STAT 269 – Winter 2009

Homework Assignment 3

Assignment 3 — due Friday, January 30 (in the class or in the mail-box for office 511 during the day)

1. **Problems 11.20–11.21.**

2. **Problems 11.24.** **Hint:** standard deviation = S.

   How different would the answers be when using

   a) Table on the back of the book;

   b) R.

3. Among 100 fish caught in a certain lake, 18 were inedible as a result of the chemical pollution of the environment. Construct an approximate 99% confidence interval for the corresponding proportion.

4. Exhibit two-sided $100(1-\alpha)\%$ confidence intervals based on the MLE for the unknown parameters of

   a) geometric distribution;

   b) Poisson distribution;

   c) exponential distribution;

   d) Laplace distribution.

5. **The growth model.** Suppose the size of a plant is measured at time periods $t_1, t_2, ..., t_n$, and the measurements are described by

   $$y_i = \theta t_i + e_i$$

   where θ is the unknown speed of growth and $e_1, ..., e_n$ are independent measurement errors having normal distribution $\mathcal{N}(0, \sigma^2)$, with a given variance σ^2. The following estimator of θ was derived in the class:

   $$\hat{\theta} = \frac{\sum_{i=1}^n t_i y_i}{\sum_{i=1}^n t_i^2}.$$

   a) What is the distribution of the random measurements $y_i,\ i = 1, ..., n$?

   b) Is $\hat{\theta}$ an unbiased estimator of θ? Explain.

   c) What is the variance of $\hat{\theta}$?

   d) What is the distribution of $\hat{\theta}$?

   e) Exhibit a two-sided $100(1-\alpha)\%$ confidence interval for θ based on the estimator $\hat{\theta}$.

   f) Exhibit a one-sided $100(1-\alpha)\%$ upper confidence interval for θ.

   g) Exhibit a one-sided $100(1-\alpha)\%$ lower confidence interval for θ.

6. **(Optional)** a) For $n = 3, 5, 10, 20$, plot probability density of the t-distribution with n degrees of freedom vs. the standard normal density. **Hint:**

```
m=100
x=-4+8*(0:m)/m
par(mfrow=c(2,2))
plot(x,dnorm(x),type="l",ylab="pdf")
lines(x,dt(x,3))
title("pdf t3 vz normal")
```

b) For $n = 3, 5, 10, 20$, plot cumulative distribution function of the t-distribution with n degrees of freedom vs. the standard normal cdf. **Hint:**

```
m=100
x=-4+8*(0:m)/m
par(mfrow=c(2,2))
plot(x,pnorm(x),type="l",ylab="cdf")
lines(x,pt(x,3))
title("cdf t3 vz normal")
```