STAT 269 – Winter 2009

Homework Assignment 6

Assignment 6 — due Friday, March 6 (in the class or in the mail-box for office 511 during the day)

1. Convert the following non-linear relationships into linear ones by making transformations and defining new stimulus and response variables.

   a) $y = a/(b + cx)$

   b) $y = ae^{-bx}$

   c) $y = ab^x$

   d) $y = x/(a + bx)$

   e) $y = 1/(1 + e^{bx})$

2. a) Plot y versus x for the following pairs:

x	.34	1.38	-.65	.68	1.40	-.88	-.30	-1.18	.50	-1.75
y	.27	1.34	-.53	.35	1.28	-.98	-.72	-.81	.64	-1.59

   b) Fit a line $y = a + bx$ by the method of least squares, and sketch it on the plot.

   c) Fit a line $x = c + dy$ by the method of least squares, and sketch it on the plot.

   d) Are the lines in parts b) and c) the same? If not, why not?

3. Consider fitting a quadratic regression $y = \alpha + \beta x^2$ to points (x_i, y_i), where $i = 1, ..., n$.

   a) Explain how results for the standard linear model $y = \alpha + \beta x$ can be applied in this case.

   b) Find the *least squares estimates* $\hat{\alpha}$ and $\hat{\beta}$.

4. Suppose that $y_i = \alpha + \varepsilon_i$, where $i = 1, ..., n$ and ε_i are independent errors with mean zero and variance σ^2. Show that $\bar{y}$ is the least squares estimate of α.

5. Let $e_1, ..., e_n$ be the *residuals* of a linear regression model $y = \alpha + \beta x_i + \varepsilon_i$ where $i = 1, ..., n$. Show that

$$\sum_{i=1}^{n} e_i = 0.$$

6. Suppose that in the linear regression model $y_i = \alpha + \beta x_i + \varepsilon_i$ where $i = 1, ..., n$, the stimulus variables x_i satisfy

$$\sum_{i=1}^{n} x_i = 0.$$

a) Simplify the formulas for the least squares estimates $\hat{\alpha}$ and $\hat{\beta}$ in this case.

b) Suppose further that the errors ε_i are independent normal random variables $\mathcal{N}(0, \sigma^2)$. Show that $\hat{\alpha}$ and $\hat{\beta}$ are normal random variables and find their expected values and variances.

c) Conclude that the least squares estimates $\hat{\alpha}$ and $\hat{\beta}$ are unbiased.

d) **(optional)** Show that random variables $\hat{\alpha}$ and $\hat{\beta}$ are independent.

Hint: Two Gaussian random variables are independent if and only if they are uncorrelated.