sample. Calculate the method of moments estimates using R. Compare the obtained estimates with the "true" values of the parameters α and β. **Hint:**

```
par(mfrow=c(1,1))
a=2
b=3
n=100
X=rbeta(n,a,b)
hist(X)
mu1=mean(X)
mu2=mean(X^2)
```