MAP Decoding of Correlated Sources over Soft-Decision Orthogonal Multiple Access Fading Channels with Memory

by

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Abstract

We consider the joint source-channel coding (JSCC) problem where the real valued outputs of two correlated memoryless Gaussian sources are scalar quantized, bit assigned, and transmitted, without applying any error correcting code, over a multiple access channel (MAC) which consists of two orthogonal point-to-point time-correlated Rayleigh fading sub-channels with soft-decision demodulation. At the receiver side, a joint sequence maximum a posteriori (MAP) detector is used to exploit the correlation between the two sources as well as the redundancy left in the quantizers' indices, the channel's soft-decision outputs, and noise memory. The MAC's sub-channels are modeled via non-binary Markov noise discrete channels recently shown to effectively represent point-to-point fading channels. Two scenarios are studied. In the first scenario, the sources are memoryless and generated according to a bivariate Gaussian distribution with a given correlation parameter. In the second scenario, the sources have memory captured by a changing correlation parameter governed by a two state first order Markov process.

In each scenario, for the simple case of quantizing the sources with two levels, we establish a necessary and a sufficient condition under which the joint sequence MAP decoder can be reduced to a simple instantaneous symbol-by-symbol decoder. Then, using numerical results obtained by system simulation, the theorems are illustrated

and it is also verified that JSCC can harness the correlation between sources, redundancies in the source symbols, and statistics of the channel noise to achieve improved signal-to-distortion ratio (SDR) performance. For example, when the memoryless sources are highly correlated and soft-decision quantization is used, JSCC can profit from high correlation in the channel noise process and provide significant SDR gains of up to 6.3 dB over a fully interleaved channel.

Acknowledgments

I would like to express my gratitude to all who have supported me and had their contributions in making this thesis possible.

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List of Acronyms

BPSK Binary Phase-Shift Keying

BSC Binary Symmetric Channel

CC Centroid Condition

DFC Discrete Fading Channel

DJSCC Distributed Joint Source-Channel Coding

DMC Discrete Memoryless Channel

FBC Folded Binary Code

I.I.D. Independent and Identically Distributed

JSCC Joint Source-Channel Coding

LBG-VQ Linde-Buzo-Gray Vector Quantizer

MAC Multiple access channels

MAP Maximum A Posteriori

MMSE Minimum Mean Square Error

MSE Mean Square Error

NBNDC Non-Binary Noise Discrete Channel

NBNDC-QB Non-Binary Noise Discrete Channel with Queue Based noise

NNC Nearest Neighbor Condition

SDR Signal-to-Distortion Ratio

SER Symbol Error Rate

SNR Signal-to-Distortion Ratio

SQ Scalar Quantizer

VQ Vector Quantizer

Chapter 1

Introduction

1.1 Communication System Models

In a communication system, the source is usually data or a multimedia signal modelled as a random process which can be discrete (finite or countable alphabet) or continuous (uncountable alphabet) in value and in time. Continuous time sources are often sampled to form discrete time sources which are more efficient to work with. In a general digital communication system, some processing (called encoding) takes place on the source data to make it suitable for transmission. In the receiver part, the reverse operation (called decoding) must be done to recover the original information (or its approximation) and deliver it to the destination. Fig. 1.1 depicts a simple block diagram of a *tandem coding system* in which the encoding/decoding process is performed in two separately designed steps, referred to as source coding/decoding and channel coding/decoding. In order to have an efficient transmission under restricted bandwidth and storage capacity, the source encoder attempts to remove the unnecessary or redundant content in the source and represent it in a compressed

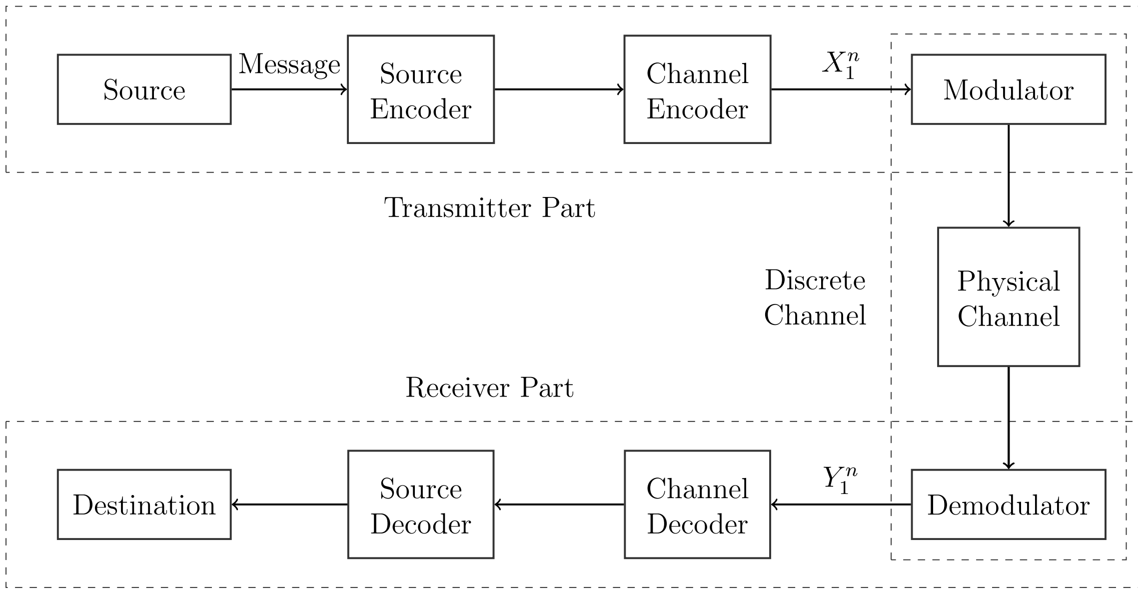


Figure 1.1: Block diagram of a general point-to-point digital communication system with tandem source-channel coding.

format. The compressed information at the output of the source encoder is more vulnerable to the errors caused by the channel noise. The channel encoder enables detection and/or correction of errors which results in a reliable reproduction of the source encoder outputs after transmission through a noisy communication channel by adding controlled redundancy (usually using an algebraic structure). The resulting digital (discrete time, discrete value) signal needs to be sent over the physical channel which is a noisy (or unreliable) medium that is only capable of transmitting analog signals. The modulator transforms the channel encoder output into a waveform suitable for transmission, typically by varying the parameters of a sinusoidal signal in proportion with the digital data. At the receiver side, the demodulator converts the received analog signal to a digital signal which goes thorough the channel decoder and the source decoder to retrieve or approximate the original message produced by the source.

In a tandem coding system, the source coder is designed under the assumption of perfect error correction by the channel coder and the channel coder is designed for an ideal source coder with uniformly distributed outputs. The optimality of the separate design of the source and the channel coder is justified by Shannon's source-channel separation theorem, with the use of asymptotically long block lengths of data in the coding procedures [30]. Hence, in many practical applications with delay and complexity constraints, tandem coding is not optimal [30, 31]. Furthermore, Shannon's theorem states that in a single-user system as long as the entropy rate of the source is less than the capacity of the channel a coding/decoding scheme can be found to provide a lossless communication and the coding design can be done in two separate steps (source coding and channel coding) without loss of optimality. However, for a multi-user system this theorem does not hold anymore and jointly designing the source and channel codes is the only reliable solution. Thus, various schemes of *joint source-channel coding* (JSCC) have been developed to address this problem. A generic point-to-point joint source-channel coding system is shown in Fig. 1.2.

1.2 Background and Literature Review

JSCC can be more advantageous than tandem coding in many situations. In [39], it is information theoretically proved that the error exponent for JSCC can be twice as large as the exponent for separate source and channel coding; this implies that JSCC would need half the (encoding and decoding) delay of separate coding to achieve the same overall probability of error and consequently a 2-dB power saving is realized in a wide class of source-channel pairs. In [14], new tight finite-blocklength bounds for

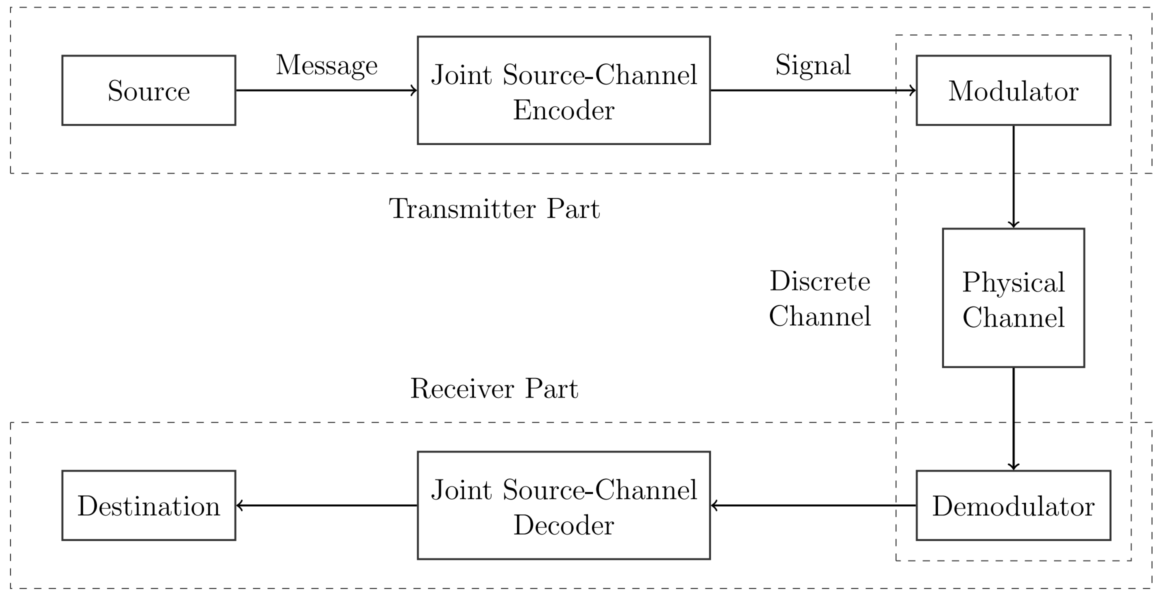


Figure 1.2: Block diagram of a general point-to-point digital communication system with joint source-channel coding.

the best achievable lossy joint source-channel code rate are derived, demonstrating that joint source-channel code design can bring considerable performance advantage over separate code design one in the nonasymptotic regime. Some more advantages of JSCC over separate source-channel coding have been quantitatively characterized in [16]. Hence, JSCC has a great potential to be adopted in practical wireless communications systems. JSCC can be done using a variety of different methods. One approach is to design a JSCC system based on lossy coding which is resilient against channel noise; see [2–4,9,10,21,23,27,29,33], and other works. Some of these works focus on decreasing complexity and delay on the transmitter side by designing a simple lossy encoder and a more complex decoder. For example, in [29], sequence maximum a posteriori (MAP) decoding is studied for a system with no algebraic channel coding and channel interleaving. It is demonstrated that the residual redundancy in the source (in the form of non-uniform distribution and/or memory) and the channel noise can be used to improve performance in terms of the signal-to-distortion ratio (SDR). It is also observed in [29] that using the channel's soft-decision information can result in significant SDR gain over hard-decision decoding, which is in line with other works showing that soft-decision decoding can increase channel capacity and system performance (e.g., see [3, 22, 32, 35]).

Furthermore, JSCC has applications in common communication systems with multiple access channels (MAC), where multiple users are sending over a shared medium (such as an uplink channel). Some works consider JSCC for users sending over a MAC without using any multiplexing. For example, [15] and [11] investigate JSCC for transmission of two correlated Gaussian memoryless sources over a Gaussian MAC. However, in practice usually orthogonal multiple access schemes (such as orthogonal frequency-division multiple access and code division multiple access) are used. A MAC consisting of orthogonal sub-channels is a suitable model for this situation. Moreover, orthogonals MACs can model distributed transmission of nodes to a common destination. Hence, studying a practical JSCC problem on the orthogonal MAC is interesting, although little work has been done in this area. [40] proposes a distributed joint source-channel coding (DJSCC) scheme for multiple correlated sources.

1.3 Thesis Contribution

In this thesis, we extend the results of [29], where a single-user system was considered, and study the JSCC problem of sending two correlated Gaussian sources over an orthogonal MAC. A practical situation where two sensors separately measure a pair of correlated parameters, such as temperature and humidity, and send them to a fusion center is one motivation for considering such a problem. Our MAC channel is defined by two orthogonal sub-channels. Each sub-channel is a point-to-point correlated Rayleigh discrete fading channel (DFC) used with antipodal signaling and q-bit output quantization. However, as the correlated Rayleigh DFC is hard to treat analytically [24, 29], we will instead use the recently introduced non-binary noise discrete channel with queue based noise (NBNDC-QB) which has been shown to efficiently model such DFCs [25,26]. The NBNDC-QB is a binary input 2^q-ary output channel with 2^q-ary stationary ergodic Mth order Markov noise.

We design a joint sequence MAP decoder (which is optimal in terms of sequence error probability) and implement it using a modified version of the Viterbi algorithm. Numerical results confirm that our joint MAP decoder can well exploit of the statistics of the correlated sources in addition to the channel's soft-decision information and statistical memory and thus produce better SDR. Our main theoretical contribution is an easy-to-check analytical condition in terms of the sources and channel parameters, under which the costly delay-prone joint MAP decoder can be replaced by a straightforward instantaneous decoder of identical performance.

1.4 Thesis Overview

In Chapter 2, we give an overview of digital communication channel models and describe our MAC channel model by introducing the Rayleigh DFC and the NBNDC-QB models. At the end of the chapter, we briefly review source coding theory. In Chapter 3, we design a joint sequence MAP decoder for independent and identically distributed (i.i.d.) correlated analog-valued Gaussian sources which are scalar quantized and sent over a MAC with orthogonal NBNDC-QB sub-channels. For the case of hard (2-level) quantized sources and the NBNDC-QB sub-channels with memory order $M = 1$, we find a necessary and sufficient condition under which the sequence MAP detector can be replaced with an instantaneous symbol-by-symbol mapping. We numerically illustrate the condition in our theorem and also evaluate the performance of the system in terms of SDR to show how well it can exploit the correlation between sources, memory in the channel noise, and soft-decision information. Furthermore, we numerically validate the NBNDC-QB model as an effective approximation of the Rayleigh DFC in the proposed system. In Chapter 4, the same JSCC problem is studied for a system with memory in which the correlation between sources is a two state Markov process. For the special case of correlated binary sources, we establish a necessary and a sufficient condition under which the joint MAP decoder can be simplified to the symbol-by-symbol decoder. The theoretical results are numerically illustrated and the system SDR performance is assessed. Finally, the conclusion and ideas for future works are presented in Chapter 5.

Chapter 2

Preliminaries

2.1 Communication Channel Models

In this thesis, we focus on digital communication systems. As it can be seen from Fig. 1.1, the data symbols at the input of the modulator and the output of the demodulator are discrete. Hence, the concatenation of the modulator, physical channel, and demodulator can be modeled via a discrete channel with a given sequence of conditional (or transition) probability distributions of receiving an output given that a specific input was sent. A discrete channel is characterized by a finite input alphabet $\mathcal{X}$ and a finite output alphabet $\mathcal{Y}$ and a sequence of n-dimensional transition distributions $\{P_{Y^n|X^n}(Y_1^n = y_1^n|X_1^n = x_1^n)\}_{n=1}^{\infty}$, where $x_1^n \triangleq (x_1, x_2, \ldots, x_n) \in \mathcal{X}^N$ is the n-tuple input and $y_1^n \triangleq (y_1, y_2, \ldots, y_n) \in \mathcal{Y}^N$ is the n-tuple received output.

2.1.1 Discrete Memoryless Channels

A discrete memoryless channel (DMC) is a discrete channel which is fully described by the channel transition matrix $Q \triangleq \left[P_{Y|X}(y|x)\right]$ of size $|\mathcal{X}| \times |\mathcal{Y}|$, where $x \in \mathcal{X}$ and

$y \in \mathcal{Y}$. For every $n = 1, 2, \ldots$, the transition probabilities are as follows

$$P_{Y^n|X^n}(Y_1^n = y_1^n | X_1^n = x_1^n) = \prod_{i=1}^{n} P_{Y|X}(y_i|x_i). \tag{2.1}$$

Equation (2.1) implies that the current output Y_n only depends on the current input X_n and is independent of the past inputs X_1^{n-1} and outputs Y_1^{n-1}. Furthermore, the past outputs Y_1^{n-1} do not depend on the current input X_n. Also, given the past inputs X_1^{n-1}, the current input X_n is independent of the past outputs Y_1^{n-1}. Binary symmetric channels and binary erasure channels are well-known examples of DMCs.

2.1.2 Discrete Channels with Memory

Discrete channels with memory model practical situations in which errors tend to occur in bursts rather than independently. In these channels, each output symbol depends statistically both on the current input and on the past inputs and outputs (with the assumption that the current output is independent of future inputs, given current input and the input and output histories) [12].

Markov channel models are able to characterize fairly well the complex physical phenomena in discrete channels with memory. For example, the binary Markov noise channel models a channel with memory described by an additive Markov noise process as follows

$$Y_k = X_k \oplus Z_k, \qquad k = 1, 2, 3, \ldots \tag{2.2}$$

where X_k and Y_k are the input and output at time k and $\oplus$ represents modulo 2 addition. $\{Z_k\}_{k=1}^{\infty}$ is a binary stationary ergodic Markov noise process with memory order M, independent of the input and produced by a 2^M by 2^M transition matrix which in general is characterized by 2^M independent parameters. The transition matrix describes the process for $k > M$, where the corresponding states are defined as $S_k = (z_{k-M+1}, \ldots, z_k)$. As a result each row of the transition matrix has only two nonzero element which justifies the number of independent parameters. This model may result in excessive complexity when the memory order is high.

In [1], a finite memory contagion urn model is proposed for generating the noise process. Since the noise process is independent of the input, the transition distributions can be written as

$$\begin{aligned} P\left(Y_k = y_k | X_1^k = x_1^k, Y_1^{k-1} = y_1^{k-1}\right) &= P\left(Z_k = z_k | Z_{k-M}^{k-1} = z_{k-M}^{k-1}\right) \\ &= P\left(Z_k = z_k \;\middle|\; \sum_{i=k-M}^{k-1} Z_i = \sum_{i=k-M}^{k-1} z_i\right) \\ &= \frac{\rho + \delta\left(\sum_{i=k-M}^{k-1} z_i\right)}{1 + M\delta}, \end{aligned} \tag{2.3}$$

where $z_i^k \triangleq (z_i, z_{i+1}, \ldots, z_k)$, $\rho = P[Z_i = 1]$ is the channel bit error rate (BER) and δ is a correlation parameter. As illustrated in [1], this model can be fully described using only three parameters: memory order M, channel BER ρ, and noise correlation coefficient $Cor = \delta/(\delta + 1)$.

Rayleigh discrete fading channel

The single-user Rayleigh discrete fading channel (DFC), shown in Fig. 2.1, is a binary-input and 2^q-ary output channel defined as follows. First, a binary phase-shift keying (BPSK) modulator takes the DFC's binary input process $\{X_k\}_{k=1}^{\infty}$, $X_k \in \mathcal{X} = \{0, 1\}$, and generates $S_k = 2X_k - 1 \in \{-1, 1\}$ for $k = 1, 2, \ldots$. Then, the modulated signal S_k is transmitted over a time-correlated flat Rayleigh fading channel with additive

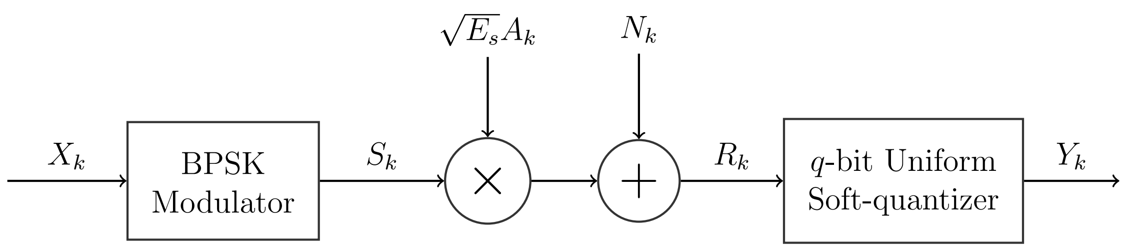


Figure 2.1: Rayleigh discrete fading channel.

white Gaussian noise which produces the output

$$R_k = \sqrt{E_s} A_k S_k + N_k, \qquad k = 1, 2, \ldots,$$

where E_s is the energy of signal sent over the channel, and the additive noise $\{N_k\}_{k=1}^{\infty}$ is a sequence of independent and identically distributed (i.i.d.) Gaussian random variables of variance $N_0/2$. Furthermore, $\{A_k\}$ is the channel's Rayleigh fading process (assumed to be independent of $\{N_k\}$ and the input process) with $A_k = |G_k|$, where $\{G_k\}$ is a time-correlated complex wide-sense stationary Gaussian process with Clarke's autocorrelation function [6] given as $R[k] = J_0(2\pi f_D T|k|)$, where $J_0(.)$ is the zeroth-order Bessel function of the first kind and $f_D T$ is the maximum Doppler frequency normalized by the signaling rate $1/T$. As a result, each fading random variable A_k is Rayleigh distributed with unit second moment which causes an attenuation in the signal. Finally, a soft-decision demodulator consisting of a q-bit uniform quantizer with step size Δ processes the output R_k and produces the DFC's channel output $Y_k \in \mathcal{Y} = \{0, 1, \ldots 2^q - 1\}$ given by

$$Y_k = j, \qquad \text{if} \quad R_k \in (T'_{j-1}, T'_j],$$

where $j \in \mathcal{Y}$ and the uniformly spaced thresholds T'_j satisfy

$$T'_j = \begin{cases} -\infty, & \text{if} \quad j = -1 \\ (j+1-2^{q-1})\Delta, & \text{if} \quad j = 0, 1, \ldots, 2^q - 2 \\ \infty, & \text{if} \quad j = 2^q - 1 \end{cases} .$$

Defining $\delta \triangleq \Delta/\sqrt{E_s}$ and $T_j \triangleq T'_j/\sqrt{E_s}$ as the normalized step-size and thresholds, we have $T_j = (j+1-2^{q-1})\delta$, for $j = 0, 1, \ldots, 2^q - 2$. The DFC's conditional probabilities $q_{x_k,y_k}(a_k) \triangleq \Pr(Y_k = y_k | X_k = x_k, A_k = a_k)$ can be found as follows

$$\begin{aligned} q_{x_k,y_k}(a_k) &= \Pr(T_{y_k-1} < R_k < T_{y_k} | X_k = x_k, A_k = a_k) \\ &= \Pr(T_{y_k-1} - (2x_k - 1)a_k < \frac{N_k}{\sqrt{E_s}} < T_{y_k} - (2x_k - 1)a_k) \\ &= Q\Big(\sqrt{2\,\mathrm{SNR}}(T_{y_k-1} - (2x_k - 1)a_k)\Big) - Q\Big(\sqrt{2\,\mathrm{SNR}}(T_{y_k} - (2x_k - 1)a_k)\Big), \end{aligned} \tag{2.4}$$

where SNR $= E_s/N_0$ is the Rayleigh DFC's signal-to-noise ratio (SNR) and $Q(x) = 1/\sqrt{2\pi}\int_x^\infty \exp\{-t^2/2\}dt$ is the Gaussian Q-function. Because of the symmetry in the quantizer thresholds and the BPSK constellation, it can be observed from (2.4) that

$$q_{x_k,y_k}(a_k) = q_{1-x_k,2^q-1-y_k} = q_{0,\frac{y_k-(2^q-1)x_k}{(-1)^{x_k}}}(a_k). \tag{2.5}$$

For integer $n \geq 1$, the n-fold transition probability of the DFC can be calculated via [26]

$$P_{DFC}^{(n)}(y_1^n | x_1^n) \triangleq \Pr\{Y_1^n = y_1^n | X_1^n = x_1^n\} = \mathbf{E}_{A_1 \ldots A_n}\Big[\prod_{k=1}^{n} q_{x_k,y_k}(A_k)\Big], \tag{2.6}$$

where $\mathbf{E}_X[.]$ denotes expectation with respect to the random variable X. Unfortunately, $P_{DFC}^{(n)}(y_1^n | x_1^n)$ can be expressed in closed form only for $n \leq 3$; as the joint probability density function of arbitrarily correlated Rayleigh random variable, which is required for direct calculation of the expected value in (2.6), is only known in closed form for $n \leq 3$ [5]. Consider $n = 1$ as an example. The closed-form expression for (2.6) is given by [34]

$$P^{(1)}_{DFC}(y|x) = P^{(1)}_{DFC}(j) = \eta(-T_{j-1}) - \eta(-T_j), \tag{2.7}$$

where $x \in \mathcal{X}$, $y \in \mathcal{Y}$, $j = \frac{y-(2^q-1)x}{(-1)^x} \in \mathcal{Y}$, and

$$\eta(T_j) = 1 - Q(T_j\sqrt{2\ \mathrm{SNR}}) - \frac{\left[1 - Q\left(\frac{T_j\sqrt{2\ \mathrm{SNR}}}{\sqrt{\frac{1}{\mathrm{SNR}}+1}}\right)\right] e^{-\frac{T_j^2}{\frac{1}{\mathrm{SNR}}+1}}}{\sqrt{\frac{1}{\mathrm{SNR}}+1}}. \tag{2.8}$$

For $n > 3$, the expected value in (2.6) can only be found numerically. Therefore, the non-binary noise discrete channel with queue based noise (NBNDC-QB) is introduced as a more tractable alternative model for the DFC.

Non-Binary Noise Discrete Channel

The queue-based channel (QBC) is introduced in [38] to model a binary additive noise communication channel with memory. The channel noise process is a stationary ergodic Mth-order Markov source which is generated according to a ball sampling mechanism involving a queue of finite length M. This model has evolved from the binary additive communication channel with memory introduced in [1], in which the noise process is based on the contagion model of G. Polya. Numerical results in [38] indicate that the QBC provides a good approximation of the well-known Gilbert Elliott channel while remaining mathematically tractable. Finally, the idea of increasing the capacity by softly quantizing the channel output has led to the NBNDC-QB model, introduced in [26].

The non-binary noise discrete channel is a binary-input and 2^q-ary-output channel model which is described by

$$Y_k = (2^q - 1)X_k + (-1)^{X_k} Z_k, \qquad k = 1, 2, \ldots, \tag{2.9}$$

where $q \geq 1$ is an integer, $X_k \in \{0, 1\}$ is the input data bit, $Y_k \in \mathcal{Y} = \{0, 1, \ldots 2^q - 1\}$ is the channel output, and $Z_k \in \mathcal{Y}$ is the corresponding noise symbol which is assumed to be independent of the input. From (2.9), we can write Z_k in terms of the corresponding input and output symbols,

$$Z_k = \frac{Y_k - (2^q - 1)X_k}{(-1)^{X_k}}, \qquad k = 1, 2, \ldots, \tag{2.10}$$

The noise process $\{Z_k\}_{k=1}^{\infty}$ can in general be any stochastic process such as a binary stationary memoryless process ($q = 1$) which reduces the NBNDC to a binary symmetric channel (BSC). In this thesis, $\{Z_k\}$ is considered to be a generalization of the queue-based (QB) noise introduced in [38]. Consequently, the non-binary noise discrete channel with queue based noise is referred to as NBNDC-QB [26]. The noise model is a 2^q-ary stationary and ergodic M^{th}-order Markov process which can be described using only $2^q + 2$ independent parameters (typically, $q = 2$ or 3 for most systems; hence, the exponential complexity in q is not a concern): the memory order M, the marginal probability distribution $(\rho_0, \rho_1, \ldots, \rho_{2^q-1})$, and correlation parameters $0 \leq \epsilon < 1$ and $\alpha \geq 0$. Since the NBNDC model's complexity is determined

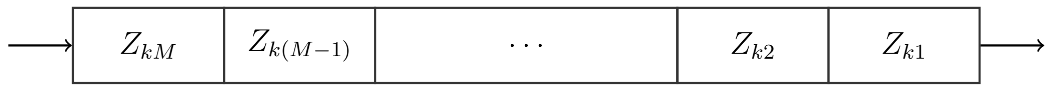


Figure 2.2: A queue of length M.

by the number of model parameters which is independent of the channel noise memory order M, we can implement noise models with arbitrarily large memory orders without adding further complexity. Here, we present the generating procedure of the queue-based non-binary noise process. At each sample time k, one of the two following generating mechanisms is selected via flipping a biased coin. Assume that the first mechanism is selected with probability ϵ. For this case, we have a queue of M balls labeled with error symbols $Z_{kj} \in \mathcal{Y}$ as shown in Fig. 2.2; where $k \geq 1$ is the experiment time index and $j = 1, 2, \ldots, M$ specifies the location in the queue. From the queue, a ball is chosen randomly with the probability of the j-th ball being selected given as

$$\begin{cases} \dfrac{1}{M-1+\alpha}, & \text{if} \quad j = 1, 2, \ldots, M-1; \\ \dfrac{\alpha}{M-1+\alpha}, & \text{if} \quad j = M. \end{cases}, \qquad \alpha \geq 0.$$

In the second mechanism, executed with probability of $1-\epsilon$, a ball is taken out from an urn containing a very large number of balls labeled with symbols in $\mathcal{Y}$ in a way that the probability distribution $(\rho_0, \rho_1, \ldots, \rho_{2^q-1})$ is satisfied; i.e., with probability $1-\epsilon$, the noise symbol Z_k is independent of the past noise symbols and is picked according to $\Pr\{Z_k = j\} = \rho_j, j \in \mathcal{Y}$. Assume that z_k is the noise symbol generated according to the explained procedure. Before producing symbol z_{k+1}, a ball with the symbol z_k on it will be pushed into the queue, pushing out the ball labeled z_{k1}. This procedure implies that the probability of $Z_k = j, j \in \mathcal{Y}$ depends on the bias parameter α and increases in proportion to the number of times that the value j occurred in the past M noise symbols. As in [38] and [26], the cell bias parameter is set to $\alpha = 1$ when $M = 1$ (i.e., the queue has only one cell).

The state process $\{S_k\}$ of the QB noise, defined by $S_k = (Z_k, Z_{k-1}, \ldots, Z_{k-M+1})$ for $k \geq M$, is a homogeneous first-order Markov process taking values in $\{0, 1, \ldots, 2^q - 1\}^M$. The noise state transition probability is defined as

$$Q(s_k|s_{k-1}) \triangleq \Pr\{S_k = s_k | S_{k-1} = s_{k-1}\},$$

where $s_k = (z_k, z_{k-1}, \ldots, z_{k-M+1})$ and $s_{k-1} = (z'_k, z'_{k-1}, \ldots, z'_{k-M+1})$. It is shown in [26] that for $k \geq M + 1$,

$$Q(s_k|s_{k-1}) = \begin{cases} \left(\sum_{\ell=1}^{M-1} \delta_{z_k, z_{k-\ell}} + \alpha \delta_{z_k, z_{k-M}} \right) \dfrac{\epsilon}{M - 1 + \alpha} + (1 - \epsilon)\rho_{z_k} & \\ \quad \text{if} \quad z_{\ell-1} = z'_\ell \quad \text{for } \ell = k, \ldots, k - M + 2 & \\ & \\ 0 \qquad\qquad \text{otherwise} & \end{cases} \tag{2.11}$$

where $\delta_{i,i'} = 1$ if $i = i'$ and $\delta_{i,i'} = 0$ if $i \neq i'$, and $\sum_{\ell=1}^{0} \triangleq 0$.

The n-fold channel transition probabilities are

$$\Pr\{Z_1^n = z_1^n\} = \Pr\{Y_1^n = y_1^n | X_1^n = x_1^n\} \triangleq P_{QB}^{(n)}(z_1^n), \tag{2.12}$$

where y_1^n is the output sequence, x_1^n the input sequence, and $z_1^n = (z_1, \ldots, z_n)$ is the sequence of corresponding noise symbols related to x_1^n and y_1^n according to (2.10). $P_{QB}^{(n)}(z_1^n)$ can be determined as follows [26]:

- For $n \leq M$:

$$P_{QB}^{(n)}(z_1^n) = \frac{\prod_{\ell=0}^{2^q-1} \prod_{j=0}^{\xi_\ell - 1} \left((1-\epsilon)\rho_\ell + j\frac{\epsilon}{M-1+\alpha} \right)}{\prod_{k=0}^{n-1} \left((1-\epsilon) + k\frac{\epsilon}{M-1+\alpha} \right)} \tag{2.13}$$

where $\prod_{k=0}^{-1}(.) = 1$ and $\xi_\ell = \sum_{k=1}^{n} \delta_{z_k,\ell}$. As a result, for $n = 1$, $P_{QB}^{(1)}(z_1) = \rho_{z_1}$ for all $z_1 \in \mathcal{Y}$; and also for $n = M$, we get the stationary distribution components of the Markov process $\{S_k\}$ as $\pi_{(z_1,z_2,\ldots,z_M)} = P_{QB}^{(M)}(z_1^M)$.

- For $n > M$:

$$\begin{aligned} P_{QB}^{(n)}(z_1^n) = \prod_{i=M+1}^{n} &\left[\left(\sum_{\ell=i-M+1}^{i-1} \delta_{z_i,z_\ell} + \alpha\delta_{z_i,z_{iM}} \right) \right. \\ &\times \left. \frac{\epsilon}{M-1+\alpha} + (1-\epsilon)\rho_{z_i} \right] \pi_{(z_1,z_2,\ldots,z_M)}. \end{aligned} \tag{2.14}$$

The correlation coefficient for the NBNDC-QB noise is a non-negative quantity given by

$$Cor = \frac{E[Z_k Z_{k+1}] - E[Z_k]^2}{Var(Z_k)} = \frac{\frac{\epsilon}{M-1+\alpha}}{1-(M-2+\alpha)\frac{\epsilon}{M-1+\alpha}}, \tag{2.15}$$

where $Var(Z_k)$ denotes the variance of Z_k.

Fitting the NBNDC-QB model to the Raleigh DFC model

The introduced NBNDC-QB model can mimic the statistical behavior of the Rayleigh DFC channels which are hard to treat analytically. Hence, the NBNDC-QB model must be fitted to a given Rayleigh DFC with fixed parameters $(\mathrm{SNR}, q, \delta, f_D T)$ via the following steps [26].

- The QB noise one-dimensional probability distributions are matched to the first order statistics of the the underlying fading channel by setting $\rho_j = P^{(1)}_{DFC}(j)$ for $j \in \mathcal{Y}$. Noting (2.7) in computing $P^{(1)}_{DFC}(j)$, it can be seen that ρ_j is a function of δ, q, and SNR. In [29, Table I], corresponding ρ_j's for some typical values of the Rayleigh DFC parameters are presented.

- The DFC noise correlation coefficients, calculated using $P^{(n)}_{DFC}(y_1^n|x_1^n)$ in (2.6) with $n = 2$, is matched to the noise correlation coefficient of the NBNDC-QB model. Hence, α can be written in terms of M and ϵ.

- Finally, the remaining QB parameters (M, ϵ) are estimated by minimizing the Kullback-Leibler divergence rate between the two (2^q-ary) noise processes. We can observe that the memory and correlation parameters (M, ϵ, α) are coupled with $f_D T$.

The values of the fitting NBNDC-QB parameters obtained as outlined above are given in [26, Table II] for different DFC's. Notice that since the first order statistics of the NBNDC-QB and the underlying fading channel are guaranteed to be matched, for the memoryless case (with $Cor = 0$), the NBNDC-QB is statistically identical to the ideally interleaved DFC (by ideal interleaving, we mean that the block interleaver has sufficiently large depth compared with the transmission block length [17, 36]).

2.1.3 Multiple Access Channels

Intuitively the MAC is a multi-point communication medium via which two (or more) senders transmit information to a common receiver. In general, the source symbols from all users will interfere with each other during transmission and a single symbol is received at the output of the MAC. Hence, channel coding is a common method to protect the data against user interference and noise of the channel. Fig. 2.3 shows a general discrete memoryless MAC with two users and input alphabets $\mathcal{X}_1, \mathcal{X}_2$, output alphabet $\mathcal{Y}$ and a probability transition matrix $P(y \mid x_1, x_2)$. $W_1 \in \mathcal{W}_1 = \{1, 2, \ldots, 2^{nR_1}\}$ and $W_2 \in \mathcal{W}_2 = \{1, 2, \ldots, 2^{nR_2}\}$ are the source mes-

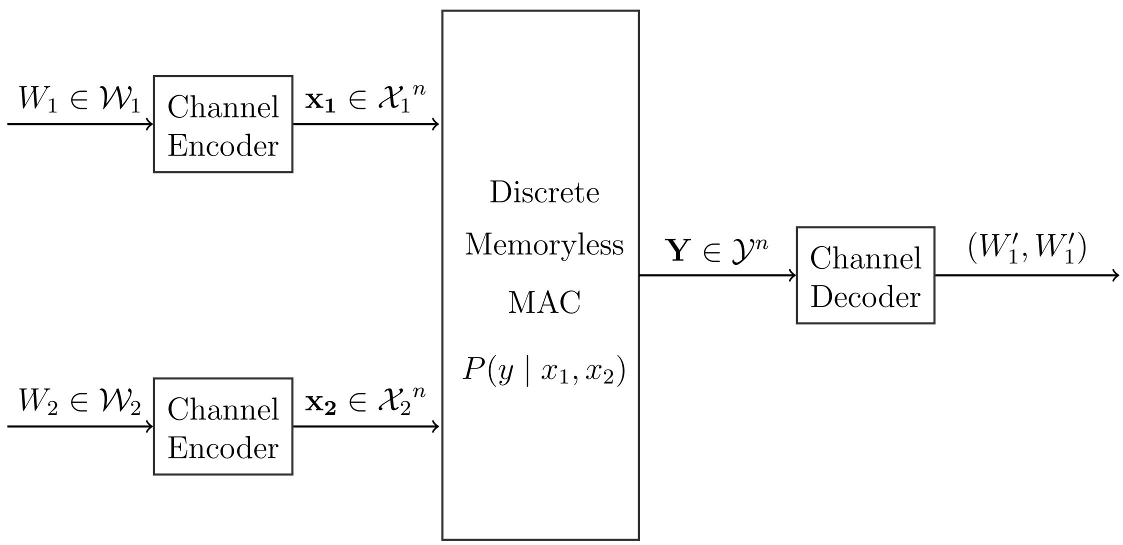


Figure 2.3: The discrete memoryless MAC model

sages which can be considered as outputs of ideal source encoders. Also, R_1 and R_2 are the corresponding channel encoder rates given in bits per channel input symbol. Recovering original messages using only channel codes is very difficult due to the user interference in the channel; hence, in many practical communication systems where the available channel bandwidth must be efficiently shared among several users, various orthogonal multiple access schemes such as frequency division multiple access, time division multiple access, and code division multiple access are employed to avoid unrecoverable collision of messages from different users. Fig. 2.4 represents an orthogonal MAC channel with two users. An orthogonal MAC channel is also

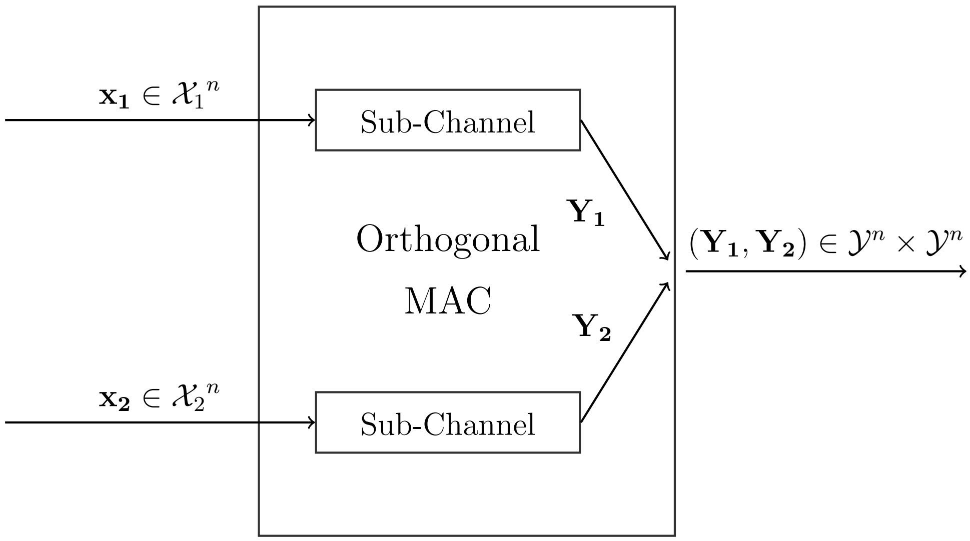


Figure 2.4: The orthogonal MAC model with two independent sub-channels

suitable for the distributed source coding (DSC) problem where multiple correlated information sources are compressed and independently transmitted without any inter-communication. In many applications such as sensor networks and video/multimedia compression, senders have complexity constrains. DSC aims to reduce the complexity at the transmitter by using the correlation between multiple sources in the design of a joint decoder which can carry the computational burden. This motivates us to consider an orthogonal MAC channel consisting of two independent single-user Rayleigh DFC sub-channels which are modeled via NBNDC-QB channels.

2.2 Source Coding and Quantization

The first step in the encoding process of a tandem coding system is source coding in which the source information is compressed as much as possible by eliminating the redundancy in the source symbols. Source coding can be divided into two categories; lossless and lossy source coding. In lossless source coding, the sequence of source symbols can be completely recovered from the compressed data while in lossy source coding, source data is reconstructed within some distortion. Shannon's lossless source coding theorem (or noiseless coding theorem) shows that there exists a lossless fixed-to-variable length source coding which can achieve any code rate (average number of coded symbols per source symbol) greater than or equal to the Shannon entropy of the source. Conversely, compressing the data with a code rate less than the Shannon entropy results in inevitable data loss and the probability of decoding error goes arbitrarily close to one, for sufficiently large source blocks [30].

For a discrete source X with alphabet $\mathcal{X} = \{0, 1, \ldots, N-1\}$, the entropy rate $H_\infty(\mathcal{X})$ which represents the amount of uncertainty or information in the source, is defined as

$$H_\infty(\mathcal{X}) = \lim_{n\to\infty} \frac{1}{n}(-\mathbf{E}_{X_1,X_2,\ldots,X_n}[\log p(X_1, X_2, \ldots, X_n)]),$$

where $\{X_i\}, i = 1, 2, \ldots$ is the source stochastic process and $p(X_1, X_2, \ldots, X_n) \triangleq \Pr\{X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n\}$. Setting the base of the logarithms to 2, the unit of these measures is expressed in bits. It can be shown that

$$H_\infty(\mathcal{X}) \leq \log_2 N,$$

where N is the size of the source alphabet.

The redundant information in a source can be due to its memory (ρ_M) or non-uniformity of its marginal probability distribution (ρ_D). For a discrete source $\{X_i\}, i =$

$1, 2, \ldots$ with alphabet size N, the total redundancy ρ_T can be written as

$$\rho_T = \rho_D + \rho_M,$$

where

$$\rho_D = \log_2 N - H(X_1),$$
$$\rho_M = H(X_1) - H_\infty(\mathcal{X}),$$

and

$$H(X_1) = -\sum_{x_1 \in \mathcal{X}} p(x_1) \log p(x_1) = -E_{X_1}[\log p(X_1)].$$

For continuous sources, the entropy rate $H_\infty(\mathcal{X})$ is theoretically infinite which indicates that compressing a continuous source without incurring any loss or distortion indeed requires an infinite number of bits. Thus, lossy source coding is the only practical solution. The corresponding process is called quantization in which the analog source symbols are mapped to discrete (digital) symbols from a finite alphabet at the cost of some distortion with respect to the original source.

In general, a quantizer partitions the continuous domain of the analog source into a finite number of regions and represents all the members of each region with a value called output level or reconstruction point. The set of output levels is known as the codebook.

A scalar quantizer (SQ) takes only one source symbol at a time. In this thesis, the SQ is assumed to produce a sequence of binary outputs for each input source symbol; hence, a SQ with the rate R bits/sample has 2^R output levels.

On the other hand, a vector quantizer (VQ) with rate R bits/sample accepts k

source symbols at a time and maps them to a binary sequence of length 2^{Rk} representing the corresponding output level.

The quality of the quantizer Q applied to a random source X is measured via the expected distortion defined as

$$D(Q) = E[d(X, Q(X))], \tag{2.16}$$

where $Q(X)$ is the quantized value and $d : \mathbb{R} \times \mathbb{R} \to [0, \infty)$ is the distortion measure; typically considered to be the rth power of the magnitude error $d(X, \hat{X}) = |x, \hat{x}|^r, r > 0$. For the popular square error distortion, $E[d(X, Q(X))]$ is called mean square error (MSE) expected distortion and is calculated via $D(Q) = E[d(X, Q(X))] = E[(X - Q(X))^2]$.

The optimum quantizer (encoder) and dequantizer (decoder), in terms of minimizing the expected distortion, must satisfy the following necessary conditions.

- The centroid condition (CC): Given the output levels or partitions of the quantizer, the reconstruction point corresponding to each region must be set as the centroid of the part of source that lies in that region. This choice minimizes the conditional expected distortion over the assignment region [7, p. 303].

- The Nearest neighbor condition (NNC): Given the reproduction points (codebook), the optimum encoder selects the partition boundaries in a way that all the values being mapped to a reproduction point have the minimum distortion with respect to it; i.e., each x is mapped to its "nearest" reproduction point [13, p. 176-185].

The Lloyd-Max algorithm is an iterative algorithm to design an optimal SQ - called the Lloyd-Max quantizer [19]. It repeatedly applies the CC and NNC conditions. For a given codebook, the partition set is optimized according to the NNC. For the resulting partition, the optimum codebook is found according to the CC. Since the distortion is nonnegative and each iteration either reduces the distortion or leaves it unchanged, the sequence of distortions produced by Lloyd-Max algorithm finally converges. The initial codebook is usually selected by the splitting algorithm, [8, 18] which quickly converges into a well-designed final quantizer. The generalized Lloyd algorithm, also known as Linde-Buzo-Gray vector quantizer algorithm (LBG-VQ) can be used for VQ design [18].

Chapter 3

Joint Sequence MAP Decoding of I.I.D. Correlated Sources over the Orthogonal MAC

In this chapter, we study a source-channel decoding scheme which is designed to take advantage of the channel memory. We extend the work in [28], where a single-user system was considered. In particular, we examine the joint sequence MAP decoding problem for two quantized sources transmitted over an orthogonal MAC with underlying NBNDC-QB sub-channels modeling time-correlated Rayleigh DFCs (as described in Chapter 2). Each real-valued source is followed by a SQ designed for a noiseless channel. The SQ output is passed through an index assignment mapping and then sent over one of the underlying sub-channels. The sub-channel outputs are soft-demodulated with resolutions q and q', respectively, (which in general can be different) and fed to a joint sequence MAP detector to combat channel errors. We refer to such a coding scheme as SQ-MAC-MAP. The reason for choosing scalar instead of vector quantization is the ability of former to preserve more redundancy in the index codewords at the quantizer output, which can be later used, in conjunction with channel's characteristics, by the joint MAP decoder for the purpose of robust error correction. We note, however, that the more complex VQ may result in better SDR performance because of the space-filling gain [20].

It is important to mention that the SQ-MAC-MAP scheme is designed to minimize the sequence error probability, while we evaluate its performance in terms of SDR with the MSE distortion measure. Hence, the SQ-MAC-MAP is not necessarily optimal in terms of achieving minimum mean square error (MMSE). MMSE optimal and suboptimal MAP decoding metrics are studied in [21, 33]. However, according to simulations results, our system improves SDR performance by exploiting residual source redundancy as well as noise correlation and soft-decision information of the NBNDC model. We also numerically observe that an orthogonal MAC with NBNDC-QB sub-channels can effectively model Rayleigh DFC sub-channels when measured in terms of joint SDR performance under some conditions. Furthermore, we prove a theorem for a specific case of our system setup (SQ-MAC-MAP with the NBNDC-QB sub-channels), in which we provide necessary and sufficient condition for a joint sequence MAP decoder to be replaceable with simple instantaneous (symbol-by-symbol) decoding rules, hence significantly reducing decoder delay.

3.1 System setup

Consider the communication system depicted in Fig. 3.1. Two correlated zero-mean and unit-variance Gaussian sources $\mathcal{V}$ and $\mathcal{V}'$ generate a sequence of input pairs $\{(V_i, {V'}_i)\}_{i=1}^{\infty}$ which are i.i.d. real-valued samples taken according to the bivariate

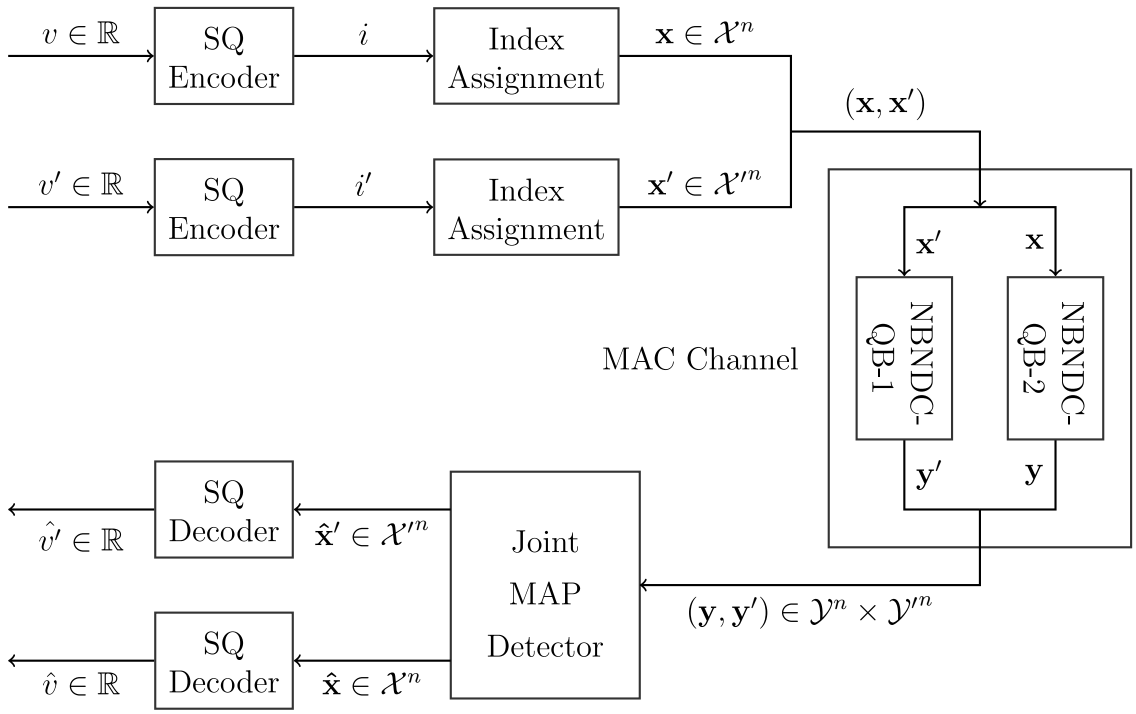


Figure 3.1: Block diagram of a JSCC system using scalar quantization and joint MAP decoder over an orthogonal MAP channel with memory.

normal density

$$f_{V,V'}(v, v') = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{v^2 + v'^2 - 2\rho v v'}{2(1-\rho^2)}\right), \tag{3.1}$$

where $-1 \leq \rho \leq 1$ is the correlation between the two sources.

The above system is a generalization of the single-user system presented in [29]. The output samples of the first source are encoded using a rate-n SQ. The SQ utilizes the Lloyd-Max algorithm [19], with the initial codebook selection obtained via the splitting algorithm [18] and produces an index $i \in \{0, 1, \ldots, 2^n - 1\}$. As explained in [29], because of its simplicity and good performance, the folded binary code (FBC) [23] is chosen as the one-to-one index assignment method to map the index i to a binary vector $\mathbf{x} \in \{0,1\}^n$. The same encoding process is separately done for the second source which results in the codeword $\mathbf{x}' \in \{0,1\}^n$. Then, the vector pair $(\mathbf{x}, \mathbf{x}')$ is transmitted through the orthogonal Rayleigh DFC MAC channel and the corresponding vectors $\mathbf{y} \in \mathcal{Y}^n = \{0, 1, \dots, 2^q - 1\}^n$ and $\mathbf{y}' \in \mathcal{Y}'^n = \{0, 1, \dots, 2^{q'} - 1\}^n$ are received. This communication is modeled as sending the n-tuple codeword $\mathbf{x}$ bit-by-bit over the first NBNDC-QB sub-channel with 2^q-ary noise symbols $z \in \mathcal{Y} = \{0, 1, \dots, 2^q - 1\}$ and noise memory M which will result in the output sequence $\mathbf{y}$. Similarly, $\mathbf{x}'$ and $\mathbf{y}'$ are the input and output vectors of the second NBNDC-QB sub-channel with $2^{q'}$-ary noise symbols $z' \in \mathcal{Y}' = \{0, 1, \dots, 2^{q'} - 1\}$ and noise memory M'. At the receiver side, the MAC channel's output $(\mathbf{y}, \mathbf{y}')$ is fed to a joint MAP decoder. Finally, two SQ decoders map the decoder outputs $(\hat{\mathbf{x}}, \hat{\mathbf{x}}')$ into output levels of the quantizer codebook.

It can be observed that in the described system, which is referred to as the SQ-MAC-MAP system, the receiver carries most of the complexity load.

3.2 Joint MAP decoder design

The residual redundancy of the source and channel statistics can be harnessed by a MAP decoder which is designed to minimize the sequence error probability [23]. In general, the total redundancy ρ_T can be written as $\rho_T = \rho_D + \rho_M$ where ρ_D is the redundancy due to the non-uniformity of the distribution and ρ_M denotes the redundancy in the form of memory. Since the input sequence $\{(V_i, V'_i)\}$ is an i.i.d. process, the SQ encoder output process $\{(\mathbf{X}_i, \mathbf{X}'_i)\}$ is also i.i.d.; which implies that $\rho_M = 0$ and as a result the only source of redundancy comes from the non-uniformity of the encoders' outputs and the correlation between two sources.

Suppose that each source produces N symbols. The sequence $(\mathbf{x},\mathbf{x}')^N = \big((\mathbf{x}_1,\mathbf{x}'_1),\ldots,(\mathbf{x}_N,\mathbf{x}'_N)\big) \in \big(\{0,1\}\times\{0,1\}\big)^{nN}$ at the output of the SQ encoders is transmitted over the MAC channel in nN channel uses. The independent NBNDC-QB sub-channels contaminate the bit streams related to the first and second source with noise sequences $z_1^{nN} \in \mathcal{Y}^{nN}$ and $z'^{nN}_1 \in \mathcal{Y}'^{nN}$, respectively. In other words, the input n-tuples x_{i+1} and x'_{i+1}, $i = 0,1,\ldots,N-1$, are transmitted in a bit-by-bit fashion over the first and second sub-channels with the corresponding noise symbols $(z_{ni+1}, z_{ni+1},\ldots,z_{n(i+1)})$ and $(z'_{ni+1}, z'_{ni+1},\ldots,z'_{n(i+1)})$ which will result in the output n-tuples y_{i+1} and y'_{i+1}. Receiving the channel output $(\mathbf{y},\mathbf{y}')^N = \big((\mathbf{y}_1,\mathbf{y}'_1),\ldots,(\mathbf{y}_N,\mathbf{y}'_N)\big) \in (\mathcal{Y}\times\mathcal{Y}')^{nN}$, the MAP decoder estimates $(\mathbf{x},\mathbf{x}')^N$ by $(\hat{\mathbf{x}},\hat{\mathbf{x}}')^N$ as

$$
\begin{aligned}
(\hat{\mathbf{x}},\hat{\mathbf{x}}')^N &= \arg\max_{(\mathbf{x},\mathbf{x}')^N} \big(\Pr\{(\mathbf{X},\mathbf{X}')^N = (\mathbf{x},\mathbf{x}')^N | (\mathbf{Y},\mathbf{Y}')^N = (\mathbf{y},\mathbf{y}')^N\}\big) \\
&= \arg\max_{(\mathbf{x},\mathbf{x}')^N} \big(\Pr\{(\mathbf{Y},\mathbf{Y}')^N = (\mathbf{y},\mathbf{y}')^N | (\mathbf{X},\mathbf{X}')^N = (\mathbf{x},\mathbf{x}')^N\} \\
&\qquad\qquad \times \Pr\{(\mathbf{X},\mathbf{X}')^N = (\mathbf{x},\mathbf{x}')^N\}\big) \\
&= \arg\max_{(\mathbf{x},\mathbf{x}')^N} \big(\Pr\{\mathbf{Y}^N = \mathbf{y}^N | \mathbf{X}^N = \mathbf{x}^N\} \Pr\{\mathbf{Y}'^N = \mathbf{y}'^N | \mathbf{X}'^N = \mathbf{x}'^N\} \\
&\qquad\qquad \times \Pr\{(\mathbf{X},\mathbf{X}')^N = (\mathbf{x},\mathbf{x}')^N\}\big) \\
&= \arg\max_{(\mathbf{x},\mathbf{x}')^N} \big(\Pr\{Z_1^{nN} = z_1^{nN}\} \Pr\{Z'^{nN}_1 = z'^{nN}_1\} \times \Pr\{(\mathbf{X},\mathbf{X}')^N = (\mathbf{x},\mathbf{x}')^N\}\big) \\
&= \arg\max_{(\mathbf{x},\mathbf{x}')^N} \Bigg(P_{QB}^{(n)}(z_1^n) P'^{(n)}_{QB}(z'^n_1) P(\mathbf{x}_1,\mathbf{x}'_1) \\
&\qquad\qquad \times \prod_{i=1}^{N-1} \Big(Q(z_{in+1}^{(i+1)n} | z_1^{in}) Q'(z'^{(i+1)n}_{in+1} | z'^{in}_1) P(\mathbf{x}_{i+1},\mathbf{x}'_{i+1}) \Big) \Bigg),
\end{aligned} \tag{3.2}
$$

where the third equation comes from the orthogonality of the two sub-channels.

$P(\mathbf{x}_{i+1}, \mathbf{x'}_{i+1}) \triangleq P(\mathbf{X}_{i+1} = \mathbf{x}_{i+1}, \mathbf{X'}_{i+1} = \mathbf{x'}_{i+1})$ is the joint probability distribution for the pair of n-tuple codewords $(\mathbf{X}_{i+1}, \mathbf{X'}_{i+1})$. For $i = 1, 2, \ldots, nN$, the noise symbols z_i and z'_i can be found applying (2.10) separately to each sub-channel input and output. For the first sub-channel, the QB noise block probability $P_{QB}^{(n)}(z_1^n)$ can be calculated via (2.13) or (2.14) and the noise transition probabilities in the last line of (3.2) are defined based on $Q(z_{i+1}^{i+j}|z_{i-k}^i) \triangleq \Pr\{Z_{i+1}^{i+j} = z_{i+1}^{i+j}|Z_{i-k}^i = z_{i-k}^i\}$, where $i, j, k \in \{1, 2, \ldots, nN-1\}, i+j \leq nN, i-k \geq 1$. Note that $z_i \triangleq 0$ if $i < 1$, $z_i^j \triangleq (z_i, z_{i+1}, \ldots, z_j), j \geq i$. For the second sub-channel, $Q(z'^{i+j}_{i+1}|z'^{i}_{i-k})$ and $P'^{(n)}_{QB}(z'^n_1)$ are defined and calculated similarly using the parameters associated with this channel.

Assuming $nN \geq \max\{M, M'\}$ (which typically holds as large values of N are usually used in practice), since the noise memory order in the first and second sub-channel is respectively M and M', we can write

$$(\mathbf{x}, \mathbf{x'})^N = \arg\max_{(\mathbf{x},\mathbf{x'})^N} \Big\{ \log \left[P_{QB}^{(n)}(z_1^n) P'^{(n)}_{QB}(z'^n_1) P(\mathbf{x}_1, \mathbf{x'}_1)\right] + \sum_{i=1}^{N-1} \log \left[Q(z_{in+1}^{(i+1)n}|z_{in-(M-1)}^{in}) Q'(z'^{(i+1)n}_{in+1}|z'^{in}_{in-(M'-1)}) P(\mathbf{x}_{i+1}, \mathbf{x'}_{i+1})\right] \Big\}, \tag{3.3}$$

where $Q(z_{in+1}^{(i+1)n}|z_{in-(M-1)}^{in})$ and $Q'(z'^{(i+1)n}_{in+1}|z'^{in}_{in-(M'-1)})$ can be calculated via the following equation, by selecting $j = in$ and considering the parameters of each sub-channel, [29]:

$$Q(z_{j+1}^{j+n}|z_{j-(M-1)}^j) = \prod_{k=j+1}^{j+n} \left[\left(\sum_{\ell=k-(M-1)}^{k-1} \delta_{z_k, z_\ell} + \alpha \delta_{z_k, z_{k-M}} \right) \times \frac{\epsilon}{M-1+\alpha} + (1-\epsilon)\rho_{z_k} \right]. \tag{3.4}$$

To implement the MAP decoder, we employ a modified version of the Viterbi algorithm similar to the one used in [28]. The corresponding trellis consists of $4^{(kn)}$ states, the set of all possible pairs of kn-tuple codewords, where k is the smallest integer which satisfies $kn \geq \max\{M, M'\}$. In the trellis, each state has $2^{(kn-M+1)} \times 2^{(kn-M'+1)}$ incoming and 4^n outgoing branches and the path metric at step i is as follows:

$$\log\left[Q(z_{in+1}^{(i+1)n}|z_{in-(M-1)}^{in})Q'({z'}_{in+1}^{(i+1)n}|{z'}_{in-(M'-1)}^{in})\right] + \log\left[P(\mathbf{x}_{i+1}, \mathbf{x'}_{i+1})\right].$$

Applying the Viterbi algorithm, the MAP decoder needs to observe the entire received sequence before deciding on the most likely message words, which results in significant decoding delay as well as storage complexity of order $\mathcal{O}(nN4^{(kn)})$ that increases with the length of the sequence. Thus it is interesting to investigate situations where MAP decoding can be replaced by a simple and fast instantaneous (symbol-by-symbol) decoding rule which exhibits the same performance in terms of symbol error rate (SER).

3.3 Case study: joint MAP detection of binary sources

For the single-user case and for a binary symmetric Markov source transmitted over a NBNDC-QB channel with noise memory $M = 1$, [29] establishes a necessary and sufficient condition under which an instantaneous symbol-by-symbol decoder can function as the MAP decoder.

In our MAC problem, we consider the special case where both correlated sources are separately quantized to two levels ($n = 1$). Using the symmetry in the joint distribution (3.1), the following equations hold for the joint distribution of the resulting binary sources:

$$P(1,1) = P(0,0) \qquad P(1,0) = P(0,1). \tag{3.5}$$

This is shown as follows. Since the marginal distribution for sources V and V' is zero-mean normal Gaussian, each scalar quantizer, using Lloyd Max algorithm with $n = 1$, selects zero as the quantization threshold. Hence, we can write

$$\begin{aligned} P(0,0) &= \Pr\{V < 0, V' < 0\} = \int_{-\infty}^{0}\int_{-\infty}^{0} f_{V,V'}(v,v')dvdv', \\ P(1,1) &= \Pr\{V \geq 0, V' \geq 0\} = \int_{0}^{\infty}\int_{0}^{\infty} f_{V,V'}(v,v')dvdv'. \end{aligned} \tag{3.6}$$

Since the density function $f_{V,V'}(v,v')$ is even, we have

$$P(0,0) = P(1,1). \tag{3.7}$$

Similarly, we can write

$$\begin{aligned} P(0,1) &= \Pr\{V < 0, V' \geq 0\} = \int_{-\infty}^{0}\int_{0}^{\infty} f_{V,V'}(v,v')dvdv', \\ P(1,0) &= \Pr\{V \geq 0, V' < 0\} = \int_{0}^{\infty}\int_{-\infty}^{0} f_{V,V'}(v,v')dvdv'. \end{aligned} \tag{3.8}$$

Since $f_{V,V'}(v,v')$ is symmetric, we can switch the role of v and v' which results in

$$P(0,1) = P(1,0). \tag{3.9}$$

3.3.1 Instantaneous symbol-by-symbol decoding rule

We next present our instantaneous symbol-by-symbol decoder. By making use of the orthogonality of the MAC channel, equation (3.5), and the assumption $\rho_0 \geq \rho_1 \geq \rho_2 \geq \cdots \geq \rho_{2^q-1}$ for each NBNDC-QB sub-channel, we will show that for the case of binary sources, the same function θ introduced in [29] can be adapted to map a 2^q-ary ($q \geq 1$) output y_i of the NBNDC-QB channel to a binary symbol $\tilde{y}_i$. In fact, among all mappings $\theta : \mathcal{Y} \mapsto \{0, 1\}$, the following mapping θ^* minimizes the symbol error probability for each sub-channel:

$$\theta^*(y_i) = \tilde{y}_i = \begin{cases} 0, & \text{if } y_i < 2^{q-1} \\ 1, & \text{otherwise} \end{cases} \quad ; 0 \leq i \leq N. \tag{3.10}$$

Lemma 3.1. *For the problem of two correlated binary sources (X, X'), with the joint distribution $P(x, x')$, sent over an orthogonal MAC consisting of two independent single-user NBNDC-QB channels, assume the output sequences are instantaneously decoded as $(\tilde{y}, \tilde{y}')^N = (\theta^*(y), \theta'^*(y'))^N$, where the mapping functions (θ^*, θ'^*) are applied component-wise to each output pair (y_i, y'_i).*

Consider the first NBNDC-QB sub-channel has the noise parameters satisfying

$$\rho_0 \geq \rho_1 \geq \rho_2 \geq \cdots \geq \rho_{2^q-1}. \tag{3.11}$$

Among all mappings $\theta : \mathcal{Y} \mapsto \{0, 1\}$, where $\mathcal{Y} = \{0, 1, \ldots 2^q - 1\}$, the following *mapping $\theta^\star$ yields the lowest symbol probability of error defined as* $\Pr(\tilde{y} \neq x)$

$$\theta^\star(y_i) = \tilde{y}_i = \begin{cases} 0, & \text{if } y_i < k^* \\ 1, & \text{otherwise} \end{cases}, \tag{3.12}$$

where $k^* \in \{0, 1, \ldots, 2^q\}$ is the smallest value satisfying

$$\frac{\rho_{k^\star}}{\rho_{2^q-k^*-1}} \leq \frac{P_X(1)}{P_X(0)}, \tag{3.13}$$

and $\rho_{-1} \triangleq \infty$, $\rho_{2^q} \triangleq 0$, and $P_X(x) \triangleq \sum_{x' \in \mathcal{X}'} P(x, x')$ is the marginal distribution of the first source (similarly, $P_{X'}(x') \triangleq \sum_{x \in \mathcal{X}} P(x, x')$).

Considering the second NBNDC-QB sub-channel, the instantaneous mapping function $\theta'^* : \mathcal{Y}' = \{0, 1, \ldots 2^{q'} - 1\} \mapsto \{0, 1\}$ will have the same format as (3.12) with the parameters of the second source and the second sub-channel being used.

Having two correlated memoryless Gaussian sources with zero-means and unit-variances and quantizing each with a two level LloydMax quantizer ($n = 1$), we showed that the joint distribution of the resulted binary sources will have the symmetry

$$P(1, 1) = P(0, 0), \qquad P(1, 0) = P(0, 1). \tag{3.14}$$

Having assumption (3.14), it can be seen from (3.13) that $k^* = 2^{(q-1)}$ and $k'^* = 2^{(q'-1)}$. Note that for $q = 1$ and $q' = 1$, the binary output sequences can be accepted without any further processing by decoder ("decode-what-you-see").

Proof of Lemma 3.1. Considering the first NBNDC-QB sub-channel with input $X \in \mathcal{X} = \{0, 1\}$ and output $Y \in \mathcal{Y} = \{0, 1, \ldots, 2^q - 1\}$, we will show that any mapping $\theta : \mathcal{Y} \mapsto \{0, 1\}$ can be transformed to θ^*, presented in (3.12), through a finite sequence of simple modifications, none of which can increase the error probability. Hence, the mapping θ^* has the minimum error probability.

In general, a mapping function $\theta : \mathcal{Y} \mapsto \{0,1\}$ is a classification rule that classifies 2^q different output symbols from $\{0,1,\ldots,2^q-1\}$ into two classes $\mathcal{Y}_0$ and $\mathcal{Y}_1$. Thus θ is defined by

$$\theta(y) = \tilde{y} = \begin{cases} 0, & \text{if } y \in \mathcal{Y}_0 \\ 1, & \text{if } y \in \mathcal{Y}_1, \end{cases} \tag{3.15}$$

where $\mathcal{Y}_1 \subset \mathcal{Y}$ and $\mathcal{Y}_0 = \mathcal{Y} \setminus \mathcal{Y}_1$.

Having (3.12) for θ^*, we can write $\mathcal{Y}_0^* = \{0,1,\ldots,k^*-1\}$ and $\mathcal{Y}_0^* = \{k^*, k^*+1,\ldots,2^q-1\}$. The error probability under mapping θ is defined as $P_e \triangleq \Pr\{\theta(Y) \neq X\}$, where (X,Y) has the common joint distribution of the pairs (X_i,Y_i). In other words,

$$\begin{aligned} P_e &= \Pr\{Y \in \mathcal{Y}_0 | X=1\} Pr\{X=1\} + \Pr\{Y \in \mathcal{Y}_1 | X=1\} \Pr\{X=0\} \\ &= \Pr\{X=1\} \sum_{y \in \mathcal{Y}_0} \Pr\{y|X=1\} + \Pr\{X=0\} \sum_{y \in \mathcal{Y}_1} \Pr\{y|X=0\}. \end{aligned} \tag{3.16}$$

If $\theta \neq \theta^*$, one of the two following cases happen:

i) There exists an element $a \in \mathcal{Y}_1$, such that $a < k^*$. Removing a from $\mathcal{Y}_1$ and adding it to $\mathcal{Y}_0$ yields a mapping with error probability $\tilde{P}_e$, such that

$$
\begin{aligned}
&\tilde{P}_e - P_e = \\
&Pr\{Y = a|X = 1, X' = 0\}P(1,0) + Pr\{Y = a|X = 1, X' = 1\}P(1,1) \\
&- Pr\{Y = a|X = 0, X' = 0\}P(0,0) - Pr\{Y = a|X = 0, X' = 1\}P(0,1) \\
&= Pr\{Y = a|X = 1\}\left(P(1,0) + P(1,1)\right) \\
&\quad - Pr\{Y = a|X = 0\}\left(P(0,0) + P(0,1)\right) \\
&= Q(2^q - 1 - a)\left(P(1,0) + P(1,1)\right) - Q(a)\left(P(0,0) + P(0,1)\right) \\
&= \rho_{2^q-1-a}\left(P(1,0) + P(1,1)\right) - \rho_a\left(P(0,0) + P(0,1)\right),
\end{aligned}
\tag{3.17}
$$

where the second equality follows from the fact that the two sub-channels are orthogonal; as a result the output Y is independent of the input X' (similarly, Y' is independent of X). According to (3.11), $\rho_a \geq \rho_{k^*-1}$ and $\rho_{2^q-k^*} \geq \rho_{2^q-1-a}$. Hence by (3.13),

$$\frac{P(1,0) + P(1,1)}{P(0,0) + P(0,1)} \leq \frac{\rho_{k^*-1}}{\rho_{2^q-k^*}} \leq \frac{\rho_a}{\rho_{2^q-1-a}} \tag{3.18}$$

and therefore $\tilde{P}_e - P_e \leq 0$. Thus, removing a from $\mathcal{Y}_1$, and adding it to $\mathcal{Y}_0$ does not increase the error probability.

ii) There exists an element $b \in \mathcal{Y}_0$, such that $b \geq k^*$. Removing b from $\mathcal{Y}_0$ and adding it to $\mathcal{Y}_1$, similar to (3.17), it can be shown that

$$\tilde{P}_e - P_e = -\rho_{2^q-1-b}\left(P(1,0) + P(1,1)\right) + \rho_b\left(P(0,0) + P(0,1)\right). \tag{3.19}$$

According to (3.11), $\rho_b \le \rho_{k^*}$ and $\rho_{2^q-k^*-1} \le \rho_{2^q-1-b}$. Hence by (3.13),

$$\frac{P(1,0)+P(1,1)}{P(0,0)+P(0,1)} \ge \frac{\rho_{k^*}}{\rho_{2^q-k^*-1}} \ge \frac{\rho_b}{\rho_{2^q-1-b}} \tag{3.20}$$

and thus $\tilde{P}_e - P_e \le 0$. Hence, removing b from $\mathcal{Y}_0$, and adding it to $\mathcal{Y}_1$ does not increase the error probability.

It can be concluded that, for any given mapping θ, by applying at most $2^q - 1$ replacement steps, as described above, θ^* can be obtained. Since each step either decreases or does not change the error probability, θ^* is the optimal symbol-by-symbol decoder in the sense of minimizing the error probability of decoding the first source bits. This completes the proof.

For the other sub-channel, considering its characteristics, the same proof holds with only swapping the role of X and X' (for example, $P(1,0)$ and $P(0,1)$ will be swapped in the previous equations).

■

Note that we independently apply the same function (3.10) to y and y', the demodulated outputs of the orthogonal MAC, and acquire binary symbols $\tilde{y}$ and $\tilde{y}'$, respectively. Since the parameters of the NBNDC-QB sub-channels can be different, we denote the first instantaneous decoder by θ^* and the second by θ'^* with the q in (3.10) changed to q'. Hence, a joint symbol (y_i, y_i') is decoded correctly when $(\tilde{y}_i, \tilde{y}_i') = (x_i, x_i')$.

3.3.2 Equivalence between joint MAP and instantaneous decoding

The following result presents a necessary and sufficient condition for the mappings (θ^*, θ'^*) to form an optimal sequence detection rule in the sense of minimizing the sequence error probability. In this case, the MAP decoder is unnecessary and can be replaced by the singlet decoders (θ^*, θ'^*), without increasing the error probability.

Theorem 3.1. *Consider two correlated memoryless binary sources having joint distribution $P(x, x')$ with the symmetry assumption (3.5) and an orthogonal MAC channel consisting of two independent NBNDC-QB sub-channels where the first one has the correlation parameter $\epsilon \geq 0$, memory order $M = 1$, $q \geq 1$, and a noise one-dimensional probability distribution satisfying $\rho_0 \geq \rho_1 \geq \rho_2 \geq \cdots \geq \rho_{2^q-1}$. Similarly, assume that in the second channel $\epsilon' \geq 0, q' \geq 1, M' = 1$, and $\rho'_0 \geq \rho'_1 \geq \rho'_2 \geq \cdots \geq \rho'_{2^{q'}-1}$. Let $(x, x')^N$ be a source sequence of length $N \geq 2$, $(y, y')^N$ the channel output sequence, and let $(\tilde{y}, \tilde{y}')^N = (\theta^*(y), \theta'^*(y'))^N$ be obtained by applying the mapping functions component-wise to the corresponding output sequences of the underlying channels.*

Then, decoding as $(\hat{x}, \hat{x}')^N = (\tilde{y}, \tilde{y}')^N$ is an optimal sequence MAP detection rule for all possible received sequences if

$$\min\left\{\left(\frac{P(0,0)}{\frac{1}{2} - P(0,0)}\right), \left(\frac{\frac{1}{2} - P(0,0)}{P(0,0)}\right)\right\} A \geq 1, \tag{3.21}$$

where

$$A = \min\left\{\frac{\epsilon' + (1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon' + (1-\epsilon')\rho'_{2^{q'}-1}}, \frac{\epsilon + (1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon + (1-\epsilon)\rho_{2^{q}-1}}\right\}. \tag{3.22}$$

Conversely, if (3.21) does not hold, then for all N large enough there is at least one *sequence for which* $(\hat{x}, \hat{x}')^N = (\tilde{y}, \tilde{y}')^N$ *is not an optimal sequence MAP detection rule. A necessary condition which holds for any* $N \geq 1$ *is as follows*

$$\min\left\{\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}, \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}}\right\} \min\left\{\left(\frac{P(0,0)}{\frac{1}{2}-P(0,0)}\right), \left(\frac{\frac{1}{2}-P(0,0)}{P(0,0)}\right)\right\} \geq 1. \tag{3.23}$$

Proof. See Appendix A □

It can be observed that, if (3.23) does not hold, we have

$$\begin{aligned} &A \min\left\{\left(\frac{P(0,0)}{\frac{1}{2}-P(0,0)}\right), \left(\frac{\frac{1}{2}-P(0,0)}{P(0,0)}\right)\right\} \leq \\ &\min\left\{\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}, \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}}\right\} \min\left\{\left(\frac{P(0,0)}{\frac{1}{2}-P(0,0)}\right), \left(\frac{\frac{1}{2}-P(0,0)}{P(0,0)}\right)\right\} < 1, \end{aligned} \tag{3.24}$$

which implies that (3.23) is a loose necessary condition for the case of $N \to \infty$.

In order to illustrate Theorem 3.1, we have simulated the system under various channel and source conditions, by generating $N = 10^5$ independent samples of two correlated binary sources according to a joint distribution satisfying (3.5). These binary sources can represent two correlated Gaussian sources quantized with two level quantizers ($n = 1$). Each simulation is repeated 10 times and the average joint symbol error rate is computed to ensure the results are consistent. Denoting the left-hand term of (3.21) by C, when $C \geq 1$ it can be observed from Tables 3.1-3.3 that the performance of the instantaneous decoding (θ^*, θ'^*) and the joint MAP decoder are identical, while for $C < 1$ the joint MAP decoder outperforms the instantaneous decoder. In Table 3.1. (a), we present results for the case when the MAC's sub-channels have identical parameters and in Table 3.1. (b) to (e) we use sub-channels with different parameters in order to further illustrate the theoretical result.

Theorem 3.1 states that when $C < 1$, there exists an input-output sequence pair for which the joint MAP decoder performs better than instantaneous decoders; this may not hold for all input-output sequences. As a result, the tables are just illustrations of Theorem 3.1 and not a verification. Looking at the tables, we find situations where $C < 1$ and the instantaneous decoders still perform as well as the joint MAP decoder. It can be seen that as C approaches one, the chance of getting equal joint SER increases. Note that the joint distribution between two binary sources can be defined using only one parameter $0 \leq P(0,0) \leq 0.5$, where $P(0,0) = 0.25$ represents two independent sources and moving away from this value increases the correlation between the sources. As explained, for a given $P(0,0)$, there exists a corresponding pair of correlated Gaussian sources with correlation parameter ρ which will result in joint binary sources with the same joint distribution if quantized with two level quantizers. Hence, we use $P(0,0)$ and ρ interchangeably.

Table 3.1 presents simulation results for joint MAP decoding and symbol-by-symbol decoding when two correlated binary sources are transmitted over an orthogonal MAC consisting of two sub-channels, one is fully interleaved ($Cor = 0.0$) and the other one has a high noise correlation. Simulation results when both sub-channels have high noise correlations has been shown in Table 3.2; it can be observed that, due to the fact that both sources have the same marginal distribution and the noise correlations in the both sub-channels are identical, there is a symmetry between the situations (SNR_1, SNR'_1, q_1, q'_1) and (SNR_2, SNR'_2, q_2, q'_2), where $\text{SNR}_1 = \text{SNR}'_2$, $\text{SNR}'_1 = \text{SNR}_2$, $q_1 = q'_2$ and $q'_1 = q_2$. In Table 3.3, we present simulation results corresponding to the case when a fully-interleaved memoryless channel ($Cor = Cor' = 0$) is used. Finally, in Table 3.4, we use sub-channels with $(Cor, Cor') = (2.5 \times 10^{-3}, 0.5)$

in order to further illustrate the theoretical results.

Looking at Tables 3.1-3.3, the following observations can be made.

- The joint SER of the instantaneous decoder (θ, θ') does not change significantly with (q, q'), (Cor, Cor') and ρ; however, it increases when lowering (SNR, SNR$'$); i.e., with noisier sub-channels. This behavior can be intuitively explained by writing the definition of SER and noting that $\rho_0 + \cdots + \rho_{2^{q-1}-1} = \rho_{2^{q-1}} + \cdots + \rho_{2^q}$ for the marginal distributions given in [29, TABLE I].

- In general, the joint SER of the MAP decoder improves when the parameters (q, q'), (Cor, Cor'), (SNR, SNR$'$) and ρ are increased. This implies that the joint MAP decoder is taking advantage of these parameters to decode the outputs.

- The results show that (q, q'), (Cor, Cor'), and ρ constructively contribute in helping the joint MAP decoder to combat channel errors; i.e., each individual parameter is more effective in the presence of other helpful parameters with high values. Furthermore, increasing these parameters makes more significant improvements in the sub-channels with low SNR.

- The improvement of the joint MAP SER with increasing (SNR, SNR$'$) is more visible when the parameters (q, q'), (Cor, Cor'), and ρ are small.

- When (SNR $\leq$ SNR$'$), comparing the effect of q and q' through the results of Tables 3.1-3.3 (b) and (e), we can conclude that it is usually more beneficial, in terms of the joint MAP SER improvement, to increase q instead of q'. However, in the case when the sources are not highly correlated to each other and sub-channels have $Cor < Cor'$, increasing q' results in slightly better results.

- When (SNR $>$ SNR$'$), according to Tables 3.1-3.3 (d), it can be observed that increasing q' instead of q usually results in a better joint SER improvements.
- It can be observed from Tables 3.1-3.3 (c) to (e) that having sub-channels with $Cor < Cor'$, increasing SNR rather than SNR$'$ (by investing power in the corresponding source) has more significant effect on improving the joint SER. When $Cor = Cor'$, improving the SNR of the sub-channel with less resolution leads to better results. Furthermore, the joint SER improvement is more visible when two sources are highly correlated.

3.4 Simulation Results

3.4.1 SQ-MAC-MAP system simulation

We next simulate the SQ-MAC-MAP system for sending two correlated Gaussian sources (generated by (3.1)). First, the SQ is designed and the joint distribution of the sources $P(X, X')$ is calculated using a training set of 10^6 paired source symbols. Then, 10^5 source symbols are transmitted for simulation and the average SDR with the mean square error distortion is measured after repeating each simulation 10 times (for getting consistent results).

Table 3.5 shows the average SDR (in dB) in the SQ-MAC-MAP system simulation, where SDR is defined as

$$SDR \triangleq \frac{\sum_{i=1}^{2} E[(\mathbf{X}_i)^2]}{\sum_{i=1}^{2} E[(\mathbf{X}_i - \hat{\mathbf{X}}_i)^2]}. \tag{3.25}$$

It can be verified that in general the system performs better with highly correlated sources and high noise correlation which means that the joint MAP decoder successfully exploits the channel noise correlation and the correlation between sources. For example, when the correlation between sources is high (0.81), a 5.4 dB (at $q = 2, n = 3$, SNR = 2) SDR gain is achieved by having a system with high noise correlation Cor (instead of a fully-interleaved memoryless channel); also, in such a system, increasing the correlation between sources (from -0.31 to 0.81) leads to significant improvements as high as 4.2 dB (at $q = 2, n = 3$, SNR = 2) in SDR.

Furthermore, it can be observed that incorporating more soft-decision information has a positive effect on the performance of the system under joint MAP decoding. For example using a 3-bit soft-decision quantizer rather a hard-decision quantizer ($q = 1$) results in a 4.28 dB gain (at $n = 3$, SNR = 2, and $\rho = 0.81$).

Considering Table 3.6, increasing soft-decision information and also the correlation between the sources do not have any significant effect on the performance of the instantaneous decoder. These results are predictable because according to (3.10), for $0 \leq i \leq N$, the outputs of the instantaneous symbol-by-symbol decoder $(\theta^*(y_i), \theta'^*(y'_i))$ can be written as functions of R_i and R'_i, the unquantized outputs of the Rayleigh fading underlying sub-channels

$$\tilde{y}_i = \begin{cases} 0, & \text{if } R_i \leq 0 \\ 1, & \text{otherwise} \end{cases}, \qquad \tilde{y}'_i = \begin{cases} 0, & \text{if } R'_i \leq 0 \\ 1, & \text{otherwise} \end{cases}, \tag{3.26}$$

which shows no dependence on q, q', and $P(x, x')$.

Furthermore, considering a system with a 2-level quantizer ($n = 1$) and a fully interleaved channel ($Cor = 0$), we can verify Theorem 3.1 by comparing Tables 3.2

and 3.3 and observing that whenever $C \geq 1$ in Table 3.1, the instantaneous symbol-by-symbol decoder is performing as well as the joint MAP decoder.

The joint SDR (for both joint MAP decoder and symbol-by-symbol decoder) of a system with 2-level quantizers ($n = 1$) shows the same behavior as its joint SER which we examined in the previous section. Hence, we only fully present the results for a system with 4-level quantizers ($n = 2$), simulated under various conditions; see Tables 3.7 and 3.8 . It can be observed that all the arguments regarding the joint MAP SER of the binary input system also hold for the join MAP SDR of a system with more quantizer levels ($n = 2$). Unlike the system with binary sources, the SDR results of the instantaneous decoder improve with the increase in the noise correlations of the sub-channels. Intuitively, this is due to having symbols which are made of n bits ($n > 1$) and have the higher probability of being received correctly because of the correlation between the bits. It is also observed that increasing the noise correlation in the sub-channel with lower SNR results in a more significant SDR improvement.

Furthermore, looking at Table 3.5, we observe that there are situations when increasing the sub-channels noise correlations does not increase the SDR of the binary system ($n = 1$). This behavior can be explained by looking at the corresponding SER performance. If $C \geq 1$ for two sets of source-channel parameters that only differ in the value of Cor, the joint MAP decoder becomes useless and can be replaced by the instantaneous decoder whose performance does not change with Cor. As already noted, the joint MAP decoder is not optimal in terms of SDR; as a result, the SDR performance of the binary system may decrease if increasing Cor results in $C < 1$.

To further illustrate this issue, we rewrite (3.21). Without loss of generality, assume that

$$\min\left\{\frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}}},\frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}}\right\}=\frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}},$$

and let

$$B\triangleq\min\left\{\left(\frac{P(0,0)}{\frac{1}{2}-P(0,0)}\right),\left(\frac{\frac{1}{2}-P(0,0)}{P(0,0)}\right)\right\}.$$

As a result, (3.21) can be written as

$$\frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}}B\geq 1. \tag{3.27}$$

Thus, noting that $Cor=\epsilon$ (since $M=\alpha=1$ in the sub-channels), (3.27) implies that if

$$\max\{Cor,Cor'\}\leq\min\left\{\frac{\rho_{2^{q-1}}-B\rho_{2^{q-1}-1}}{B(1-\rho_{2^{q-1}-1})+\rho_{2^{q-1}}-1},\frac{\rho'_{2^{q'-1}}-B\rho'_{2^{q'-1}-1}}{B(1-\rho'_{2^{q'-1}-1})+\rho'_{2^{q'-1}}-1}\right\}, \tag{3.28}$$

changing the sub-channels noise correlations does not change the system SDR. Note that for (3.28) to hold, since the denominator is always non-positive; the numerator must be non-positive (i.e., $B\geq\frac{\rho_{2^{q-1}}}{\rho_{2^{q-1}-1}}$ or $B\geq\frac{\rho'_{2^{q'-1}}}{\rho'_{2^{q'-1}-1}}$).

3.4.2 Validating the NBNDC-QB model for the orthogonal Rayleigh discrete fading MAC

Although it has been shown in [29] that NBNDC-QB is a good model for a point-to-point time-correlated Rayleigh DFC in terms of the MAP decoder SDR, it is interesting to verify that the NBNDC-QB model can effectively represent the time-correlated Rayleigh DFC sub-channels of an orthogonal MAC in terms of the joint SDR performance of the SQ-MAC-MAP system. As described in Section 3.2, we design the joint MAP decoder based on the NBNDC-QB parameters matched to a given Rayleigh DFC (fixed SNR, $f_D T$, q and δ) using the techniques described in Section 2.1.2. Then, we run the SQ-MAC-MAP system simulations using the Rayleigh DFC and the fitted NBNDC-QB (values in [26, Table II] are considered as examples) and compare their SDR performance. The fading coefficients for the Rayleigh DFC are generated according to the modified Clarke's method in [37]. As explained in Section 3.2, the joint MAP decoder is designed using a modified version of the Viterbi algorithm which consists of $4^{(kn)}$ states and $2^{(kn-M+1)} \times 2^{(kn-M'+1)}$ incoming and 4^n outgoing branches, where k is the smallest integer satisfying $kn \geq \max\{M, M'\}$. We simulate the SQ-MAC-MAP system with an input sequence of length $N = 2 \times 10^5$ and repeat each simulation for at least 4 times to find a trustable average result. Even for relatively small memory orders such as $M = 4$, the joint sequence decoder has a complexity which can result in very long simulation times. Hence, the validation is done only for a Rayleigh DFC with noise memory $M = 4$ under SNR=15 dB and $q = 2$.

Looking at Table 3.9, we can see that the difference between the joint SDR performances of the two channel models is very minor when the sources are quantized with two level quantizers ($n = 1$) but this difference becomes more significant as n increases. The mismatch is more visible at low source correlation (ρ) since having high correlation helps joint MAP decoder correct more errors. The joint SDR agreement for low coding rate can be explained by noting that for the case of binary inputs, the sub-channel inputs are almost i.i.d. uniform, the capacity-achieving distribution for these two symmetric channel models. Thus, the two channel models will behave similarly in this situation.

Furthermore, the conformity of the two channel models in a low rate SQ system with instantaneous decoders (θ^*, θ'^*) can be verified from Table 3.10.

On the other hand, it can be seen from Tables 3.11 and 3.12 that there is a significant mismatch between the two channel models in terms of the joint symbol error rate.

We can observe that an orthogonal MAC with DFC-fitted NBNDC-QB sub-channels can efficiently model the orthogonal MAC with original Rayleigh DFC sub-channels in terms of the joint SDR when the system is operating under a low coding rate. It is also interesting to note that in a system with a Rayleigh DFC orthogonal MAC, the instantaneous decoders may outperform our joint MAP decoder because the latter is designed based on the DFC-fitted NBNDC-QB parameters and used with the original Rayleigh DFC orthogonal MAC which is not completely matched with the NBNDC-QB model.

Table 3.1: Joint symbol error rate (in %) of joint MAP decoding and instantaneous mapping (θ^*, θ'^*) for two correlated Gaussian sources with the joint distributions $P(0,0) = 0.2$ and 0.4. The channel model is a MAC channel with two orthogonal NBNDC-QBs, with $M = \alpha = 1$, $Cor = 0.0$, $Cor' = 0.9$ and $q = 1, 2, 3$.

Part (a): Two sub-channels with identical parameters (SNR, q).

$P(0,0)$	(q, q')	(SNR, SNR′) (dB)							
		(15,15)		(10,10)		(5,5)		(2,2)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.2	(1,1)	$C = 0.74 < 1$		$C = 0.74 < 1$		$C = 0.73 < 1$		$C = 0.72 < 1$	
		1.61	1.61	4.63	4.63	12.46	12.47	20.63	20.66
	(2,2)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		1.30	1.48	4.00	4.66	11.28	12.42	19.17	20.59
	(3,3)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		1.28	1.54	3.87	4.58	10.84	12.44	18.49	20.53
0.4	(1,1)	$C = 0.28 < 1$		$C = 0.28 < 1$		$C = 0.27 < 1$		$C = 0.27 < 1$	
		1.09	1.49	3.21	4.53	8.81	12.39	14.86	20.25
	(2,2)	$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$	
		0.79	1.57	2.23	4.62	6.43	12.56	10.41	20.67
	(3,3)	$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$	
		0.66	1.52	2.00	4.60	5.72	12.39	9.75	20.32

Table 3.1 (b): Two sub-channels with identical parameter SNR.

$P(0,0)$	(q,q')	(SNR, SNR$'$) (dB)							
		(15,15)		(10,10)		(5,5)		(2,2)	
		MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)
0.2	(1,2)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		1.35	1.50	3.95	4.43	11.24	12.36	18.94	20.67
	(1,3)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		1.29	1.55	3.87	4.48	10.67	12.19	18.45	20.55
	(2,3)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		1.27	1.51	3.95	4.56	10.90	12.51	18.46	20.57
	(3,2)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		1.32	1.57	4.02	4.56	11.25	12.62	19.01	20.56
	(3,1)	$C=0.74<1$		$C=0.74<1$		$C=0.73<1$		$C=0.72<1$	
		1.58	1.58	4.61	4.61	12.27	12.27	20.42	20.48
	(2,1)	$C=0.74<1$		$C=0.74<1$		$C=0.73<1$		$C=0.72<1$	
		1.55	1.55	4.53	4.53	12.49	12.49	20.65	20.67
0.4	(1,2)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		0.89	1.56	2.75	4.62	7.70	12.48	13.40	20.50
	(1,3)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		0.87	1.53	2.57	4.54	7.40	12.34	12.77	20.44
	(2,3)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		0.74	1.59	2.16	4.73	6.11	12.29	9.92	20.52
	(3,2)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		0.70	1.51	2.13	4.57	6.05	12.42	10.46	20.51
	(3,1)	$C=0.28<1$		$C=0.28<1$		$C=0.28<1$		$C=0.27<1$	
		0.94	1.54	2.71	4.62	7.22	12.36	12.00	20.33
	(2,1)	$C=0.28<1$		$C=0.28<1$		$C=0.27<1$		$C=0.27<1$	
		0.98	1.57	2.77	4.50	7.59	12.34	11.99	20.38

Table 3.1 (c): Two sub-channels with identical parameter q.

$P(0,0)$	(q,q')	(SNR, SNR′) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)
0.2	(1,1)	$C = 0.74 < 1$		$C = 0.73 < 1$		$C = 0.72 < 1$		$C = 0.73 < 1$	
		3.21	3.21	7.38	7.38	11.42	11.47	8.45	8.46
	(2,2)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		2.52	3.17	5.67	7.24	9.12	11.58	7.20	8.73
	(3,3)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		2.36	3.27	5.22	7.12	8.16	11.47	6.73	8.53
0.4	(1,1)	$C = 0.28 < 1$		$C = 0.27 < 1$		$C = 0.27 < 1$		$C = 0.27 < 1$	
		1.69	2.98	2.97	7.31	4.13	11.47	4.45	8.46
	(2,2)	$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$	
		1.03	3.08	1.67	7.02	2.50	11.34	2.98	8.72
	(3,3)	$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$	
		0.83	3.09	1.35	7.24	1.97	11.36	2.54	8.57

$P(0,0)$	(q,q')	(SNR, SNR′) (dB)							
		(5,10)		(2,15)		(5,15)		(10,15)	
		MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)
0.2	(1,1)	$C = 0.74 < 1$		$C = 0.74 < 1$		$C = 0.74 < 1$		$C = 0.74 < 1$	
		8.59	8.59	11.50	11.50	7.17	7.17	3.05	3.05
	(2,2)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		8.24	8.81	11.46	11.64	7.04	7.18	2.86	3.04
	(3,3)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		8.01	8.61	11.39	11.61	6.94	7.13	2.81	3.07
0.4	(1,1)	$C = 0.28 < 1$		$C = 0.28 < 1$		$C = 0.28 < 1$		$C = 0.28 < 1$	
		7.41	8.60	11.25	11.53	6.78	7.14	2.66	3.09
	(2,2)	$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$	
		5.63	8.64	8.17	11.57	5.29	7.09	2.02	3.06
	(3,3)	$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$	
		5.11	8.57	8.09	11.44	4.99	7.16	1.81	3.05

Table 3.1 (d): First sub-channel (with $Cor = 0.0$) has higher SNR.

$P(0,0)$	(q,q')	(SNR, SNR′) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)
0.2	(1,2)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		2.47	3.17	5.50	7.06	8.97	11.52	7.13	8.65
	(1,3)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		2.38	3.21	5.19	7.24	8.19	11.59	6.78	8.61
	(2,3)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		2.46	3.16	5.23	7.16	7.99	11.53	6.86	8.65
	(3,2)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		2.43	3.01	5.56	7.09	9.14	11.57	7.20	8.67
	(3,1)	$C = 0.74 < 1$		$C = 0.73 < 1$		$C = 0.72 < 1$		$C = 0.73 < 1$	
		3.07	3.07	7.31	7.32	11.75	11.80	8.67	8.70
	(2,1)	$C = 0.74 < 1$		$C = 0.73 < 1$		$C = 0.72 < 1$		$C = 0.73 < 1$	
		3.07	3.07	6.97	6.97	11.36	11.40	8.56	8.56
0.4	(1,2)	$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$	
		1.12	3.03	1.84	7.24	2.64	11.57	3.41	8.65
	(1,3)	$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$	
		1.01	2.95	1.54	7.13	2.12	11.83	3.16	8.61
	(2,3)	$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$	
		0.92	3.08	1.41	7.08	2.03	11.63	2.69	8.46
	(3,2)	$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$	
		0.97	3.10	1.65	7.19	2.46	11.59	2.84	8.75
	(3,1)	$C = 0.28 < 1$		$C = 0.27 < 1$		$C = 0.27 < 1$		$C = 0.27 < 1$	
		1.42	2.99	2.75	7.13	3.87	11.51	3.94	8.54
	(2,1)	$C = 0.28 < 1$		$C = 0.27 < 1$		$C = 0.27 < 1$		$C = 0.27 < 1$	
		1.55	3.16	2.81	6.96	4.00	11.47	4.12	8.59

Table 3.1 (e): Second sub-channel (with $Cor = 0.9$) has higher SNR.

$P(0,0)$	(q,q')	(SNR, SNR$'$) (dB)							
		(5,10)		(2,15)		(5,15)		(10,15)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.2	(1,2)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		8.10	8.54	11.40	11.45	6.96	7.14	2.88	3.11
	(1,3)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		7.93	8.50	11.39	11.54	6.95	7.13	2.88	3.08
	(2,3)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		7.98	8.52	11.47	11.59	6.99	7.17	2.83	3.10
	(3,2)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		8.14	8.53	11.43	11.60	7.01	7.17	2.93	3.14
	(3,1)	$C = 0.74 < 1$		$C = 0.74 < 1$		$C = 0.74 < 1$		$C = 0.74 < 1$	
		8.56	8.56	11.49	11.49	7.08	7.08	3.11	3.11
	(2,1)	$C = 0.74 < 1$		$C = 0.74 < 1$		$C = 0.74 < 1$		$C = 0.74 < 1$	
		8.65	8.65	11.51	11.51	7.16	7.16	3.12	3.12
0.4	(1,2)	$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$	
		6.86	8.65	10.93	11.48	6.53	7.14	2.46	3.15
	(1,3)	$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$	
		6.73	8.52	10.97	11.55	6.55	7.16	2.39	3.05
	(2,3)	$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$	
		5.24	8.63	8.13	11.54	4.93	7.09	1.90	3.11
	(3,2)	$C = 0.28 < 1$		$C = 0.28 < 1$		$C = 0.28 < 1$		$C = 0.28 < 1$	
		5.78	8.58	8.41	11.60	5.19	7.11	2.08	3.08
	(3,1)	$C = 0.28 < 1$		$C = 0.28 < 1$		$C = 0.28 < 1$		$C = 0.28 < 1$	
		6.17	8.68	8.39	11.54	5.57	7.14	2.22	3.06
	(2,1)	$C = 0.28 < 1$		$C = 0.28 < 1$		$C = 0.28 < 1$		$C = 0.28 < 1$	
		6.17	8.68	8.39	11.54	5.57	7.14	2.22	3.06

Table 3.2: Joint symbol error rate (in %) of joint MAP decoding and instantaneous mapping (θ^*, θ'^*) for two correlated Gaussian sources with the joint distributions $P(0,0) = 0.2$ and 0.4. The channel model is a MAC channel with two orthogonal NBNDC-QBs, with $M = \alpha = 1$, $Cor = 0.9$, $Cor' = 0.9$ and $q = 1, 2, 3$.

Part (a): Two sub-channels with identical parameters (SNR, q).

$P(0,0)$	(q, q')	(SNR, SNR′) (dB)							
		(15,15)		(10,10)		(5,5)		(2,2)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.2	(1,1)	$C = 0.74 < 1$		$C = 0.74 < 1$		$C = 0.73 < 1$		$C = 0.72 < 1$	
		1.52	1.52	4.50	4.50	12.63	12.63	20.32	20.34
	(2,2)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		1.13	1.54	3.44	4.56	9.56	12.33	16.80	20.67
	(3,3)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		1.03	1.50	3.25	4.58	8.95	12.21	15.00	20.60
0.4	(1,1)	$C = 0.28 < 1$		$C = 0.28 < 1$		$C = 0.27 < 1$		$C = 0.27 < 1$	
		1.08	1.53	3.23	4.55	8.61	12.27	14.29	20.34
	(2,2)	$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$	
		0.28	1.57	0.93	4.55	2.89	12.51	6.08	20.65
	(3,3)	$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$		$C = 0.25 < 1$	
		0.19	1.51	0.59	4.57	1.99	12.09	4.21	20.48

Table 3.2 (b): Two sub-channels with identical parameter SNR.

$P(0,0)$	(q,q')	(SNR, SNR′) (dB)							
		(15,15)		(10,10)		(5,5)		(2,2)	
		MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)
0.2	(1,2)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		1.32	1.51	3.97	4.53	10.98	12.28	18.52	20.41
	(1,3)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		1.28	1.53	3.81	4.55	10.43	12.35	18.23	20.88
	(2,3)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		1.02	1.54	3.22	4.57	9.29	12.56	16.03	20.65
	(3,2)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		1.03	1.46	3.21	4.41	9.28	12.24	16.03	20.98
	(3,1)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		1.28	1.62	3.81	4.40	10.44	12.04	18.23	20.55
	(2,1)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		1.32	1.54	3.96	4.36	10.97	12.18	18.51	20.31
0.4	(1,2)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		0.54	1.40	1.69	4.62	4.66	12.45	8.60	20.63
	(1,3)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		0.46	1.52	1.30	4.78	3.98	12.41	7.47	20.76
	(2,3)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		0.24	1.48	0.74	4.57	2.44	12.42	4.87	20.28
	(3,2)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		0.24	1.62	0.74	4.76	2.43	12.58	4.87	20.51
	(3,1)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		0.45	1.59	1.30	4.47	3.98	12.48	7.48	20.12
	(2,1)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		0.53	1.58	1.68	4.70	4.66	12.42	8.61	20.48

Table 3.2 (c): Two sub-channels with identical parameter q.

$P(0,0)$	(q,q')	(SNR, SNR′) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)
0.2	(1,1)	$C=0.74<1$		$C=0.73<1$		$C=0.72<1$		$C=0.73<1$	
		3.06	3.06	7.21	7.27	11.60	11.72	8.59	8.60
	(2,2)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		2.19	3.04	5.28	7.30	8.77	11.50	6.50	8.27
	(3,3)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		2.08	3.07	4.82	6.99	7.90	11.50	5.85	8.45
0.4	(1,1)	$C=0.28<1$		$C=0.27<1$		$C=0.27<1$		$C=0.27<1$	
		1.66	3.01	2.84	6.95	4.10	11.35	4.51	8.63
	(2,2)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		0.56	2.94	1.26	7.22	2.05	11.28	1.79	8.69
	(3,3)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		0.36	3.13	0.85	7.23	1.50	11.56	1.17	8.68

$P(0,0)$	(q,q')	(SNR, SNR′) (dB)							
		(5,10)		(2,15)		(5,15)		(10,15)	
		MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)
0.2	(1,1)	$C=0.73<1$		$C=0.72<1$		$C=0.73<1$		$C=0.74<1$	
		8.65	8.98	11.51	11.54	7.13	7.14	3.07	3.07
	(2,2)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		6.53	8.51	8.76	11.59	5.26	6.86	2.20	3.04
	(3,3)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		5.88	8.35	7.89	11.46	4.81	6.98	2.09	3.13
0.4	(1,1)	$C=0.27<1$		$C=0.27<1$		$C=0.27<1$		$C=0.28<1$	
		4.49	8.40	4.08	11.47	2.85	7.12	1.67	3.05
	(2,2)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		1.81	8.57	2.06	11.56	1.26	7.31	0.56	3.01
	(3,3)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		1.17	8.66	1.48	11.61	0.86	6.95	0.37	3.09

Table 3.2 (d): First sub-channel has higher SNR.

$P(0,0)$	(q,q')	(SNR, SNR′) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)
0.2	(1,2)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		2.49	3.05	5.61	7.30	8.96	11.56	7.03	8.69
	(1,3)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		2.31	3.06	4.98	7.02	8.17	11.43	6.65	8.66
	(2,3)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		2.20	3.29	5.01	7.09	7.94	11.38	6.11	8.59
	(3,2)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		2.29	3.16	5.24	7.05	8.70	11.37	6.48	8.60
	(3,1)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		2.76	2.94	6.96	7.18	11.24	11.64	7.89	8.55
	(2,1)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		2.94	3.13	7.10	7.30	11.28	11.50	8.01	8.49
0.4	(1,2)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		1.03	3.04	1.72	6.93	2.61	11.62	2.75	8.45
	(1,3)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		0.72	3.03	1.37	7.08	2.06	11.40	2.24	8.61
	(2,3)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		0.41	3.15	0.95	7.21	1.51	11.78	1.43	8.83
	(3,2)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		0.46	2.99	1.16	7.04	2.10	11.85	1.50	8.52
	(3,1)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		1.02	3.10	2.32	7.33	3.48	11.38	2.74	8.60
	(2,1)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		1.06	3.11	2.31	7.13	3.63	11.76	2.90	8.55

Table 3.2 (e): Second sub-channel has higher SNR.

$P(0,0)$	(q,q')	(SNR, SNR$'$) (dB)							
		(5,10)		(2,15)		(5,15)		(10,15)	
		MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)
0.2	(1,2)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		8.05	8.82	11.23	11.40	7.08	7.23	2.98	3.21
	(1,3)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		7.90	8.50	11.20	11.42	7.01	7.24	2.78	3.05
	(2,3)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		6.51	8.72	8.67	11.44	5.20	6.99	2.32	3.28
	(3,2)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		6.12	8.60	7.98	11.72	4.98	7.22	2.17	3.22
	(3,1)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		6.68	8.62	8.20	11.52	5.01	6.97	2.28	3.04
	(2,1)	$C=0.67<1$		$C=0.67<1$		$C=0.67<1$		$C=0.67<1$	
		7.04	8.64	8.91	11.56	5.58	7.07	2.51	3.07
0.4	(1,2)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		3.06	8.77	3.57	11.63	2.35	7.05	1.14	3.19
	(1,3)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		2.69	8.52	3.63	11.49	2.23	7.14	0.99	3.07
	(2,3)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		1.43	8.59	2.02	11.55	1.15	7.30	0.49	2.95
	(3,2)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		1.41	8.72	1.59	11.55	0.99	7.29	0.45	3.09
	(3,1)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		2.29	8.65	2.03	11.77	1.34	7.17	0.67	3.04
	(2,1)	$C=0.25<1$		$C=0.25<1$		$C=0.25<1$		$C=0.25<1$	
		2.88	8.68	2.53	11.54	1.76	7.01	0.95	3.04

Table 3.3: Joint symbol error rate (in %) of joint MAP decoding and instantaneous mapping (θ^*, θ'^*) for two correlated Gaussian sources with the joint distributions $P(0,0) = 0.2$ and 0.4. The channel model is a MAC channel with two orthogonal NBNDC-QBs, with $M = \alpha = 1$, $Cor = Cor' = 0$ and $q = 1, 2, 3$.

Part (a): Two sub-channels with identical parameters (SNR, q).

$P(0,0)$	(q, q')	(SNR, SNR′) (dB)							
		(15,15)		(10,10)		(5,5)		(2,2)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.2	(1,1)	$C = 85.66 > 1$		$C = 27.98 > 1$		$C = 9.72 > 1$		$C = 5.48 > 1$	
		1.51	1.51	4.63	4.63	12.44	12.44	20.52	20.52
	(2,2)	$C = 2.08 > 1$		$C = 1.88 > 1$		$C = 1.90 > 1$		$C = 1.54 > 1$	
		1.53	1.53	4.60	4.60	12.44	12.44	20.52	20.52
	(3,3)	$C = 1.20 > 1$		$C = 1.19 > 1$		$C = 1.09 > 1$		$C = 1.03 > 1$	
		1.53	1.53	4.61	4.61	12.48	12.48	20.54	20.54
0.4	(1,1)	$C = 32.12 > 1$		$C = 10.49 > 1$		$C = 3.65 > 1$		$C = 2.05 > 1$	
		1.53	1.53	4.61	4.61	12.48	12.48	20.56	20.56
	(2,2)	$C = 0.78 < 1$		$C = 0.70 < 1$		$C = 0.71 < 1$		$C = 0.58 < 1$	
		1.32	1.53	3.77	4.60	10.28	12.44	15.94	20.45
	(3,3)	$C = 0.45 < 1$		$C = 0.45 < 1$		$C = 0.41 < 1$		$C = 0.39 < 1$	
		1.17	1.54	3.47	4.58	9.50	12.43	15.66	20.48

Table 3.3 (b): Two sub-channels with identical parameter SNR.

$P(0,0)$	(q,q')	(SNR, SNR′) (dB)							
		(15,15)		(10,10)		(5,5)		(2,2)	
		MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)
0.2	(1,2)	$C=2.08>1$		$C=1.88>1$		$C=1.90>1$		$C=1.54>1$	
		1.54	1.54	4.60	4.60	12.39	12.39	20.54	20.54
	(1,3)	$C=1.20>1$		$C=1.19>1$		$C=1.09>1$		$C=1.03>1$	
		1.57	1.57	4.59	4.59	12.48	12.48	20.50	20.50
	(2,3)	$C=1.20>1$		$C=1.19>1$		$C=1.09>1$		$C=1.03>1$	
		1.55	1.55	4.57	4.57	12.44	12.44	20.55	20.55
0.4	(1,2)	$C=0.78<1$		$C=0.70<1$		$C=0.71<1$		$C=0.58<1$	
		1.43	1.54	4.18	4.62	11.30	12.47	17.95	20.50
	(1,3)	$C=0.45<1$		$C=0.45<1$		$C=0.41<1$		$C=0.39<1$	
		1.37	1.56	4.05	4.62	10.96	12.44	17.94	20.55
	(2,3)	$C=0.45<1$		$C=0.45<1$		$C=0.41<1$		$C=0.38<1$	
		1.24	1.53	3.60	4.59	9.89	12.48	15.71	20.54

Table 3.3 (c): Two sub-channels with identical parameter q.

$P(0,0)$	(q,q')	(SNR, SNR′) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)
0.2	(1,1)	$C=27.98>1$		$C=9.72>1$		$C=5.48>1$		$C=9.72>1$	
		3.10	3.10	7.14	7.14	11.48	11.48	8.55	8.55
	(2,2)	$C=1.88>1$		$C=1.90>1$		$C=1.54>1$		$C=1.88>1$	
		3.09	3.09	7.11	7.11	11.54	11.54	8.57	8.57
	(3,3)	$C=1.19>1$		$C=1.09>1$		$C=1.03>1$		$C=1.09>1$	
		3.06	3.06	7.17	7.17	11.56	11.56	8.61	8.61
0.4	(1,1)	$C=10.49>1$		$C=3.65>1$		$C=2.05>1$		$C=3.65>1$	
		3.08	3.08	7.16	7.16	11.53	11.53	8.62	8.62
	(2,2)	$C=0.70<1$		$C=0.71<1$		$C=0.58<1$		$C=0.70<1$	
		2.51	3.08	5.79	7.17	8.64	11.59	7.06	8.63
	(3,3)	$C=0.45<1$		$C=0.41<1$		$C=0.39<1$		$C=0.41<1$	
		2.33	3.09	5.43	7.12	8.53	11.52	6.54	8.61

Table 3.3 (d): First sub-channel has higher SNR.

$P(0,0)$	(q,q')	(SNR, SNR′) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)
0.2	(1,2)	$C=1.88>1$		$C=1.90>1$		$C=1.54>1$		$C=1.90>1$	
		3.08	3.08	7.15	7.15	11.50	11.50	8.63	8.63
	(1,3)	$C=1.19>1$		$C=1.09>1$		$C=1.03>1$		$C=1.09>1$	
		3.11	3.11	7.22	7.22	11.53	11.53	8.63	8.63
	(2,3)	$C=1.19>1$		$C=1.09>1$		$C=1.03>1$		$C=1.09>1$	
		3.11	3.11	7.14	7.14	11.53	11.53	8.56	8.56
	(3,2)	$C=1.20>1$		$C=1.20>1$		$C=1.20>1$		$C=1.19>1$	
		3.09	3.09	7.13	7.13	11.47	11.47	8.55	8.55
	(3,1)	$C=1.20>1$		$C=1.20>1$		$C=1.20>1$		$C=1.19>1$	
		3.07	3.07	7.16	7.16	11.54	11.54	8.61	8.61
	(2,1)	$C=2.08>1$		$C=2.08>1$		$C=2.08>1$		$C=1.88>1$	
		3.07	3.07	7.13	7.13	11.48	11.48	8.58	8.58
0.4	(1,2)	$C=0.70<1$		$C=0.71<1$		$C=0.58<1$		$C=0.71<1$	
		2.65	3.11	5.91	7.11	8.65	11.50	7.38	8.58
	(1,3)	$C=0.45<1$		$C=0.41<1$		$C=0.39<1$		$C=0.41<1$	
		2.50	3.07	5.56	7.15	8.68	11.51	7.08	8.60
	(2,3)	$C=0.45<1$		$C=0.41<1$		$C=0.39<1$		$C=0.41<1$	
		2.42	3.06	5.44	7.08	8.56	11.47	6.66	8.59
	(3,2)	$C=0.45<1$		$C=0.45<1$		$C=0.45<1$		$C=0.45<1$	
		2.50	3.12	5.78	7.16	8.58	11.61	6.88	8.58
	(3,1)	$C=0.45<1$		$C=0.45<1$		$C=0.45<1$		$C=0.45<1$	
		2.87	3.06	7.02	7.19	11.39	11.56	8.03	8.56
	(2,1)	$C=0.78<1$		$C=0.78<1$		$C=0.78<1$		$C=0.70<1$	
		2.95	3.05	7.05	7.16	11.36	11.46	8.21	8.63

Table 3.4: Joint symbol error rate (in %) of joint MAP decoding and instantaneous mapping (θ^*, θ'^*) for two correlated Gaussian sources with the joint distribution $P(0,0) = 0.2$. The channel model is a MAC channel with two orthogonal NBNDC-QBs, with $M = \alpha = 1$, $Cor = 2.5 \times 10^{-3}$, $Cor' = 0.5$ and $q = 1, 2, 3$.

Part (a): Two sub-channels with identical parameters (SNR, q).

$P(0,0)$	(q, q')	(SNR, SNR′) (dB)							
		(15,15)		(10,10)		(5,5)		(2,2)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.2	(1,1)	$C = 1.32 > 1$		$C = 1.29 > 1$		$C = 1.21 > 1$		$C = 1.14 > 1$	
		1.51	1.51	4.67	4.67	12.37	12.37	20.59	20.59
	(2,2)	$C = 0.68 < 1$		$C = 0.69 < 1$		$C = 0.73 < 1$		$C = 0.74 < 1$	
		1.54	1.55	4.51	4.53	12.42	12.49	20.40	20.41
	(3,3)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.68 < 1$		$C = 0.69 < 1$	
		1.45	1.50	4.43	4.57	12.16	12.44	20.15	20.47

Table 3.4 (b): Two sub-channels with identical parameter SNR.

$P(0,0)$	(q, q')	(SNR, SNR′) (dB)							
		(15,15)		(10,10)		(5,5)		(2,2)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.2	(1,2)	$C = 0.68 < 1$		$C = 0.69 < 1$		$C = 0.73 < 1$		$C = 0.74 < 1$	
		1.55	1.57	4.46	4.50	12.41	12.43	20.51	20.52
	(1,3)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.68 < 1$		$C = 0.69 < 1$	
		1.57	1.61	4.44	4.55	12.00	12.30	20.15	20.53
	(2,3)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.68 < 1$		$C = 0.69 < 1$	
		1.49	1.54	4.45	4.56	12.10	12.48	20.15	20.42
	(3,2)	$C = 0.68 < 1$		$C = 0.69 < 1$		$C = 0.73 < 1$		$C = 0.74 < 1$	
		1.49	1.50	4.51	4.54	12.47	12.54	20.59	20.68
	(3,1)	$C = 1.00 > 1$		$C = 1.11 > 1$		$C = 1.06 > 1$		$C = 1.01 > 1$	
		1.52	1.52	4.53	4.53	12.31	12.31	20.78	20.78
	(2,1)	$C = 1.32 > 1$		$C = 1.29 > 1$		$C = 1.21 > 1$		$C = 1.14 > 1$	
		1.49	1.49	4.60	4.60	12.37	12.37	20.42	20.42

Table 3.4 (c): Two sub-channels with identical parameter q.

$P(0,0)$	(q,q')	(SNR, SNR$'$) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.2	(1,1)	$C = 1.29 > 1$		$C = 1.21 > 1$		$C = 1.14 > 1$		$C = 1.21 > 1$	
		2.95	2.95	7.20	7.20	11.27	11.27	8.71	8.71
	(2,2)	$C = 0.69 < 1$		$C = 0.73 < 1$		$C = 0.74 < 1$		$C = 0.73 < 1$	
		3.09	3.13	6.97	7.07	11.30	11.48	8.46	8.49
	(3,3)	$C = 0.67 < 1$		$C = 0.68 < 1$		$C = 0.69 < 1$		$C = 0.68 < 1$	
		2.98	3.10	6.70	7.02	10.81	11.38	8.29	8.65

$P(0,0)$	(q,q')	(SNR, SNR$'$) (dB)							
		(5,10)		(2,15)		(5,15)		(10,15)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.2	(1,1)	$C = 1.29 > 1$		$C = 1.32 > 1$		$C = 1.32 > 1$		$C = 1.32 > 1$	
		8.53	8.53	11.51	11.51	7.20	7.20	3.11	3.11
	(2,2)	$C = 0.69 < 1$		$C = 0.68 < 1$		$C = 0.68 < 1$		$C = 0.68 < 1$	
		8.57	8.64	11.60	11.62	7.01	7.02	3.05	3.07
	(3,3)	$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$		$C = 0.67 < 1$	
		8.52	8.64	11.55	11.59	7.10	7.13	3.07	3.14

Table 3.4 (d): First sub-channel (with $Cor = 2.5 \times 10^{-3}$) has higher SNR.

$P(0,0)$	(q,q')	(SNR, SNR′) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.2	(1,2)	$C = 0.69 < 1$		$C = 0.73 < 1$		$C = 0.74 < 1$		$C = 0.73 < 1$	
		3.07	3.14	6.90	7.01	11.39	11.66	8.48	8.56
	(1,3)	$C = 0.67 < 1$		$C = 0.68 < 1$		$C = 0.69 < 1$		$C = 0.68 < 1$	
		2.88	3.03	6.79	7.16	11.11	11.69	8.33	8.63
	(2,3)	$C = 0.67 < 1$		$C = 0.68 < 1$		$C = 0.69 < 1$		$C = 0.68 < 1$	
		2.90	3.02	6.82	7.09	10.89	11.41	8.29	8.57
	(3,2)	$C = 0.69 < 1$		$C = 0.73 < 1$		$C = 0.74 < 1$		$C = 0.73 < 1$	
		2.97	3.04	7.04	7.16	11.56	11.68	8.49	8.56
	(3,1)	$C = 1.00 > 1$		$C = 1.00 > 1$		$C = 1.00 > 1$		$C = 1.11 > 1$	
		3.10	3.10	7.21	7.21	11.52	11.52	8.46	8.46
	(2,1)	$C = 1.29 > 1$		$C = 1.21 > 1$		$C = 1.14 > 1$		$C = 1.21 > 1$	
		3.07	3.07	7.23	7.23	11.66	11.66	8.55	8.55

Table 3.5: SQ-MAC-MAP simulation SDR results (in dB) for two correlated Gaussian sources with correlation parameters $\rho = -0.31$ and $\rho = 0.81$ and corresponding joint distributions $P(0,0) = 0.2$ and $P(0,0) = 0.4$ sent over the orthogonal MAC channel with memoryless NBNDC-QB sub-channels ($Cor = 0.0$) ,moderately and highly correlated NBNDC-QB sub-channels ($Cor = 0.5$ and $Cor = 0.9$) with identical parameters (SNR, q) and $M = \alpha = 1$.

Sources Correlation	q	n	Fully interleaved (Cor=0) SNR (dB)				Cor=0.5 SNR (dB)				Cor=0.9 SNR (dB)			
			15	10	5	2	15	10	5	2	15	10	5	2
-0.31	1	1	4.17	3.74	2.78	1.94	4.17	3.74	2.78	1.94	4.17	3.74	2.78	1.94
		2	8.15	6.48	3.83	2.13	8.21	6.64	4.09	2.40	8.32	6.91	4.65	3.19
		3	11.02	7.74	3.85	1.89	11.06	7.83	4.10	1.99	11.47	8.51	5.17	3.34
	2	1	4.17	3.74	2.78	1.94	4.18	3.76	2.81	1.98	4.22	3.91	3.09	2.31
		2	8.23	6.60	3.97	2.45	8.36	6.92	4.44	2.85	8.80	7.87	6.03	4.56
		3	11.26	8.16	4.45	2.57	11.66	8.68	4.95	2.91	12.92	10.54	7.27	5.19
	3	1	4.17	3.74	2.78	1.94	4.18	3.79	2.87	2.04	4.24	3.93	3.20	2.43
		2	8.23	6.68	4.15	2.53	8.42	7.05	4.66	3.03	8.89	8.11	6.42	5.08
		3	11.43	8.31	4.68	2.70	11.79	8.91	5.24	3.19	13.14	11.14	8.03	6.02
0.81	1	1	4.17	3.74	2.78	1.94	4.16	3.73	2.75	1.90	4.16	3.72	2.75	1.90
		2	8.11	6.46	4.25	2.66	8.28	6.82	4.39	2.82	8.90	8.19	6.75	5.53
		3	11.01	7.64	4.01	2.34	11.54	8.67	5.11	3.15	13.22	11.47	8.44	6.46
	2	1	4.28	4.03	3.33	2.59	4.29	4.08	3.41	2.67	4.34	4.22	3.90	3.32
		2	8.73	7.67	5.52	3.70	8.88	8.05	6.16	4.38	9.20	8.94	8.24	7.33
		3	12.64	10.02	6.33	3.99	13.20	11.05	7.52	4.95	14.25	13.41	11.47	9.42
	3	1	4.26	3.98	3.24	2.65	4.30	4.10	3.51	2.86	4.37	4.31	4.07	3.72
		2	8.84	7.94	5.82	3.99	8.99	8.31	6.57	4.87	9.23	9.12	8.61	8.01
		3	13.00	10.63	6.87	4.40	13.51	11.70	8.17	5.60	14.40	13.92	12.44	10.74

Table 3.6: SQ with instantaneous decoder (θ^*, θ'^*) - simulation SDR results (in dB) for two correlated Gaussian sources with correlation parameters $\rho = -0.31$ and $\rho = 0.81$ and corresponding joint distributions $P(0,0) = 0.2$ and $P(0,0) = 0.4$ sent over the orthogonal MAC channel with memoryless NBNDC-QB sub-channels ($Cor = 0.0$), moderately and highly correlated NBNDC-QB sub-channels ($Cor = 0.5$ and $Cor = 0.9$) with identical parameters (SNR, q) and $M = \alpha = 1$.

Sources Correlation	q	n	Fully interleaved (Cor=0)				Cor=0.5				Cor=0.9			
			SNR (dB)				SNR (dB)				SNR (dB)			
			15	10	5	2	15	10	5	2	15	10	5	2
-0.31	1	1	4.17	3.74	2.78	1.94	4.17	3.74	2.78	1.94	4.17	3.74	2.78	1.94
		2	8.15	6.48	3.83	2.13	8.21	6.64	4.09	2.40	8.28	6.77	4.30	2.64
		3	11.02	7.74	3.96	1.85	11.06	7.83	4.10	1.97	11.25	8.09	4.41	2.29
	2	1	4.17	3.74	2.78	1.94	4.17	3.74	2.79	1.96	4.17	3.74	2.80	1.97
		2	8.16	6.47	3.86	2.15	8.21	6.63	4.08	2.41	8.26	6.79	4.35	2.63
		3	11.02	7.75	3.95	1.84	11.07	7.82	4.07	1.98	11.28	8.06	4.36	2.29
	3	1	4.17	3.74	2.78	1.94	4.16	3.75	2.79	1.94	4.15	3.72	2.77	1.96
		2	8.13	6.48	3.84	2.13	8.25	6.68	4.09	2.39	8.32	6.81	4.28	2.67
		3	11.05	7.76	3.95	1.86	11.09	7.84	4.09	1.98	11.28	8.04	4.41	2.27
0.81	1	1	4.17	3.74	2.78	1.94	4.16	3.73	2.78	1.94	4.17	3.73	2.77	1.94
		2	8.14	6.47	3.84	2.14	8.23	6.66	4.12	2.42	8.31	6.74	4.32	2.65
		3	11.01	7.74	3.96	1.86	11.08	7.84	4.12	1.96	11.21	8.12	4.40	2.28
	2	1	4.17	3.74	2.78	1.94	4.16	3.75	2.79	1.96	4.18	3.75	2.77	1.94
		2	8.14	6.46	3.85	2.14	8.22	6.65	4.11	2.40	8.32	6.79	4.24	2.62
		3	11.05	7.73	3.95	1.85	11.06	7.82	4.08	1.98	11.26	8.03	4.33	2.27
	3	1	4.17	3.74	2.78	1.94	4.15	3.74	2.80	1.94	4.15	3.72	2.78	1.92
		2	8.14	6.48	3.84	2.12	8.24	6.65	4.10	2.41	8.27	6.77	4.36	2.66
		3	11.02	7.74	3.95	1.86	11.09	7.86	4.10	1.98	11.32	8.04	4.37	2.29

Table 3.7: SQ-MAC-MAP simulation SDR results (in dB) for two correlated Gaussian sources with correlation parameters $\rho = -0.31$ and $\rho = 0.81$ and corresponding joint distributions $P(0,0) = 0.2$ and $P(0,0) = 0.4$ sent over the orthogonal MAC channel consisting of NBNDC-QB sub-channels with $n = 2$, $M = \alpha = 1$ and various parameters set $(q, Cor$, SNR).

Part (a): Two sub-channels with identical parameters (SNR, q).

ρ	(q,q')	$Cor = Cor' = 0$				$Cor = 0, Cor' = 0.9$				$Cor = Cor' = 0.9$			
		SNR (dB)				SNR (dB)				SNR (dB)			
		(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)
-0.31	(1,1)	8.13	6.47	3.84	2.11	8.23	6.73	4.23	2.63	8.31	6.90	4.68	3.19
	(2,2)	8.20	6.61	3.96	2.43	8.50	7.21	4.91	3.40	8.82	7.88	6.02	4.54
	(3,3)	8.25	6.68	4.16	2.52	8.54	7.33	5.16	3.61	8.86	8.09	6.46	5.07
0.81	(1,1)	8.12	6.44	4.27	2.65	8.76	7.85	6.32	4.86	8.90	8.23	6.79	5.50
	(2,2)	8.73	7.68	5.52	3.68	9.00	8.46	7.13	5.83	9.20	8.95	8.26	7.31
	(3,3)	8.83	7.93	5.81	3.97	9.04	8.59	7.43	6.24	9.24	9.11	8.63	8.00

Table 3.7 (b): Two sub-channels with identical parameter SNR

ρ	(q,q')	$Cor = Cor' = 0$ SNR (dB)				$Cor = 0, Cor' = 0.9$ SNR (dB)				$Cor = Cor' = 0.9$ SNR (dB)			
		(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)
-0.31	(1,2)	8.19	6.53	3.89	2.26	8.48	7.13	4.78	3.13	8.58	7.43	5.27	3.78
	(1,3)	8.19	6.58	3.97	2.29	8.48	7.20	4.94	3.27	8.59	7.46	5.42	3.96
	(2,3)	8.24	6.64	4.08	2.48	8.53	7.28	5.09	3.58	8.84	8.02	6.27	4.81
	(3,2)	8.23	6.66	4.07	2.48	8.52	7.22	4.99	3.41	8.85	7.99	6.25	4.82
	(3,1)	8.18	6.57	3.98	2.29	8.29	6.82	4.40	2.80	8.59	7.48	5.42	3.96
	(2,1)	8.18	6.54	3.89	2.27	8.27	6.71	4.30	2.78	8.58	7.37	5.23	3.76
0.81	(1,2)	8.62	7.39	5.31	3.52	8.88	8.10	6.81	5.50	9.11	8.68	7.66	6.60
	(1,3)	8.67	7.51	5.38	3.53	8.89	8.16	6.98	5.73	9.12	8.80	7.89	6.88
	(2,3)	8.80	7.86	5.76	3.96	9.04	8.50	7.28	6.07	9.23	9.05	8.49	7.64
	(3,2)	8.80	7.87	5.75	3.95	9.03	8.53	7.28	6.00	9.22	9.03	8.44	7.66
	(3,1)	8.67	7.52	5.38	3.54	8.93	8.28	6.78	5.35	9.12	8.78	7.88	6.92
	(2,1)	8.61	7.41	5.32	3.52	8.90	8.21	6.68	5.18	9.08	8.64	7.64	6.49

Table 3.7 (c): Two sub-channels with identical parameter q

ρ	(q,q')	$Cor = Cor' = 0$ SNR (dB)				$Cor = 0, Cor' = 0.9$ SNR (dB)				$Cor = Cor' = 0.9$ SNR (dB)			
		(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)
-0.31	(1,1)	7.22	5.48	4.17	4.96	7.43	6.08	5.07	5.50	7.54	6.15	5.12	5.63
	(2,2)	7.34	5.65	4.58	5.12	8.05	7.02	6.09	6.31	8.34	7.22	6.26	6.90
	(3,3)	7.39	5.78	4.63	5.27	8.15	7.27	6.48	6.54	8.47	7.52	6.68	7.18
0.81	(1,1)	7.57	6.85	6.14	5.87	8.50	8.06	7.69	7.31	8.57	8.19	7.83	7.60
	(2,2)	8.30	7.33	6.50	6.73	8.91	8.65	8.39	8.18	9.07	8.83	8.59	8.66
	(3,3)	8.42	7.51	6.73	6.98	8.99	8.80	8.64	8.39	9.19	9.00	8.84	8.87

ρ	(q,q')	$Cor = Cor' = 0$ SNR (dB)				$Cor = 0, Cor' = 0.9$ SNR (dB)				$Cor = Cor' = 0.9$ SNR (dB)			
		(5,10)	(2,15)	(5,15)	(10,15)	(5,10)	(2,15)	(5,15)	(10,15)	(5,10)	(2,15)	(5,15)	(10,15)
-0.31	(1,1)	4.98	4.17	5.48	7.23	5.10	4.18	5.53	7.28	5.66	5.11	6.17	7.55
	(2,2)	5.12	4.57	5.65	7.34	5.55	4.72	5.84	7.57	6.88	6.27	7.24	8.35
	(3,3)	5.26	4.64	5.79	7.40	5.74	4.77	5.96	7.66	7.20	6.68	7.54	8.46
0.81	(1,1)	5.87	6.15	6.85	7.58	6.91	6.44	7.23	8.10	7.61	7.84	8.18	8.60
	(2,2)	6.72	6.51	7.36	8.30	7.49	6.78	7.63	8.57	8.65	8.59	8.87	9.09
	(3,3)	6.97	6.72	7.51	8.41	7.68	6.99	7.77	8.65	8.91	8.82	8.99	9.18

Table 3.7 (d): The first sub-channel has higher SNR

ρ	(q,q')	$Cor = Cor' = 0$ SNR (dB)				$Cor = 0, Cor' = 0.9$ SNR (dB)				$Cor = Cor' = 0.9$ SNR (dB)			
		(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)
-0.31	(1,2)	7.33	5.65	4.58	5.09	8.04	6.98	6.11	6.29	8.11	7.08	6.14	6.46
	(1,3)	7.34	5.77	4.63	5.19	8.15	7.24	6.45	6.41	8.23	7.30	6.47	6.65
	(2,3)	7.38	5.78	4.63	5.24	8.15	7.26	6.48	6.50	8.46	7.49	6.63	7.14
	(3,2)	7.35	5.69	4.58	5.17	8.09	7.02	6.16	6.39	8.38	7.25	6.31	6.95
	(3,1)	7.26	5.51	4.18	5.02	7.58	6.11	5.03	5.60	7.76	6.31	5.20	6.04
	(2,1)	7.25	5.49	4.16	4.98	7.53	6.12	5.05	5.51	7.80	6.28	5.19	5.97
0.81	(1,2)	8.01	7.08	6.31	6.29	8.78	8.47	8.21	7.83	8.96	8.61	8.30	8.27
	(1,3)	8.21	7.20	6.49	6.49	8.82	8.61	8.40	7.92	9.02	8.80	8.56	8.41
	(2,3)	8.35	7.47	6.70	6.85	8.97	8.80	8.59	8.31	9.16	8.97	8.81	8.79
	(3,2)	8.35	7.39	6.53	6.86	8.93	8.70	8.43	8.25	9.13	8.89	8.65	8.78
	(3,1)	8.01	7.11	6.33	6.53	8.73	8.29	7.96	7.81	8.91	8.48	8.15	8.35
	(2,1)	7.97	7.09	6.30	6.51	8.66	8.25	7.86	7.70	8.85	8.47	8.10	8.20

Table 3.7 (e): The second sub-channel has higher SNR

ρ	(q,q')	$Cor = Cor' = 0$ SNR (dB)				$Cor = 0, Cor' = 0.9$ SNR (dB)				$Cor = Cor' = 0.9$ SNR (dB)			
		(5,10)	(2,15)	(5,15)	(10,15)	(5,10)	(2,15)	(5,15)	(10,15)	(5,10)	(2,15)	(5,15)	(10,15)
-0.31	(1,2)	4.98	4.15	5.48	7.26	5.38	4.28	5.65	7.48	5.98	5.20	6.28	7.75
	(1,3)	5.03	4.17	5.50	7.26	5.46	4.29	5.66	7.50	6.06	5.22	6.26	7.79
	(2,3)	5.16	4.59	5.67	7.37	5.61	4.72	5.86	7.62	6.95	6.30	7.25	8.38
	(3,2)	5.24	4.62	5.78	7.38	5.69	4.75	5.95	7.63	7.09	6.62	7.47	8.45
	(3,1)	5.20	4.62	5.75	7.35	5.33	4.65	5.83	7.44	6.68	6.48	7.30	8.21
	(2,1)	5.09	4.59	5.64	7.32	5.27	4.63	5.69	7.40	6.46	6.11	7.08	8.11
0.81	(1,2)	6.50	6.30	7.10	7.96	7.19	6.58	7.37	8.23	8.19	8.13	8.47	8.86
	(1,3)	6.54	6.32	7.11	7.99	7.28	6.59	7.39	8.24	8.34	8.18	8.51	8.90
	(2,3)	6.87	6.55	7.39	8.36	7.54	6.80	7.66	8.60	8.78	8.64	8.89	9.13
	(3,2)	6.85	6.70	7.47	8.36	7.61	6.97	7.76	8.64	8.80	8.77	8.97	9.13
	(3,1)	6.49	6.49	7.20	8.23	7.34	6.83	7.62	8.54	8.45	8.55	8.76	9.05
	(2,1)	6.25	6.33	7.07	8.01	7.22	6.66	7.50	8.46	8.25	8.34	8.61	8.92

Table 3.8: SQ with instantaneous decoder (θ^*, θ'^*) - simulation SDR results (in dB) for two correlated Gaussian sources with correlation parameters $\rho = -0.31$ and $\rho = 0.81$ and corresponding joint distributions $P(0,0) = 0.2$ and $P(0,0) = 0.4$ sent over the orthogonal MAC channel consisting of NBNDC-QB sub-channels with $n = 2$, $M = \alpha = 1$ and various parameters set $(q, Cor$, SNR).

Part (a): Two sub-channels with identical parameters (SNR, q).

ρ	(q,q')	$Cor = Cor' = 0$				$Cor = 0, Cor' = 0.9$				$Cor = Cor' = 0.9$			
		SNR (dB)				SNR (dB)				SNR (dB)			
		(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)
-0.31	(1,1)	8.13	6.47	3.84	2.11	8.21	6.67	4.07	2.39	8.28	6.77	4.33	2.65
	(2,2)	8.14	6.48	3.85	2.13	8.23	6.62	4.08	2.40	8.32	6.75	4.30	2.64
	(3,3)	8.15	6.47	3.85	2.13	8.20	6.64	4.08	2.40	8.28	6.80	4.36	2.65
0.81	(1,1)	8.14	6.47	3.84	2.13	8.20	6.65	4.09	2.38	8.30	6.81	4.35	2.64
	(2,2)	8.13	6.48	3.84	2.13	8.22	6.62	4.07	2.38	8.30	6.78	4.30	2.65
	(3,3)	8.14	6.47	3.84	2.12	8.19	6.64	4.08	2.41	8.29	6.75	4.30	2.68

Table 3.8 (b): Two sub-channels with identical parameter SNR

ρ	(q,q')	$Cor=Cor'=0$				$Cor=0,Cor'=0.9$				$Cor=Cor'=0.9$			
		SNR (dB)				SNR (dB)				SNR (dB)			
		(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)
-0.31	(1,2)	8.15	6.46	3.85	2.13	8.24	6.64	4.06	2.39	8.31	6.83	4.32	2.63
	(1,3)	8.14	6.48	3.84	2.13	8.22	6.64	4.09	2.36	8.29	6.80	4.34	2.65
	(2,3)	8.15	6.47	3.85	2.12	8.23	6.63	4.10	2.38	8.29	6.84	4.35	2.64
	(3,2)	8.14	6.48	3.85	2.13	8.24	6.62	4.05	2.37	8.27	6.81	4.36	2.65
	(3,1)	8.13	6.47	3.85	2.13	8.22	6.66	4.10	2.36	8.29	6.81	4.33	2.64
	(2,1)	8.15	6.47	3.84	2.14	8.22	6.57	4.08	2.37	8.31	6.79	4.31	2.63
0.81	(1,2)	8.16	6.46	3.86	2.13	8.23	6.63	4.05	2.38	8.33	6.84	4.31	2.68
	(1,3)	8.14	6.46	3.86	2.13	8.23	6.63	4.09	2.35	8.30	6.83	4.32	2.63
	(2,3)	8.14	6.47	3.84	2.13	8.24	6.62	4.07	2.38	8.32	6.81	4.36	2.63
	(3,2)	8.14	6.47	3.84	2.14	8.21	6.62	4.07	2.39	8.27	6.77	4.30	2.66
	(3,1)	8.15	6.48	3.85	2.13	8.23	6.65	4.11	2.38	8.26	6.85	4.31	2.62
	(2,1)	8.15	6.49	3.84	2.14	8.23	6.59	4.11	2.38	8.31	6.68	4.33	2.63

Table 3.8 (c): Two sub-channels with identical parameter q

ρ	(q,q')	$Cor=Cor'=0$ SNR (dB)				$Cor=0, Cor'=0.9$ SNR (dB)				$Cor=Cor'=0.9$ SNR (dB)			
		(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)
-0.31	(1,1)	7.22	5.48	4.17	4.96	7.35	5.82	4.60	5.28	7.44	5.88	4.62	5.34
	(2,2)	7.22	5.48	4.17	4.96	7.42	5.84	4.55	5.25	7.48	5.86	4.59	5.38
	(3,3)	7.22	5.47	4.17	4.97	7.43	5.82	4.60	5.24	7.50	5.90	4.64	5.38
0.81	(1,1)	7.23	5.48	4.17	4.97	7.35	5.81	4.63	5.26	7.40	5.89	4.67	5.38
	(2,2)	7.23	5.48	4.16	4.97	7.43	5.82	4.57	5.26	7.50	5.87	4.60	5.36
	(3,3)	7.23	5.48	4.16	4.97	7.42	5.81	4.59	5.24	7.47	5.88	4.64	5.40

ρ	(q,q')	$Cor=Cor'=0$ SNR (dB)				$Cor=0, Cor'=0.9$ SNR (dB)				$Cor=Cor'=0.9$ SNR (dB)			
		(5,10)	(2,15)	(5,15)	(10,15)	(5,10)	(2,15)	(5,15)	(10,15)	(5,10)	(2,15)	(5,15)	(10,15)
-0.31	(1,1)	4.98	4.17	5.48	7.23	5.06	4.17	5.52	7.27	5.40	4.62	5.90	7.46
	(2,2)	4.95	4.16	5.49	7.23	5.06	4.21	5.52	7.29	5.39	4.62	5.84	7.51
	(3,3)	4.95	4.17	5.48	7.23	5.10	4.18	5.52	7.32	5.39	4.62	5.88	7.44
0.81	(1,1)	4.97	4.17	5.48	7.23	5.07	4.18	5.52	7.27	5.40	4.64	5.93	7.50
	(2,2)	4.97	4.17	5.49	7.22	5.08	4.21	5.52	7.29	5.38	4.64	5.88	7.47
	(3,3)	4.96	4.17	5.48	7.22	5.08	4.19	5.53	7.32	5.41	4.59	5.89	7.49

Table 3.8 (d): the first sub-channel has higher SNR

ρ	(q,q')	$Cor=Cor'=0$ SNR (dB)				$Cor=0, Cor'=0.9$ SNR (dB)				$Cor=Cor'=0.9$ SNR (dB)			
		(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)
-0.31	(1,2)	7.24	5.49	4.18	4.97	7.41	5.77	4.60	5.30	7.47	5.87	4.63	5.39
	(1,3)	7.21	5.49	4.17	4.96	7.41	5.82	4.60	5.26	7.52	5.85	4.61	5.35
	(2,3)	7.22	5.49	4.17	4.97	7.44	5.80	4.59	5.26	7.49	5.85	4.59	5.41
	(3,2)	7.22	5.49	4.16	4.97	7.42	5.84	4.60	5.28	7.49	5.87	4.63	5.37
	(3,1)	7.22	5.48	4.16	4.95	7.47	5.81	4.56	5.29	7.47	5.88	4.60	5.39
	(2,1)	7.23	5.49	4.17	4.96	7.43	5.83	4.55	5.24	7.51	5.86	4.62	5.38
0.81	(1,2)	7.23	5.49	4.17	4.97	7.43	5.76	4.58	5.31	7.48	5.83	4.63	5.41
	(1,3)	7.22	5.49	4.16	4.96	7.43	5.84	4.60	5.24	7.47	5.88	4.66	5.39
	(2,3)	7.22	5.49	4.18	4.96	7.46	5.83	4.57	5.27	7.49	5.84	4.63	5.39
	(3,2)	7.23	5.48	4.18	4.96	7.42	5.84	4.60	5.28	7.48	5.89	4.67	5.43
	(3,1)	7.24	5.49	4.18	4.97	7.44	5.78	4.56	5.28	7.52	5.82	4.62	5.41
	(2,1)	7.22	5.48	4.16	4.97	7.42	5.83	4.53	5.25	7.51	5.89	4.58	5.33

Table 3.8 (e): the second sub-channel has higher SNR

ρ	(q,q')	$Cor=Cor'=0$ SNR (dB)				$Cor=0, Cor'=0.9$ SNR (dB)				$Cor=Cor'=0.9$ SNR (dB)			
		(5,10)	(2,15)	(5,15)	(10,15)	(5,10)	(2,15)	(5,15)	(10,15)	(5,10)	(2,15)	(5,15)	(10,15)
-0.31	(1,2)	4.96	4.16	5.48	7.23	5.08	4.20	5.54	7.30	5.37	4.63	5.87	7.48
	(1,3)	4.97	4.16	5.48	7.23	5.08	4.20	5.52	7.28	5.39	4.63	5.84	7.49
	(2,3)	4.97	4.17	5.47	7.24	5.06	4.18	5.54	7.30	5.34	4.62	5.86	7.48
	(3,2)	4.96	4.17	5.48	7.23	5.08	4.21	5.53	7.29	5.41	4.60	5.88	7.52
	(3,1)	4.98	4.17	5.47	7.22	5.06	4.20	5.54	7.29	5.39	4.61	5.89	7.45
	(2,1)	4.96	4.18	5.48	7.23	5.08	4.21	5.51	7.30	5.39	4.60	5.87	7.51
0.81	(1,2)	4.96	4.18	5.48	7.23	5.08	4.21	5.52	7.31	5.39	4.63	5.93	7.45
	(1,3)	4.96	4.17	5.48	7.22	5.09	4.19	5.52	7.29	5.45	4.58	5.90	7.44
	(2,3)	4.97	4.17	5.48	7.23	5.06	4.18	5.56	7.31	5.41	4.62	5.84	7.53
	(3,2)	4.96	4.16	5.49	7.24	5.09	4.21	5.53	7.30	5.38	4.64	5.83	7.43
	(3,1)	4.97	4.16	5.48	7.24	5.07	4.20	5.54	7.29	5.36	4.56	5.87	7.49
	(2,1)	4.95	4.17	5.48	7.23	5.09	4.22	5.53	7.29	5.39	4.65	5.88	7.44

Table 3.9: SQ-MAC-MAP simulation SDR results (in dB) for the DFC-fitted NBNDC-QB and the DFC; both sub-channels have the same parameters SNR=15 dB, $q = 2$.

$f_D T$	Correlation between sources (ρ)	n	Channel model	
			NBNDC-QB	Rayleigh DFC
0.01	-0.31	1	4.19	4.34
		2	8.45	9.045
	0.81	1	4.30	4.34
		2	8.96	9.08
0.005	-0.31	1	4.23	4.33
		2	8.57	9.10
	0.81	1	4.32	4.31
		2	8.94	9.15

Table 3.10: SQ with instantaneous decoder (θ^*, θ'^*) - simulation SDR results (in dB) for the DFC-fitted NBNDC-QB and the DFC; both sub-channels have the same parameters SNR=15 dB, $q = 2$.

$f_D T$	Correlation between sources (ρ)	n	Channel model	
			NBNDC-QB	Rayleigh DFC
0.01	-0.31	1	4.17	4.34
		2	8.22	9.05
	0.81	1	4.16	4.34
		2	8.19	9.05
0.005	-0.31	1	4.20	4.33
		2	8.16	9.10
	0.81	1	4.17	4.31
		2	8.17	9.08

Table 3.11: SQ-MAC-MAP simulation results in joint symbol error rate(%) for the DFC-fitted NBNDC-QB and the DFC; both sub-channels have the same parameters SNR=15 dB, $q = 2$.

$f_D T$	Correlation between sources (ρ)	n	Channel model	
			NBNDC-QB	Rayleigh DFC
0.01	-0.31	1	1.41	0.29
		2	2.01	0.53
	0.81	1	0.72	0.28
		2	1.12	0.50
0.005	-0.31	1	1.15	0.40
		2	1.80	0.50
	0.81	1	0.65	0.36
		2	0.94	0.51

Table 3.12: SQ with instantaneous decoder (θ^*, θ'^*) - simulation results in joint symbol error rate(%) for the DFC-fitted NBNDC-QB and the DFC; both sub-channels have the same parameters SNR=15 dB, $q = 2$.

$f_D T$	Correlation between sources (ρ)	n	Channel model	
			NBNDC-QB	Rayleigh DFC
0.01	-0.31	1	1.53	0.27
		2	2.49	0.50
	0.81	1	1.54	0.27
		2	2.53	0.49
0.005	-0.31	1	1.35	0.38
		2	2.71	0.46
	0.81	1	1.50	0.35
		2	2.41	0.44

Chapter 4

Transmitting Correlated Binary Markov Sources over the Orthogonal MAC

In this chapter, we generalize the problem investigated in Chapter 3. We assume a practical scenario which leads to a correlated source with memory. Then, we design a sequence joint MAP decoder to exploit the memory and statistics in the source symbols as well as statistics in the channel noise when the quantized sources are sent over the orthogonal MAC.

4.1 System setup

We consider two correlated zero-mean and unit-variance Gaussian sources $\mathcal{V}$ and $\mathcal{V}'$, where the correlation parameter is changing over time. These sources generate a sequence of input pairs $\{(V_i, {V'}_i)\}_{i=1}^{\infty}$ which are real-valued samples taken according to the bivariate normal density

$$f_{V_i,V_i'}(v,v') = \frac{1}{2\pi\sqrt{1-\rho_i^2}} \exp\left(-\frac{v^2+{v'}^2-2\rho_i v v'}{2(1-\rho_i^2)}\right), \tag{4.1}$$

where $\rho_i \in \{\rho, \rho'\}, -1 \leq \rho, \rho' \leq 1$, is the correlation between the two sources at time $i \geq 1$. The process $\{\rho_i\}_{i=1}^{\infty}$ is a first order Markov process described by a two state transition matrix

$$T_\rho = \begin{bmatrix} t_{\rho\rho} & 1 - t_{\rho\rho} \\ 1 - t_{\rho'\rho'} & t_{\rho'\rho'} \end{bmatrix}, \tag{4.2}$$

where $t_{\rho\rho}$ and $t_{\rho'\rho'}$ are the probabilities of the correlation parameter staying in the same state for the next time slot given the current state ρ and ρ', respectively.

This process can model a practical scenario where the correlation between two environmental parameters changes over time according to weather conditions. We model the memory for the correlation process using a first order Markov model.

The generated sequences $\{V_i\}_{i=1}^{\infty}$ and $\{V_i'\}_{i=1}^{\infty}$ are used as inputs to the SQ-MAC-MAP system depicted in Fig. 3.1 in which the real-valued sequences are separately quantized by rate-n SQs and the the resulting bit sequences are sent over the orthogonal NBNDC-QB MAC channel followed by the joint MAP decoder described in Section 4.2.

4.2 Joint MAP decoder design

Similar to Section 3.2, we design the joint MAP decoder to harness the residual redundancy of the sources (which is now caused by memory in the correlation parameter ρ and non-uniformity of the distribution after quantization) and channel statistics to minimize the sequence error probability when decoding the received channel output $(\mathbf{y}, \mathbf{y}')^N \in (\mathcal{Y} \times \mathcal{Y}')^{nN}$. Assuming $nN \geq \max\{M, M'\}$, the MAP decoder estimates binary input sequences $(\mathbf{x}, \mathbf{x}')^N \in \big(\{0,1\} \times \{0,1\}\big)^{nN}$ by $(\hat{\mathbf{x}}, \hat{\mathbf{x}}')^N$ as

$$
\begin{aligned}
(\hat{\mathbf{x}}, \hat{\mathbf{x}}')^N &= \arg\max_{(\mathbf{x},\mathbf{x}')^N} \Big(\Pr\{(\mathbf{X}, \mathbf{X}')^N = (\mathbf{x}, \mathbf{x}')^N \mid (\mathbf{Y}, \mathbf{Y}')^N = (\mathbf{y}, \mathbf{y}')^N\}\Big) \\
&= \arg\max_{(\mathbf{x},\mathbf{x}')^N} \Big\{ \log \big[P_{QB}^{(n)}(z_1^n) P'^{(n)}_{QB}(z'^n_1)\pi(\mathbf{x}_1, \mathbf{x}'_1)\big] + \\
&\sum_{i=1}^{N-1} \log \big[Q(z_{in+1}^{(i+1)n} \mid z_{in-(M-1)}^{in}) Q'(z'^{(i+1)n}_{in+1} \mid z'^{in}_{in-(M'-1)}) P\left((\mathbf{x}_{i+1}, \mathbf{x}'_{i+1}) \mid (\mathbf{x}_i, \mathbf{x}'_i)\right)\big]\Big\},
\end{aligned}
\tag{4.3}
$$

where $Q(z_{in+1}^{(i+1)n} \mid z_{in-(M-1)}^{in})$ and $Q'(z'^{(i+1)n}_{in+1} \mid z'^{in}_{in-(M'-1)})$ can be computed via (3.4). Looking at scenario explained in Section 4.1, one can realize that the process $\{(\mathbf{X_i}, \mathbf{X'_i})\}_{i=1}^{\infty}$ is in fact generated by a hidden Markov model (HMM) with $\{\rho, \rho'\}$ as the hidden states and T_ρ as its corresponding transition matrix which emits $(\mathbf{x}, \mathbf{x}') \in \big(\{0,1\} \times \{0,1\}\big)^n$ with different probabilities. Due to the nature of the HMM, implementing the joint MAP decoder based on the Viterbi algorithm is very complicated and computationally expensive; hence, we approximate the input process with a first-order Markov model. $\pi(\mathbf{x}_1, \mathbf{x}'_1) \triangleq \Pr\{(\mathbf{X}_1, \mathbf{X}'_1) = (\mathbf{x}_1, \mathbf{x}'_1)\}$ is the stationary joint distribution corresponding to the approximated first-order Markov model and $P\left((\mathbf{x}_{i+1}, \mathbf{x}'_{i+1}) \mid (\mathbf{x}_i, \mathbf{x}'_i)\right) \triangleq \Pr\{(\mathbf{X}_{i+1}, \mathbf{X}'_{i+1}) = (\mathbf{x}_{i+1}, \mathbf{x}'_{i+1}) \mid (\mathbf{X}_i, \mathbf{X}'_i) = (\mathbf{x}_i, \mathbf{x}'_i)\}$ denotes the corresponding transition probabilities.

4.3 Case study: joint MAP detection of binary sources

Using two-level quantizers ($n = 1$) in the SQ-MAC-MAP of Section 4.1, we obtain sequences of binary symbols $\{X_n\}_{n=1}^{\infty}$ and $\{X'_n\}_{n=1}^{\infty}$. We approximate these two binary sources with a first order Markovian super-source $S_{XX'} \in \{0, 1, 2, 3\}$, where $S_{XX'_n} \triangleq$

$2X_n + X'_n$; and the corresponding transition matrix is represented as the general form of

$$T = \begin{bmatrix} P_{00} & P_{01} & P_{02} & P_{03} \\ P_{10} & P_{11} & P_{12} & P_{13} \\ P_{20} & P_{21} & P_{22} & P_{23} \\ P_{30} & P_{31} & P_{32} & P_{33} \end{bmatrix} \tag{4.4}$$

where $P_{ij} \triangleq P_{S_{XX'_n}|S_{XX'_{n-1}}}(j \mid i) = P(S_{XX'_n} = j \mid S_{XX'_{n-1}} = i), \;\; i, j \in \{0, 1, 2, 3\}$ are the transition probabilities found through simulation of the SQ-MAC-MAP system.

Lemma 4.1. *The following symmetry holds for the transition matrix (4.4)*

$$\begin{aligned} &P_{00} = P_{03} = P_{30} = P_{33} = a, \qquad &P_{01} = P_{02} = P_{31} = P_{32} = b, \\ &P_{13} = P_{10} = P_{23} = P_{20} = c, \qquad &P_{12} = P_{11} = P_{21} = P_{22} = d. \end{aligned} \tag{4.5}$$

As the result, the transition matrix can be written as

$$T = \begin{bmatrix} a & b & b & a \\ c & d & d & c \\ c & d & d & c \\ a & b & b & a \end{bmatrix}, \tag{4.6}$$

where we have $a + b = c + d = 1/2$.

Proof of Lemma 4.1. Since the pair (V_i, V'_i) is generated by the bivariate Gaussian density function (4.1), the marginal distributions are $\mathcal{N}(0, 1)$ and independent of the correlation parameter ρ_i. Hence, each SQ selects zero as the quantization threshold. Having the same proof as presented for (3.5), it can be shown that at any time $i \geq 1$, the conditional joint distribution has the properties $P(0, 0) = P(1, 1)$ and

$P(0,1) = P(1,0)$; however, the values here depend on the ρ_i. In fact, the correlation parameter ρ, changing according to a Markov with the transition matrix T_ρ, defines the hidden state of the system. Denoting the conditional joint distribution of (X_i, X'_i) as $\pi_{\rho_i} = \Big[\Pr\{(X_i, X'_i) = (0,0) \mid \rho_i\}\ ,\ \Pr\{(X_i, X'_i) = (0,1) \mid \rho_i\}\ ,\ \Pr\{(X_i, X'_i) = (1,0) \mid \rho_i\}\ ,\ \Pr\{(X_i, X'_i) = (1,1) \mid \rho_i\}\Big]$ and noting that $\rho_i \in \{\rho, \rho'\}$, these conditional joint distributions are in the form

$$\pi_\rho = [a'\ ,\ b'\ ,\ b'\ ,\ a'], \quad \text{and} \quad \pi_{\rho'} = [c'\ ,\ d'\ ,\ d'\ ,\ c']. \tag{4.7}$$

The hidden Markov model for the pair (X, X') is illustrated in Fig. 4.1.

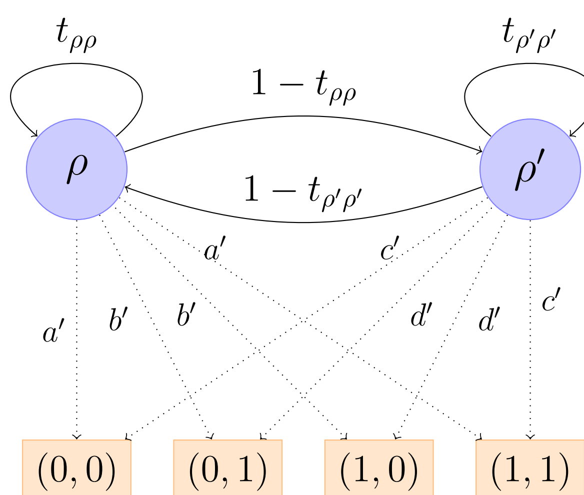


Figure 4.1: The HMM model for the sequence $\{(X_n, X'_n)\}_{n=0}^{\infty}$. The hidden transition probabilities and emission probabilities are given on the corresponding edges.

Approximating the HMM process with a first order Markov one, we find the conditional probabilities in each row of the corresponding transition matrix T. The conditional distribution in each row can be denoted as

$$\begin{aligned}\pi_{(x_{i-1},x'_{i-1})} = \Big[&\Pr\{(X_i, X'_i) = (0,0) \mid (X_{i-1}, X'_{i-1}) = (x_{i-1}, x'_{i-1})\}, \\ &\Pr\{(X_i, X'_i) = (0,1) \mid (X_{i-1}, X'_{i-1}) = (x_{i-1}, x'_{i-1})\}, \\ &\Pr\{(X_i, X'_i) = (1,0) \mid (X_{i-1}, X'_{i-1}) = (x_{i-1}, x'_{i-1})\}, \\ &\Pr\{(X_i, X'_i) = (1,1) \mid (X_{i-1}, X'_{i-1}) = (x_{i-1}, x'_{i-1})\}\Big],\end{aligned} \tag{4.8}$$

where $(x_{i-1}, x'_{i-1}) \in \{0,1\}^2$. According to our model, we can write

$$\begin{aligned}&\Pr\{\rho_i = \rho \mid (x_{i-1}, x'_{i-1}) = (0,0)\} = \\ &\Pr\left\{\rho_i = \rho \mid \rho_{i-1} = \rho, (x_{i-1}, x'_{i-1}) = (0,0)\right\} \times \Pr\left\{\rho_{i-1} = \rho \mid (x_{i-1}, x'_{i-1}) = (0,0)\right\} + \\ &\Pr\left\{\rho_i = \rho \mid \rho_{i-1} = \rho', (x_{i-1}, x'_{i-1}) = (0,0)\right\} \times \Pr\left\{\rho_{i-1} = \rho' \mid (x_{i-1}, x'_{i-1}) = (0,0)\right\} = \\ &t_{\rho\rho} \times \frac{\Pr\{(x_{i-1}, x'_{i-1}) = (0,0) \mid \rho_{i-1} = \rho\} \quad \Pr\{\rho_{i-1} = \rho\}}{\Pr\{(x_{i-1}, x'_{i-1}) = (0,0)\}} + \\ &(1 - t_{\rho'\rho'}) \times \frac{\Pr\{(x_{i-1}, x'_{i-1}) = (0,0) \mid \rho_{i-1} = \rho'\} \quad \Pr\{\rho_{i-1} = \rho'\}}{\Pr\{(x_{i-1}, x'_{i-1}) = (0,0)\}}.\end{aligned} \tag{4.9}$$

As a result,

$$\begin{aligned}\Pr\{\rho_i = \rho \mid (x_{i-1}, x'_{i-1}) = (0,0)\} &= \frac{t_{\rho\rho} \times a' \times P_\rho}{a' \times P_\rho + c' \times (1 - P_\rho)} + \frac{(1 - t_{\rho'\rho'}) \times c' \times (1 - P_\rho)}{a' \times P_\rho + c' \times (1 - P_\rho)} \\ &= \frac{t_{\rho\rho} \times a' \times P_\rho + (1 - t_{\rho'\rho'}) \times c' \times (1 - P_\rho)}{a' \times P_\rho + c' \times (1 - P_\rho)},\end{aligned} \tag{4.10}$$

where $P_\rho \triangleq \Pr\{\rho_i = \rho\} = 1 - \Pr\{\rho_i = \rho'\} = \dfrac{1 - t_{\rho'\rho'}}{2 - (t_{\rho\rho} + t_{\rho'\rho'})}$ is the stationary distribution of the Markov process $\{\rho_i\}_{i=1}^{\infty}$.

Similarly, it can be shown that

$$\Pr\{\rho_i = \rho' \mid (x_{i-1}, x'_{i-1}) = (0,0)\} = \frac{(1-t_{\rho\rho}) \times a' \times P_\rho + t_{\rho'\rho'} \times c' \times (1-P_\rho)}{a' \times P_\rho + c' \times (1-P_\rho)}. \tag{4.11}$$

Finally, we have

$$\begin{aligned} \pi_{(0,0)} &= \pi_\rho \Pr\{\rho_i = \rho \mid (x_{i-1}, x'_{i-1}) = (0,0)\} + \pi_{\rho'} \Pr\{\rho_i = \rho' \mid (x_{i-1}, x'_{i-1}) = (0,0)\} \\ &= \frac{\pi_\rho[a' t_{\rho\rho} P_\rho + c'(1-t_{\rho'\rho'})(1-P_\rho)] + \pi_{\rho'}[a'(1-t_{\rho\rho})P_\rho + c' t_{\rho'\rho'}(1-P_\rho)]}{a' P_\rho + c'(1-P_\rho)} \end{aligned} \tag{4.12}$$

In addition, (4.7) can be written as

$$\begin{aligned} \Pr\{(x_{i-1}, x'_{i-1}) = (0,0) \mid \rho_{i-1} = \rho'\} &= \Pr\{(x_{i-1}, x'_{i-1}) = (1,1) \mid \rho_{i-1} = \rho'\}, \\ \Pr\{(x_{i-1}, x'_{i-1}) = (0,0) \mid \rho_{i-1} = \rho\} &= \Pr\{(x_{i-1}, x'_{i-1}) = (1,1) \mid \rho_{i-1} = \rho\}, \end{aligned} \tag{4.13}$$

which results in

$$\begin{aligned} \Pr\{\rho_i = \rho \mid (x_{i-1}, x'_{i-1}) = (0,0)\} &= \Pr\{\rho_i = \rho \mid (x_{i-1}, x'_{i-1}) = (1,1)\}, \\ \Pr\{\rho_i = \rho' \mid (x_{i-1}, x'_{i-1}) = (0,0)\} &= \Pr\{\rho_i = \rho' \mid (x_{i-1}, x'_{i-1}) = (1,1)\}. \end{aligned} \tag{4.14}$$

Thus,

$$\pi_{(0,0)} = \pi_{(1,1)} = \frac{\pi_\rho[a' t_{\rho\rho} P_\rho + c'(1-t_{\rho'\rho'})(1-P_\rho)] + \pi_{\rho'}[a'(1-t_{\rho\rho})P_\rho + c' t_{\rho'\rho'}(1-P_\rho)]}{a' P_\rho + c'(1-P_\rho)}, \tag{4.15}$$

Repeating the same steps, we have

$$\pi_{(0,1)} = \pi_{(1,0)} = \frac{\pi_\rho[b' t_{\rho\rho} P_\rho + d'(1 - t_{\rho'\rho'})(1 - P_\rho)] + \pi_{\rho'}[b'(1 - t_{\rho\rho}) P_\rho + d' t_{\rho'\rho'}(1 - P_\rho)]}{b' P_\rho + d'(1 - P_\rho)}, \tag{4.16}$$

Hence, putting (4.7), (4.15) and (4.16) together, we can write the numerical transition matrix for the approximated first order Markov as in (4.6), where

$$\begin{aligned}
a &= \frac{a'[a' t_{\rho\rho} P_\rho + c'(1 - t_{\rho'\rho'})(1 - P_\rho)] + c'[a'(1 - t_{\rho\rho}) P_\rho + c' t_{\rho'\rho'}(1 - P_\rho)]}{a' P_\rho + c'(1 - P_\rho)}, \\
b &= \frac{b'[a' t_{\rho\rho} P_\rho + c'(1 - t_{\rho'\rho'})(1 - P_\rho)] + d'[a'(1 - t_{\rho\rho}) P_\rho + c' t_{\rho'\rho'}(1 - P_\rho)]}{a' P_\rho + c'(1 - P_\rho)}, \\
c &= \frac{a'[b' t_{\rho\rho} P_\rho + d'(1 - t_{\rho'\rho'})(1 - P_\rho)] + c'[b'(1 - t_{\rho\rho}) P_\rho + d' t_{\rho'\rho'}(1 - P_\rho)]}{b' P_\rho + d'(1 - P_\rho)}, \\
d &= \frac{b'[b' t_{\rho\rho} P_\rho + d'(1 - t_{\rho'\rho'})(1 - P_\rho)] + d'[b'(1 - t_{\rho\rho}) P_\rho + d' t_{\rho'\rho'}(1 - P_\rho)]}{b' P_\rho + d'(1 - P_\rho)}.
\end{aligned} \tag{4.17}$$

The stationary distribution of the Markov source is the stationary joint distribution of the sources denoted by $\pi = [\ P(0,0)\ ,\ P(0,1)\ ,\ P(1,0)\ ,\ P(1,1)\ ]$, where $P(x, x') \triangleq \Pr(X = x, X' = x')$. Then, the following equations must hold

$$P(1,1) = P(0,0) \tag{4.18}$$

$$P(1,0) = P(0,1) \tag{4.19}$$

$$P(0,0) = \frac{c}{1 + 2(c - a)}. \tag{4.20}$$

Using the previous equations, we can write the following equivalent relations

$$\frac{P(0,0)}{P(0,1)} b + d = \frac{1}{2}, \qquad \frac{P(0,1)}{P(0,0)} c + a = \frac{1}{2}. \tag{4.21}$$

In order to verify that π is the stationary distribution, we must show that $\pi T = \pi$.

Finding the inner product of π and the first column of T, and using (4.21), we have

$$[\,P(0,0)\,,\ P(0,1)\,,\ P(0,1)\,,\ P(0,0)\,]\times\begin{bmatrix} a \\ c \\ c \\ a \end{bmatrix} = 2P(0,0)a+2P(0,1)c = P(0,0). \quad (4.22)$$

Similarly, we can write

$$[\,P(0,0)\,,\ P(0,1)\,,\ P(0,1)\,,\ P(0,0)\,]\times\begin{bmatrix} b \\ d \\ d \\ b \end{bmatrix} = 2P(0,0)b+2P(0,1)d = P(0,1). \quad (4.23)$$

In the following we present a necessary and sufficient condition under which the mapping functions θ and θ' given in (3.10) are optimal sequence detection rules and can replace the joint MAP decoder.

Theorem 4.1. *Consider two binary sources X and X' generating a sequence of pair symbols $\{(X_i, X_i')\}_{i=1}^{\infty}$ according to a first order Markov model with a transition matrix in the form of (4.6); and an orthogonal MAC channel consisting of two independent NBNDC-QB sub-channels where the first one has the correlation parameter $\epsilon \geq 0$, memory order $M = \alpha = 1$, $q \geq 1$, and a noise one-dimensional probability distribution satisfying $\rho_0 \geq \rho_1 \geq \rho_2 \geq \cdots \geq \rho_{2^q-1}$. Similarly, assume that in the second channel $\epsilon' \geq 0, q' \geq 1, M' = 1$, and $\rho_0' \geq \rho_1' \geq \rho_2' \geq \cdots \geq \rho_{2^{q'}-1}'$. Let $(x, x')^N$ be a source sequence of length $N \geq 2$, $(y, y')^N$ the channel output sequence, and let $(\tilde{y}, \tilde{y}')^N = (\theta^*(y), \theta'^*(y'))^N$ be obtained by applying the mapping functions component-wise to the* *corresponding output sequences of the underlying channels.*

Then, assuming $(X_1, X_1') = (\tilde{Y}_1, \tilde{Y}_1')$, $(\hat{x}, \hat{x}')^N = (\tilde{y}, \tilde{y}')^N$ *is an optimal sequence MAP detection rule for all possible received sequences if*

$$A\Big(\min\left\{\frac{a}{b}, \frac{b}{a}, \frac{c}{d}, \frac{d}{c}\right\}\min\left\{\frac{a}{c}, \frac{b}{d}, \frac{d}{b}, \frac{c}{a}\right\}\Big) \geq 1, \tag{4.24}$$

where

$$A = \min\left\{\frac{\epsilon' + (1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon' + (1-\epsilon')\rho'_{2^{q'-1}}}, \frac{\epsilon + (1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon + (1-\epsilon)\rho_{2^{q-1}}}\right\} \tag{4.25}$$

and $P_{(X,X')}(x, x')$ *is the stationary distribution of the Markov process that can be calculated via (4.20).*

Furthermore, we can find a necessary condition for the joint MAP decoder being unnecessary; given as

$$\min\left\{\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}, \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\right\}\min\left\{\frac{a}{b}, \frac{b}{a}, \frac{c}{d}, \frac{d}{c}\right\} \geq 1. \tag{4.26}$$

If (4.26) does not hold, it can be shown that there is at least one pair of input and output sequences of length $N \geq 2$ *for which* $(\hat{x}, \hat{x}')^N = (\tilde{y}, \tilde{y}')^N$ *is not an optimal sequence MAP detection rule. Interestingly, this condition has no dependence on the sub-channels noise correlations.*

For a large enough N*, we have a tighter necessary condition as follows*

$$\min\left\{A\min\left\{\frac{a}{d}, \frac{d}{a}\right\}, \min\left\{\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}, \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\right\}\min\left\{\frac{a^2}{bc}, \frac{bc}{a^2}, \frac{bc}{d^2}, \frac{d^2}{bc}\right\}\right\} \geq 1. \tag{4.27}$$

Proof. See in Appendix B. □

4.3.1 Comparing the results in Theorems 3.1 and 4.1

Herein, we examine how the proposed necessary and sufficient conditions (4.26) and (4.24) relate to the ones in Theorem 3.1. It will be shown that Theorem 3.1 is a special case of Theorem 4.1 when we make the system memoryless by assuming $\rho = \rho'$. As a result, when the correlation between sources is a constant parameter in time, the transition matrix (4.6) will have identical rows which results in

$$\begin{aligned} a = c = P(0,0) = P(1,1), \\ b = d = P(0,1) = P(1,0). \end{aligned} \tag{4.28}$$

Thus, the sufficient condition (4.24) becomes

$$A\Big(\min\left\{\frac{a}{b},\frac{b}{a}\right\}\Big) \geq 1, \tag{4.29}$$

which is the same sufficient condition as given in (3.21) for optimal decoding of i.i.d. correlated sources.

Similarly, the necessary condition (4.26) reduces into

$$\min\left\{\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}},\frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\right\}\min\left\{\frac{a}{b},\frac{b}{a}\right\} \geq 1, \tag{4.30}$$

which is equivalent to the necessary condition given in (3.23) of Theorem 3.1. Furthermore, having the memoryless assumption for the sources, (4.27) can be written as

$$\min\left\{A\min\left\{\frac{a}{b},\frac{b}{a}\right\},\min\left\{\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}},\frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\right\}\min\left\{\frac{a}{b},\frac{b}{a}\right\}\right\}=A\min\left\{\frac{a}{b},\frac{b}{a}\right\}\geq 1. \tag{4.31}$$

which is equivalent to the converse part of Theorem 3.1 for the large enough N.

We illustrate Theorem 4.1 in Tables 4.1-4.3 under various source and sub-channels conditions. The system has been simulated using two binary input sequences which are jointly generated according to a first order Markov process with a transition Matrix (4.6). This matrix can be interpreted as an enumerative approximation for a bivariate Gaussian source where the correlation parameter is a two state Markov process and the sources are separately quantized using two-level quantizers ($n = 1$); see Section 4.3. It is important to mention that the assumption on (X_1, X'_1) in the theorem was not enforced in the simulations; yet the simulations indicate that the theorem's result holds without this assumption.

Furthermore, we define an average correlation parameter for this process

$$\rho_{\text{av}} = P_\rho * \rho + (1 - P_\rho) * \rho', \tag{4.32}$$

where $P_\rho = \dfrac{1 - t_{\rho'\rho'}}{2 - (t_{\rho\rho} + t_{\rho'\rho'})}$ is the stationary distribution of the Markov process $\{\rho_i\}_{i=1}^{\infty}$. The numerical results show that ρ_{av} can be a good measure for evaluating the combined effect of the correlation between two sources and the memory in the source sequences. Hence, we use ρ_{av} to address different source conditions in the simulations.

We have assumed two scenarios for the joint binary sources. In the first scenario, we choose the the transition matrix to be

$$T = \begin{bmatrix} 0.28 & 0.22 & 0.22 & 0.28 \\ 0.33 & 0.17 & 0.17 & 0.33 \\ 0.33 & 0.17 & 0.17 & 0.33 \\ 0.28 & 0.22 & 0.22 & 0.28 \end{bmatrix},$$

which has the form of (4.6) and is the transition matrix for the underlying HMM in (4.1) with $\rho = -0.31$, $\rho' = 0.81$, $t_{\rho\rho} = t_{\rho'\rho'} = 0.2$ and the corresponding $\rho_{\text{av}} = 0.25$.

In the second one,

$$T = \begin{bmatrix} 0.36 & 0.14 & 0.14 & 0.36 \\ 0.37 & 0.13 & 0.13 & 0.37 \\ 0.37 & 0.13 & 0.13 & 0.37 \\ 0.36 & 0.14 & 0.14 & 0.36 \end{bmatrix},$$

which is the transition matrix for the underlying HMM in (4.1) with $\rho = -0.31$, $\rho' = 0.81$, $(t_{\rho\rho}, t_{\rho'\rho'}) = (0.1, 0.8)$ and the corresponding $\rho_{\text{av}} = 0.61$.

Finally, in Table 4.4, we use the following transition matrix in order to further illustrate the theoretical results.

$$T = \begin{bmatrix} 0.23 & 0.27 & 0.27 & 0.23 \\ 0.27 & 0.23 & 0.23 & 0.27 \\ 0.27 & 0.23 & 0.23 & 0.27 \\ 0.23 & 0.27 & 0.27 & 0.23 \end{bmatrix},$$

which is the transition matrix for the underlying HMM in (4.1) with $\rho = -0.5$,

$\rho' = 0.5$, $(t_{\rho\rho}, t_{\rho'\rho'}) = (0.2, 0.2)$ and the corresponding $\rho_{\text{av}} = 0.0$.

Similar to Section 3.4, denoting the left hand term of (4.24) and (4.26) by C and C', respectively, it can be observed from Tables 4.1 - 4.3 that when $C \geq 1$ and consequently $C' \geq 1$, the performance of the instantaneous decoding $(\theta*, \theta'*)$ and the joint MAP decoder are identical, while for $C' < 1$, implying $C < 1$, the joint MAP decoder can outperform the instantaneous decoder. There is another possible situation which $C < 1$ and $C' \geq 1$; for this case, the instantaneous decoder can still achieve the same performance as the joint MAP decoder.

All the observations made in Section (3.3.2) are valid in the tables of the current section. The only difference is in the source correlation parameter ρ which is replaced by ρ_{av}. Furthermore, the results show that ρ_{av} is a good match for ρ and enables us to compare the system performance in Chapters 3 and 4, the system with i.i.d correlated sources and the system with correlated Markov sources. We observe that under the same sub-channels condition, i.i.d. correlated sources with the correlation parameter ρ result in a better performance, in the terms of SER and SDR, compared to when the correlated Markov sources with $\rho_{\text{av}} < \rho$ are sent over the channel. For example comparing Tables 3.5 and 4.5, it can be observed that the SDR results for the i.i.d. sources with $\rho = 0.81$ is always better than the case of correlated Markov sources with $\rho_{\text{av}} = 0.61$. This verifies that ρ_{av} efficiently represents the total redundancy (in terms of memory and non-uniform distribution) in the sources and provides a mean for comparison between different source models.

4.4 Simulation results

4.4.1 SQ-MAC-MAP system simulation

As described in Section 3.4, we simulate the SQ-MAC-MAP system for correlated Gaussian sources with the correlation parameter modeled with a Markov chain. From Table 4.5, we can observe that the joint MAP decoder successfully takes advantage of the the redundancies and statistics in the sources ($\rho_{\text{av}} < \rho$), sub-channels noise memory and the resolution bits. However, there are some cases that increasing ρ_{av} or sub-channels noise correlation does not improve the system SDR (and sometimes even worsens it). This phenomenon for $n = 1$ can be explained by rewriting the sufficient condition (4.24) as

$$\max\{Cor, Cor'\} \leq \min\left\{\frac{\rho_{2^{q-1}} - B\rho_{2^{q-1}-1}}{B(1-\rho_{2^{q-1}-1}) + \rho_{2^{q-1}} - 1}, \frac{\rho'_{2^{q'-1}} - B\rho'_{2^{q'-1}-1}}{B(1-\rho'_{2^{q'-1}-1}) + \rho'_{2^{q'-1}} - 1}\right\}, \tag{4.33}$$

where

$$B = \min\left\{\frac{a}{b}, \frac{b}{a}, \frac{c}{d}, \frac{d}{c}\right\} \min\left\{\frac{a}{c}, \frac{b}{d}, \frac{d}{b}, \frac{c}{a}\right\}. \tag{4.34}$$

In fact, while the sufficient condition (4.24) holds for two sets of source-channel parameters with the same fixed SNR (the parameters Cor, ρ_{av}, and q might vary), the SDR performance of the joint MAP decoder is identical to the SDR performance of the instantaneous decoder which does not change under these two set of parameters. These results also verify that modeling the quantized sources with a first order Markov source is a good approximation; since the input sequences for the simulations illustrating the theorem are generated by the Markov super-source while the system simulation results are based on the quantizing the original real-valued sources.

The general behavior of the system with changing the sources and sub-channels parameters is similar to the SQ-MAC-MAP system with memoryless sources.

Table 4.1: Joint symbol error rate (in %) of joint MAP decoding and instantaneous mapping (θ^*, θ'^*) for two correlated sources with Markovian correlation parameter. The channel model is a MAC channel with two orthogonal NBNDC-QB, with $M = \alpha = 1$, $Cor = Cor' = 0$ and $q = 1, 2, 3$.

Part (a): Two sub-channels with identical parameters (SNR, q).

ρ_{av}	(q, q')	(SNR, SNR′) (dB)							
		(15,15)		(10,10)		(5,5)		(2,2)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.25	(1,1)	$C = 51.00 > 1$		$C = 16.69 > 1$		$C = 5.80 > 1$		$C = 3.27 > 1$	
		$C' = 66.14 > 1$		$C' = 21.61 > 1$		$C' = 7.51 > 1$		$C' = 4.23 > 1$	
		1.52	1.52	4.56	4.56	12.49	12.49	20.59	20.59
	(2,2)	$C = 1.24 > 1$		$C = 1.12 > 1$		$C = 1.13 > 1$		$C = 0.92 < 1$	
		$C' = 1.61 > 1$		$C' = 1.45 > 1$		$C' = 1.47 > 1$		$C' = 1.19 > 1$	
		1.54	1.54	4.62	4.62	12.36	12.36	20.57	20.57
	(3,3)	$C = 0.71 < 1$		$C = 0.71 < 1$		$C = 0.65 < 1$		$C = 0.61 < 1$	
		$C' = 0.92 < 1$		$C' = 0.92 < 1$		$C' = 0.84 < 1$		$C' = 0.79 < 1$	
		1.51	1.54	4.54	4.61	12.23	12.44	20.34	20.58
0.61	(1,1)	$C = 43.90 > 1$		$C = 14.34 > 1$		$C = 4.98 > 1$		$C = 2.81 > 1$	
		$C' = 45.92 > 1$		$C' = 15.00 > 1$		$C' = 5.21 > 1$		$C' = 2.94 > 1$	
		1.51	1.51	4.54	4.54	12.37	12.37	20.55	20.55
	(2,2)	$C = 1.07 > 1$		$C = 0.96 < 1$		$C = 0.97 < 1$		$C = 0.79 < 1$	
		$C' = 1.12 > 1$		$C' = 1.01 > 1$		$C' = 1.02 > 1$		$C' = 0.83 < 1$	
		1.57	1.57	4.64	4.64	12.37	12.37	19.20	20.49
	(3,3)	$C = 0.61 < 1$		$C = 0.61 < 1$		$C = 0.56 < 1$		$C = 0.53 < 1$	
		$C' = 0.64 < 1$		$C' = 0.64 < 1$		$C' = 0.58 < 1$		$C' = 0.55 < 1$	
		1.31	1.53	3.99	4.59	10.69	12.39	17.69	20.48

Table 4.1 (b): Two sub-channels with identical parameter SNR.

ρ_{av}	(q, q')	(SNR, SNR′) (dB)							
		(15,15)		(10,10)		(5,5)		(2,2)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.25	(1,2)	$C = 1.24 > 1$		$C = 1.12 > 1$		$C = 1.13 > 1$		$C = 0.92 < 1$	
		$C' = 1.61 > 1$		$C' = 1.45 > 1$		$C' = 1.47 > 1$		$C' = 1.19 > 1$	
		1.53	1.53	4.59	4.59	12.37	12.37	20.48	20.53
	(1,3)	$C = 0.71 < 1$		$C = 0.71 < 1$		$C = 0.65 < 1$		$C = 0.61 < 1$	
		$C' = 0.92 < 1$		$C' = 0.92 < 1$		$C' = 0.84 < 1$		$C' = 0.79 < 1$	
		1.53	1.54	4.58	4.62	12.37	12.43	20.39	20.57
	(2,3)	$C = 0.71 < 1$		$C = 0.71 < 1$		$C = 0.65 < 1$		$C = 0.61 < 1$	
		$C' = 0.92 < 1$		$C' = 0.92 < 1$		$C' = 0.84 < 1$		$C' = 0.79 < 1$	
		1.55	1.56	4.60	4.64	12.38	12.44	20.44	20.59
0.61	(1,2)	$C = 1.07 > 1$		$C = 0.96 < 1$		$C = 0.97 < 1$		$C = 0.79 < 1$	
		$C' = 1.12 > 1$		$C' = 1.01 > 1$		$C' = 1.02 > 1$		$C' = 0.82 < 1$	
		1.55	1.55	4.60	4.60	12.41	12.41	19.72	20.55
	(1,3)	$C = 0.61 < 1$		$C = 0.61 < 1$		$C = 0.56 < 1$		$C = 0.53 < 1$	
		$C' = 0.64 < 1$		$C' = 0.64 < 1$		$C' = 0.58 < 1$		$C' = 0.55 < 1$	
		1.44	1.55	4.32	4.63	11.49	12.35	19.06	20.57
	(2,3)	$C = 0.61 < 1$		$C = 0.61 < 1$		$C = 0.56 < 1$		$C = 0.53 < 1$	
		$C' = 0.64 < 1$		$C' = 0.64 < 1$		$C' = 0.58 < 1$		$C' = 0.55 < 1$	
		1.44	1.56	4.33	4.61	11.59	12.44	18.37	20.53

Table 4.1 (c): Two sub-channels with identical parameter q.

$\rho_{\rm av}$	(q,q')	(SNR, SNR′) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)
0.25	(1,1)	$C=16.70>1$ $C'=21.61>1$		$C=5.80>1$ $C'=7.51>1$		$C=3.27>1$ $C'=4.23>1$		$C=5.79>1$ $C'=7.51>1$	
		3.10	3.10	7.16	7.16	11.45	11.45	8.59	8.59
	(2,2)	$C=1.12>1$ $C'=1.45>1$		$C=1.13>1$ $C'=1.47>1$		$C=0.92<1$ $C'=1.19>1$		$C=1.12>1$ $C'=1.45>1$	
		3.06	3.06	7.10	7.10	11.46	11.53	8.60	8.60
	(3,3)	$C=0.71<1$ $C'=0.92<1$		$C=0.65<1$ $C'=0.84<1$		$C=0.61<1$ $C'=0.79<1$		$C=0.65<1$ $C'=0.84<1$	
		3.04	3.09	7.03	7.16	11.22	11.51	8.44	8.60
0.61	(1,1)	$C=14.34>1$ $C'=15.00>1$		$C=4.98>1$ $C'=5.21>1$		$C=2.81>1$ $C'=2.94>1$		$C=4.98>1$ $C'=5.21>1$	
		3.08	3.08	7.14	7.14	11.51	11.51	8.63	8.63
	(2,2)	$C=0.96<1$ $C'=1.01>1$		$C=0.97<1$ $C'=1.02>1$		$C=0.79<1$ $C'=0.83<1$		$C=0.96<1$ $C'=1.01>1$	
		3.11	3.11	7.10	7.12	10.76	11.59	8.62	8.62
	(3,3)	$C=0.61<1$ $C'=0.64<1$		$C=0.56<1$ $C'=0.58<1$		$C=0.53<1$ $C'=0.55<1$		$C=0.56<1$ $C'=0.58<1$	
		2.63	3.06	6.04	7.08	9.85	11.57	7.37	8.53

Table 4.1 (d): first sub-channel has higher SNR.

ρ_{av}	(q, q')	(SNR, SNR′) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.25	(1,2)	$C = 1.12 > 1$		$C = 1.14 > 1$		$C = 0.92 < 1$		$C = 1.14 > 1$	
		$C' = 1.45 > 1$		$C' = 1.47 > 1$		$C' = 1.19 > 1$		$C' = 1.47 > 1$	
		3.10	3.10	7.19	7.19	11.54	11.55	8.65	8.65
	(1,3)	$C = 0.71 < 1$		$C = 0.65 < 1$		$C = 0.61 < 1$		$C = 0.65 < 1$	
		$C' = 0.92 < 1$		$C' = 0.84 < 1$		$C' = 0.79 < 1$		$C' = 0.84 < 1$	
		3.02	3.05	7.06	7.17	11.30	11.57	8.51	8.62
	(2,3)	$C = 0.71 < 1$		$C = 0.65 < 1$		$C = 0.61 < 1$		$C = 0.65 < 1$	
		$C' = 0.92 < 1$		$C' = 0.84 < 1$		$C' = 0.79 < 1$		$C' = 0.84 < 1$	
		3.03	3.06	7.00	7.12	11.26	11.54	8.53	8.61
	(3,2)	$C = 0.71 < 1$		$C = 0.71 < 1$		$C = 0.71 < 1$		$C = 0.71 < 1$	
		$C' = 0.92 < 1$		$C' = 0.92 < 1$		$C' = 0.92 < 1$		$C' = 0.92 < 1$	
		3.09	3.11	7.11	7.12	11.62	11.62	8.60	8.63
	(3,1)	$C = 0.71 < 1$		$C = 0.71 < 1$		$C = 0.71 < 1$		$C = 0.71 < 1$	
		$C' = 0.92 < 1$		$C' = 0.92 < 1$		$C' = 0.92 < 1$		$C' = 0.92 < 1$	
		3.08	3.09	7.15	7.17	11.49	11.50	8.55	8.59
	(2,1)	$C = 1.24 > 1$		$C = 1.24 > 1$		$C = 1.24 > 1$		$C = 1.12 > 1$	
		$C' = 1.61 > 1$		$C' = 1.61 > 1$		$C' = 1.61 > 1$		$C' = 1.45 > 1$	
		3.08	3.08	7.18	7.18	11.53	11.53	8.63	8.63
0.61	(1,2)	$C = 0.96 < 1$		$C = 0.97 < 1$		$C = 0.79 < 1$		$C = 0.97 < 1$	
		$C' = 1.01 > 1$		$C' = 1.02 > 1$		$C' = 0.82 < 1$		$C' = 1.02 > 1$	
		3.08	3.09	7.18	7.20	10.59	11.46	8.56	8.56
	(1,3)	$C = 0.61 < 1$		$C = 0.56 < 1$		$C = 0.53 < 1$		$C = 0.56 < 1$	
		$C' = 0.64 < 1$		$C' = 0.58 < 1$		$C' = 0.55 < 1$		$C' = 0.58 < 1$	
		2.77	3.08	6.26	7.21	9.90	11.52	7.57	8.51
	(2,3)	$C = 0.61 < 1$		$C = 0.56 < 1$		$C = 0.53 < 1$		$C = 0.56 < 1$	
		$C' = 0.64 < 1$		$C' = 0.58 < 1$		$C' = 0.55 < 1$		$C' = 0.58 < 1$	
		2.77	3.08	6.18	7.20	9.87	11.52	7.65	8.52
	(3,2)	$C = 0.61 < 1$		$C = 0.61 < 1$		$C = 0.61 < 1$		$C = 0.61 < 1$	
		$C' = 0.64 < 1$		$C' = 0.64 < 1$		$C' = 0.64 < 1$		$C' = 0.64 < 1$	
		2.99	3.10	7.04	7.14	10.70	11.62	8.25	8.53
	(3,1)	$C = 0.61 < 1$		$C = 0.61 < 1$		$C = 0.61 < 1$		$C = 0.61 < 1$	
		$C' = 0.64 < 1$		$C' = 0.64 < 1$		$C' = 0.64 < 1$		$C' = 0.64 < 1$	
		2.97	3.07	7.01	7.10	11.42	11.51	8.20	8.53
	(2,1)	$C = 1.07 > 1$		$C = 1.07 > 1$		$C = 1.07 > 1$		$C = 0.96 < 1$	
		$C' = 1.12 > 1$		$C' = 1.12 > 1$		$C' = 1.12 > 1$		$C' = 1.01 > 1$	
		3.04	3.04	7.12	7.12	11.56	11.56	8.55	8.55

Table 4.2: Joint symbol error rate (in %) of joint MAP decoding and instantaneous mapping (θ^*, θ'^*) for two correlated sources with Markovian correlation parameter. The channel model is a MAC channel with two orthogonal NBNDC-QB, with $M = \alpha = 1$, $Cor = Cor' = 0.5$ and $q = 1, 2, 3$.

Part (a): Two sub-channels with identical parameters (SNR, q).

ρ_{av}	(q, q')	(SNR, SNR′) (dB)							
		(15,15)		(10,10)		(5,5)		(2,2)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.25	(1,1)	$C = 0.78 < 1$ $C' = 66.14 > 1$		$C = 0.77 < 1$ $C' = 21.61 > 1$		$C = 0.72 < 1$ $C' = 7.51 > 1$		$C = 0.68 < 1$ $C' = 4.23 > 1$	
		1.53	1.53	4.61	4.61	12.49	12.49	20.55	20.55
	(2,2)	$C = 0.40 < 1$ $C' = 1.61 > 1$		$C = 0.41 < 1$ $C' = 1.45 > 1$		$C = 0.43 < 1$ $C' = 1.47 > 1$		$C = 0.44 < 1$ $C' = 1.19 > 1$	
		1.50	1.54	4.49	4.65	12.26	12.43	20.19	20.54
	(3,3)	$C = 0.40 < 1$ $C' = 0.92 < 1$		$C = 0.40 < 1$ $C' = 0.92 < 1$		$C = 0.40 < 1$ $C' = 0.84 < 1$		$C = 0.41 < 1$ $C' = 0.79 < 1$	
		1.47	1.57	4.37	4.68	11.71	12.42	19.36	20.49
0.61	(1,1)	$C = 0.68 < 1$ $C' = 46.00 > 1$		$C = 0.66 < 1$ $C' = 15.03 > 1$		$C = 0.62 < 1$ $C' = 5.22 > 1$		$C = 0.58 < 1$ $C' = 2.94 > 1$	
		1.56	1.56	4.52	4.53	12.37	12.37	20.46	20.50
	(2,2)	$C = 0.35 < 1$ $C' = 1.12 > 1$		$C = 0.35 < 1$ $C' = 1.01 > 1$		$C = 0.37 < 1$ $C' = 1.02 > 1$		$C = 0.38 < 1$ $C' = 0.83 < 1$	
		1.13	1.50	3.61	4.61	10.02	12.46	16.86	20.69
	(3,3)	$C = 0.34 < 1$ $C' = 0.64 < 1$		$C = 0.35 < 1$ $C' = 0.64 < 1$		$C = 0.35 < 1$ $C' = 0.58 < 1$		$C = 0.35 < 1$ $C' = 0.55 < 1$	
		1.04	1.55	3.18	4.56	8.94	12.44	15.30	20.42

Table 4.2 (b): Two sub-channels with identical parameter SNR.

<table>
<tr><th rowspan="3">ρ_{av}</th><th rowspan="3">(q, q')</th><th colspan="8">(SNR, SNR′) (dB)</th></tr>
<tr><th colspan="2">(15,15)</th><th colspan="2">(10,10)</th><th colspan="2">(5,5)</th><th colspan="2">(2,2)</th></tr>
<tr><th>MAP</th><th>(θ^*, θ'^*)</th><th>MAP</th><th>(θ^*, θ'^*)</th><th>MAP</th><th>(θ^*, θ'^*)</th><th>MAP</th><th>(θ^*, θ'^*)</th></tr>
<tr><td rowspan="6">0.25</td><td rowspan="2">(1,2)</td><td colspan="2">$C = 0.40 < 1$
$C' = 1.61 > 1$</td><td colspan="2">$C = 0.41 < 1$
$C' = 1.45 > 1$</td><td colspan="2">$C = 0.43 < 1$
$C' = 1.47 > 1$</td><td colspan="2">$C = 0.44 < 1$
$C' = 1.19 > 1$</td></tr>
<tr><td>1.50</td><td>1.53</td><td>4.53</td><td>4.58</td><td>12.35</td><td>12.43</td><td>20.33</td><td>20.50</td></tr>
<tr><td rowspan="2">(1,3)</td><td colspan="2">$C = 0.40 < 1$
$C' = 0.92 < 1$</td><td colspan="2">$C = 0.40 < 1$
$C' = 0.92 < 1$</td><td colspan="2">$C = 0.40 < 1$
$C' = 0.84 < 1$</td><td colspan="2">$C = 0.41 < 1$
$C' = 0.79 < 1$</td></tr>
<tr><td>1.48</td><td>1.57</td><td>4.42</td><td>4.60</td><td>12.05</td><td>12.42</td><td>19.93</td><td>20.46</td></tr>
<tr><td rowspan="2">(2,3)</td><td colspan="2">$C = 0.40 < 1$
$C' = 0.92 < 1$</td><td colspan="2">$C = 0.40 < 1$
$C' = 0.92 < 1$</td><td colspan="2">$C = 0.40 < 1$
$C' = 0.84 < 1$</td><td colspan="2">$C = 0.41 < 1$
$C' = 0.79 < 1$</td></tr>
<tr><td>1.44</td><td>1.50</td><td>4.38</td><td>4.59</td><td>11.94</td><td>12.40</td><td>19.78</td><td>20.50</td></tr>
<tr><td rowspan="6">0.61</td><td rowspan="2">(1,2)</td><td colspan="2">$C = 0.35 < 1$
$C' = 1.12 > 1$</td><td colspan="2">$C = 0.35 < 1$
$C' = 1.01 > 1$</td><td colspan="2">$C = 0.37 < 1$
$C' = 1.02 > 1$</td><td colspan="2">$C = 0.38 < 1$
$C' = 0.83 < 1$</td></tr>
<tr><td>1.37</td><td>1.54</td><td>4.01</td><td>4.52</td><td>11.33</td><td>12.54</td><td>18.55</td><td>20.57</td></tr>
<tr><td rowspan="2">(1,3)</td><td colspan="2">$C = 0.34 < 1$
$C' = 0.64 < 1$</td><td colspan="2">$C = 0.35 < 1$
$C' = 0.64 < 1$</td><td colspan="2">$C = 0.35 < 1$
$C' = 0.58 < 1$</td><td colspan="2">$C = 0.35 < 1$
$C' = 0.55 < 1$</td></tr>
<tr><td>1.26</td><td>1.52</td><td>3.93</td><td>4.58</td><td>10.70</td><td>12.52</td><td>17.97</td><td>20.50</td></tr>
<tr><td rowspan="2">(2,3)</td><td colspan="2">$C = 0.34 < 1$
$C' = 0.64 < 1$</td><td colspan="2">$C = 0.35 < 1$
$C' = 0.64 < 1$</td><td colspan="2">$C = 0.35 < 1$
$C' = 0.58 < 1$</td><td colspan="2">$C = 0.35 < 1$
$C' = 0.55 < 1$</td></tr>
<tr><td>1.13</td><td>1.54</td><td>3.41</td><td>4.70</td><td>9.49</td><td>12.40</td><td>16.03</td><td>20.67</td></tr>
</table>

Table 4.2 (c): Two sub-channels with identical parameter q.

ρ_{av}	(q, q')	(SNR, SNR′) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.25	(1,1)	$C = 0.77 < 1$ $C' = 21.61 > 1$		$C = 0.72 < 1$ $C' = 7.51 > 1$		$C = 0.68 < 1$ $C' = 4.23 > 1$		$C = 0.72 < 1$ $C' = 7.51 > 1$	
		3.13	3.13	7.12	7.12	11.55	11.55	8.60	8.60
	(2,2)	$C = 0.40 < 1$ $C' = 1.45 > 1$		$C = 0.40 < 1$ $C' = 1.47 > 1$		$C = 0.40 < 1$ $C' = 1.19 > 1$		$C = 0.41 < 1$ $C' = 1.45 > 1$	
		2.91	3.00	6.98	7.14	11.25	11.52	8.41	8.52
	(3,3)	$C = 0.40 < 1$ $C' = 0.92 < 1$		$C = 0.40 < 1$ $C' = 0.84 < 1$		$C = 0.40 < 1$ $C' = 0.79 < 1$		$C = 0.40 < 1$ $C' = 0.84 < 1$	
		2.86	3.05	6.60	7.13	10.65	11.49	7.97	8.51
0.61	(1,1)	$C = 0.66 < 1$ $C' = 15.03 > 1$		$C = 0.62 < 1$ $C' = 5.22 > 1$		$C = 0.58 < 1$ $C' = 2.94 > 1$		$C = 0.62 < 1$ $C' = 5.22 > 1$	
		3.07	3.07	7.18	7.20	11.24	11.41	8.61	8.62
	(2,2)	$C = 0.35 < 1$ $C' = 1.01 > 1$		$C = 0.35 < 1$ $C' = 1.02 > 1$		$C = 0.35 < 1$ $C' = 0.83 < 1$		$C = 0.35 < 1$ $C' = 1.01 > 1$	
		2.33	3.14	5.70	7.19	8.94	11.59	6.77	8.57
	(3,3)	$C = 0.34 < 1$ $C' = 0.64 < 1$		$C = 0.34 < 1$ $C' = 0.58 < 1$		$C = 0.34 < 1$ $C' = 0.55 < 1$		$C = 0.35 < 1$ $C' = 0.58 < 1$	
		2.10	3.01	4.92	7.11	8.08	11.72	5.99	8.52

Table 4.2 (d): first sub-channel has higher SNR.

ρ_{av}	(q,q')	(SNR, SNR′) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)
0.25	(1,2)	$C=0.41<1$ $C'=1.45>1$		$C=0.43<1$ $C'=1.47>1$		$C=0.44<1$ $C'=1.19>1$		$C=0.43<1$ $C'=1.47>1$	
		3.06	3.13	7.00	7.11	11.24	11.51	8.43	8.56
	(1,3)	$C=0.40<1$ $C'=0.92<1$		$C=0.40<1$ $C'=0.84<1$		$C=0.41<1$ $C'=0.79<1$		$C=0.40<1$ $C'=0.84<1$	
		2.90	3.05	6.71	7.17	10.80	11.58	8.15	8.58
	(2,3)	$C=0.40<1$ $C'=0.92<1$		$C=0.40<1$ $C'=0.84<1$		$C=0.40<1$ $C'=0.79<1$		$C=0.40<1$ $C'=0.84<1$	
		2.92	3.11	6.66	7.13	10.71	11.54	8.14	8.62
	(3,2)	$C=0.40<1$ $C'=0.92<1$		$C=0.40<1$ $C'=0.92<1$		$C=0.40<1$ $C'=0.92<1$		$C=0.40<1$ $C'=0.92<1$	
		2.97	3.09	6.93	7.10	11.19	11.54	8.32	8.60
	(3,1)	$C=0.40<1$ $C'=0.92<1$		$C=0.40<1$ $C'=0.92<1$		$C=0.40<1$ $C'=0.92<1$		$C=0.40<1$ $C'=0.92<1$	
		2.99	3.05	7.11	7.16	11.45	11.49	8.51	8.69
	(2,1)	$C=0.40<1$ $C'=1.61>1$		$C=0.40<1$ $C'=1.61>1$		$C=0.40<1$ $C'=1.61>1$		$C=0.41<1$ $C'=1.45>1$	
		3.05	3.06	7.07	7.09	11.61	11.62	8.55	8.57
0.61	(1,2)	$C=0.35<1$ $C'=1.01>1$		$C=0.37<1$ $C'=1.02>1$		$C=0.38<1$ $C'=0.83<1$		$C=0.37<1$ $C'=1.02>1$	
		2.51	3.09	5.79	7.20	9.04	11.55	7.15	8.61
	(1,3)	$C=0.35<1$ $C'=0.64<1$		$C=0.35<1$ $C'=0.58<1$		$C=0.35<1$ $C'=0.55<1$		$C=0.35<1$ $C'=0.58<1$	
		2.32	3.05	5.18	7.22	8.32	11.73	6.60	8.47
	(2,3)	$C=0.35<1$ $C'=0.64<1$		$C=0.35<1$ $C'=0.58<1$		$C=0.35<1$ $C'=0.55<1$		$C=0.35<1$ $C'=0.58<1$	
		2.14	3.04	5.00	7.11	8.14	11.69	6.24	8.56
	(3,2)	$C=0.34<1$ $C'=0.64<1$		$C=0.34<1$ $C'=0.64<1$		$C=0.34<1$ $C'=0.64<1$		$C=0.35<1$ $C'=0.64<1$	
		2.34	3.20	5.41	7.02	8.86	11.46	6.60	8.64
	(3,1)	$C=0.34<1$ $C'=0.64<1$		$C=0.34<1$ $C'=0.64<1$		$C=0.34<1$ $C'=0.64<1$		$C=0.35<1$ $C'=0.64<1$	
		2.87	3.08	6.67	6.90	11.29	11.63	7.89	8.51
	(2,1)	$C=0.35<1$ $C'=1.12>1$		$C=0.35<1$ $C'=1.12>1$		$C=0.35<1$ $C'=1.12>1$		$C=0.35<1$ $C'=1.01>1$	
		2.87	3.06	6.90	7.08	11.20	11.58	8.20	8.72

Table 4.3: Joint symbol error rate (in %) of joint MAP decoding and instantaneous mapping (θ^*, θ'^*) for two correlated sources with Markovian correlation parameter. The channel model is a MAC channel with two orthogonal NBNDC-QB, with $M = \alpha = 1$, $Cor = Cor' = 0.9$ and $q = 1, 2, 3$.

Part (a): Two sub-channels with identical parameters (SNR, q).

ρ_{av}	(q, q')	(SNR, SNR′) (dB)							
		(15,15)		(10,10)		(5,5)		(2,2)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.25	(1,1)	$C = 0.44 < 1$		$C = 0.44 < 1$		$C = 0.43 < 1$		$C = 0.43 < 1$	
		$C' = 66.14 > 1$		$C' = 21.61 > 1$		$C' = 7.51 > 1$		$C' = 4.23 > 1$	
		1.53	1.53	4.76	4.77	12.47	12.52	20.38	20.50
	(2,2)	$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$	
		$C' = 1.61 > 1$		$C' = 1.45 > 1$		$C' = 1.47 > 1$		$C' = 1.19 > 1$	
		1.09	1.51	3.15	4.46	9.53	12.53	16.01	20.43
	(3,3)	$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$	
		$C' = 0.92 < 1$		$C' = 0.92 < 1$		$C' = 0.84 < 1$		$C' = 0.79 < 1$	
		0.97	1.55	2.93	4.56	8.46	12.60	14.53	20.45
0.61	(1,1)	$C = 0.38 < 1$		$C = 0.38 < 1$		$C = 0.38 < 1$		$C = 0.37 < 1$	
		$C' = 46.01 > 1$		$C' = 15.03 > 1$		$C' = 5.22 > 1$		$C' = 2.94 > 1$	
		1.20	1.43	3.78	4.68	10.18	12.49	17.03	20.80
	(2,2)	$C = 0.34 < 1$		$C = 0.34 < 1$		$C = 0.35 < 1$		$C = 0.35 < 1$	
		$C' = 1.12 > 1$		$C' = 1.01 > 1$		$C' = 1.02 > 1$		$C' = 0.83 < 1$	
		0.37	1.54	1.53	4.57	4.55	12.17	8.74	20.31
	(3,3)	$C = 0.34 < 1$		$C = 0.34 < 1$		$C = 0.34 < 1$		$C = 0.34 < 1$	
		$C' = 0.64 < 1$		$C' = 0.64 < 1$		$C' = 0.58 < 1$		$C' = 0.55 < 1$	
		0.29	1.47	1.04	4.75	3.34	12.37	6.83	20.63

Table 4.3 (b): Two sub-channels with identical parameter SNR.

ρ_{av}	(q, q')	(SNR, SNR′) (dB)							
		(15,15)		(10,10)		(5,5)		(2,2)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.25	(1,2)	$C = 0.40 < 1$ $C' = 1.61 > 1$		$C = 0.40 < 1$ $C' = 1.45 > 1$		$C = 0.40 < 1$ $C' = 1.47 > 1$		$C = 0.40 < 1$ $C' = 1.19 > 1$	
		1.31	1.38	3.93	4.54	10.76	12.33	18.12	20.18
	(1,3)	$C = 0.40 < 1$ $C' = 0.92 < 1$		$C = 0.40 < 1$ $C' = 0.92 < 1$		$C = 0.40 < 1$ $C' = 0.84 < 1$		$C = 0.40 < 1$ $C' = 0.79 < 1$	
		1.26	1.63	3.85	4.66	10.54	12.34	17.41	20.23
	(2,3)	$C = 0.40 < 1$ $C' = 0.92 < 1$		$C = 0.40 < 1$ $C' = 0.92 < 1$		$C = 0.40 < 1$ $C' = 0.84 < 1$		$C = 0.40 < 1$ $C' = 0.79 < 1$	
		1.02	1.47	3.22	4.70	9.03	12.56	15.31	20.55
0.61	(1,2)	$C = 0.34 < 1$ $C' = 1.12 > 1$		$C = 0.34 < 1$ $C' = 1.01 > 1$		$C = 0.35 < 1$ $C' = 1.02 > 1$		$C = 0.35 < 1$ $C' = 0.83 < 1$	
		0.85	1.53	2.43	4.47	6.67	12.26	12.07	20.60
	(1,3)	$C = 0.34 < 1$ $C' = 0.64 < 1$		$C = 0.34 < 1$ $C' = 0.64 < 1$		$C = 0.34 < 1$ $C' = 0.58 < 1$		$C = 0.34 < 1$ $C' = 0.55 < 1$	
		0.70	1.57	2.20	4.65	6.20	12.09	10.69	20.21
	(2,3)	$C = 0.34 < 1$ $C' = 0.64 < 1$		$C = 0.34 < 1$ $C' = 0.64 < 1$		$C = 0.34 < 1$ $C' = 0.58 < 1$		$C = 0.34 < 1$ $C' = 0.55 < 1$	
		0.37	1.46	1.30	5.10	4.12	12.33	7.43	20.28

Table 4.3 (c): Two sub-channels with identical parameter q.

ρ_{av}	(q, q')	(SNR, SNR′) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.25	(1,1)	$C = 0.44 < 1$ $C' = 21.61 > 1$		$C = 0.43 < 1$ $C' = 7.51 > 1$		$C = 0.43 < 1$ $C' = 4.23 > 1$		$C = 0.43 < 1$ $C' = 7.51 > 1$	
		3.31	3.32	7.07	7.11	11.34	11.48	8.64	8.68
	(2,2)	$C = 0.40 < 1$ $C' = 1.45 > 1$		$C = 0.40 < 1$ $C' = 1.47 > 1$		$C = 0.40 < 1$ $C' = 1.19 > 1$		$C = 0.40 < 1$ $C' = 1.45 > 1$	
		2.16	3.12	5.16	7.29	8.55	11.77	6.23	8.56
	(3,3)	$C = 0.40 < 1$ $C' = 0.92 < 1$		$C = 0.40 < 1$ $C' = 0.84 < 1$		$C = 0.40 < 1$ $C' = 0.79 < 1$		$C = 0.40 < 1$ $C' = 0.84 < 1$	
		1.97	3.12	4.34	6.79	7.66	11.47	5.53	8.48
0.61	(1,1)	$C = 0.38 < 1$ $C' = 15.03 > 1$		$C = 0.38 < 1$ $C' = 5.22 > 1$		$C = 0.37 < 1$ $C' = 2.94 > 1$		$C = 0.38 < 1$ $C' = 5.22 > 1$	
		2.28	3.05	4.40	6.86	6.45	11.64	6.10	8.59
	(2,2)	$C = 0.34 < 1$ $C' = 1.01 > 1$		$C = 0.34 < 1$ $C' = 1.02 > 1$		$C = 0.34 < 1$ $C' = 0.83 < 1$		$C = 0.34 < 1$ $C' = 1.01 > 1$	
		1.00	3.21	2.14	7.21	3.57	11.42	3.01	8.58
	(3,3)	$C = 0.34 < 1$ $C' = 0.64 < 1$		$C = 0.34 < 1$ $C' = 0.58 < 1$		$C = 0.34 < 1$ $C' = 0.55 < 1$		$C = 0.34 < 1$ $C' = 0.58 < 1$	
		0.67	3.20	1.78	7.11	2.81	11.01	2.04	8.42

Table 4.3 (d): first sub-channel has higher SNR.

ρ_{av}	(q, q')	(SNR, SNR′) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.25	(1,2)	$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$	
		$C' = 1.45 > 1$		$C' = 1.47 > 1$		$C' = 1.19 > 1$		$C' = 1.47 > 1$	
		2.28	3.11	5.20	6.96	8.78	11.72	7.02	8.61
	(1,3)	$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$	
		$C' = 0.92 < 1$		$C' = 0.84 < 1$		$C' = 0.79 < 1$		$C' = 0.84 < 1$	
		2.32	3.26	4.70	6.99	7.69	11.44	6.42	8.41
	(2,3)	$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$	
		$C' = 0.92 < 1$		$C' = 0.84 < 1$		$C' = 0.79 < 1$		$C' = 0.84 < 1$	
		1.96	3.17	4.50	7.02	7.50	11.44	5.61	8.35
	(3,2)	$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$	
		$C' = 0.92 < 1$		$C' = 0.92 < 1$		$C' = 0.92 < 1$		$C' = 0.92 < 1$	
		2.13	3.20	5.15	7.23	8.14	11.29	6.04	8.68
	(3,1)	$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$	
		$C' = 0.92 < 1$		$C' = 0.92 < 1$		$C' = 0.92 < 1$		$C' = 0.92 < 1$	
		2.73	2.95	7.00	7.34	11.06	11.48	7.75	8.44
	(2,1)	$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$		$C = 0.40 < 1$	
		$C' = 1.61 > 1$		$C' = 1.61 > 1$		$C' = 1.61 > 1$		$C' = 1.45 > 1$	
		2.97	3.14	6.76	6.95	11.07	11.42	8.11	8.79
0.61	(1,2)	$C = 0.34 < 1$		$C = 0.35 < 1$		$C = 0.35 < 1$		$C = 0.35 < 1$	
		$C' = 1.01 > 1$		$C' = 1.02 > 1$		$C' = 0.83 < 1$		$C' = 1.02 > 1$	
		1.42	3.18	2.80	7.03	4.00	11.29	4.04	8.54
	(1,3)	$C = 0.34 < 1$		$C = 0.34 < 1$		$C = 0.34 < 1$		$C = 0.34 < 1$	
		$C' = 0.64 < 1$		$C' = 0.58 < 1$		$C' = 0.55 < 1$		$C' = 0.58 < 1$	
		1.12	3.32	2.25	6.88	3.39	11.39	3.58	8.77
	(2,3)	$C = 0.34 < 1$		$C = 0.34 < 1$		$C = 0.34 < 1$		$C = 0.34 < 1$	
		$C' = 0.64 < 1$		$C' = 0.58 < 1$		$C' = 0.55 < 1$		$C' = 0.58 < 1$	
		0.73	3.00	1.76	7.03	2.72	11.35	2.46	8.64
	(3,2)	$C = 0.34 < 1$		$C = 0.34 < 1$		$C = 0.34 < 1$		$C = 0.34 < 1$	
		$C' = 0.64 < 1$		$C' = 0.64 < 1$		$C' = 0.64 < 1$		$C' = 0.64 < 1$	
		0.85	3.21	2.28	7.07	3.62	11.71	2.53	8.26
	(3,1)	$C = 0.34 < 1$		$C = 0.34 < 1$		$C = 0.34 < 1$		$C = 0.34 < 1$	
		$C' = 0.64 < 1$		$C' = 0.64 < 1$		$C' = 0.64 < 1$		$C' = 0.64 < 1$	
		1.66	2.98	3.95	7.10	5.83	11.36	4.30	8.31
	(2,1)	$C = 0.34 < 1$		$C = 0.34 < 1$		$C = 0.34 < 1$		$C = 0.34 < 1$	
		$C' = 1.12 > 1$		$C' = 1.12 > 1$		$C' = 1.12 > 1$		$C' = 1.01 > 1$	
		1.77	3.22	4.15	7.48	5.92	11.40	4.67	8.73

Table 4.4: Joint symbol error rate (in %) of joint MAP decoding and instantaneous mapping (θ^*, θ'^*) for two correlated sources with Markovian correlation parameter. The channel model is a MAC channel with two orthogonal NBNDC-QB, with $M = \alpha = 1$, $Cor = 5 \times 10^{-3}, Cor' = 0.5$ and $q = 1, 2, 3$.

Part (a): Two sub-channels with identical parameters (SNR, q).

ρ_{av}	(q, q')	(SNR, SNR′) (dB)							
		(15,15)		(10,10)		(5,5)		(2,2)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.0	(1,1)	$C = 1.51 > 1$ $C' = 112.39 > 1$		$C = 1.48 > 1$ $C' = 36.72 > 1$		$C = 1.39 > 1$ $C' = 12.75 > 1$		$C = 1.31 > 1$ $C' = 7.19 > 1$	
		1.52	1.52	4.57	4.57	12.40	12.40	20.53	20.53
	(2,2)	$C = 0.78 < 1$ $C' = 2.73 > 1$		$C = 0.79 < 1$ $C' = 2.46 > 1$		$C = 0.84 < 1$ $C' = 2.50 > 1$		$C = 0.84 < 1$ $C' = 2.02 > 1$	
		1.54	1.54	4.64	4.64	12.46	12.46	20.42	20.42
	(3,3)	$C = 0.77 < 1$ $C' = 1.57 > 1$		$C = 0.77 < 1$ $C' = 1.57 > 1$		$C = 0.78 < 1$ $C' = 1.43 > 1$		$C = 0.79 < 1$ $C' = 1.35 > 1$	
		1.53	1.53	4.60	4.60	12.39	12.39	20.52	20.52

Table 4.4 (b): Two sub-channels with identical parameter SNR.

ρ_{av}	(q,q')	(SNR, SNR′) (dB)							
		(15,15)		(10,10)		(5,5)		(2,2)	
		MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)	MAP	(θ^*,θ'^*)
0.0	(1,2)	$C=0.78<1$		$C=0.79<1$		$C=0.84<1$		$C=0.85<1$	
		$C'=2.74>1$		$C'=2.47>1$		$C'=2.50>1$		$C'=2.02>1$	
		1.53	1.53	4.57	4.57	12.44	12.44	20.50	20.50
	(1,3)	$C=0.77<1$		$C=0.77<1$		$C=0.78<1$		$C=0.79<1$	
		$C'=1.57>1$		$C'=1.57>1$		$C'=1.43>1$		$C'=1.35>1$	
		1.52	1.52	4.57	4.57	12.52	12.52	20.48	20.48
	(2,3)	$C=0.77<1$		$C=0.77<1$		$C=0.78<1$		$C=0.79<1$	
		$C'=1.57>1$		$C'=1.57>1$		$C'=1.43>1$		$C'=1.35>1$	
		1.50	1.50	4.57	4.57	12.39	12.39	20.54	20.54
	(3,2)	$C=0.78<1$		$C=0.79<1$		$C=0.84<1$		$C=0.85<1$	
		$C'=1.57>1$		$C'=1.57>1$		$C'=1.43>1$		$C'=1.35>1$	
		1.58	1.58	4.62	4.62	12.46	12.46	20.52	20.52
	(3,1)	$C=1.52>1$		$C=1.48>1$		$C=1.39>1$		$C=1.31>1$	
		$C'=2.74>1$		$C'=2.47>1$		$C'=2.50>1$		$C'=2.02>1$	
		1.55	1.55	4.71	4.71	12.35	12.35	20.73	20.73
	(2,1)	$C=1.52>1$		$C=1.48>1$		$C=1.39>1$		$C=1.31>1$	
		$C'=2.74>1$		$C'=2.47>1$		$C'=2.50>1$		$C'=2.02>1$	
		1.55	1.55	4.71	4.71	12.35	12.35	20.73	20.73

Table 4.4 (c): Two sub-channels with identical parameter q.

ρ_{av}	(q, q')	(SNR, SNR′) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.0	(1,1)	$C = 1.48 > 1$ $C' = 36.72 > 1$		$C = 1.39 > 1$ $C' = 12.75 > 1$		$C = 1.31 > 1$ $C' = 7.19 > 1$		$C = 1.39 > 1$ $C' = 12.75 > 1$	
		3.05	3.05	7.16	7.16	11.39	11.39	8.65	8.65
	(2,2)	$C = 0.79 < 1$ $C' = 2.46 > 1$		$C = 0.84 < 1$ $C' = 2.50 > 1$		$C = 0.84 < 1$ $C' = 2.02 > 1$		$C = 0.84 < 1$ $C' = 2.46 > 1$	
		3.18	3.18	7.19	7.19	11.58	11.58	8.52	8.52
	(3,3)	$C = 0.77 < 1$ $C' = 1.57 > 1$		$C = 0.78 < 1$ $C' = 1.43 > 1$		$C = 0.79 < 1$ $C' = 1.35 > 1$		$C = 0.78 < 1$ $C' = 1.43 > 1$	
		3.06	3.06	7.15	7.15	11.64	11.64	8.45	8.45

ρ_{av}	(q, q')	(SNR, SNR′) (dB)							
		(5,10)		(2,15)		(5,15)		(10,15)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.0	(1,1)	$C = 1.48 > 1$ $C' = 12.77 > 1$		$C = 1.51 > 1$ $C' = 7.19 > 1$		$C = 1.52 > 1$ $C' = 12.77 > 1$		$C = 1.52 > 1$ $C' = 36.76 > 1$	
		8.59	8.59	11.47	11.47	7.15	7.15	3.11	3.11
	(2,2)	$C = 0.79 < 1$ $C' = 2.47 > 1$		$C = 0.78 < 1$ $C' = 2.02 > 1$		$C = 0.78 < 1$ $C' = 2.50 > 1$		$C = 0.78 < 1$ $C' = 2.47 > 1$	
		8.59	8.59	11.48	11.48	7.15	7.15	3.11	3.11
	(3,3)	$C = 0.77 < 1$ $C' = 1.43 > 1$		$C = 0.77 < 1$ $C' = 1.35 > 1$		$C = 0.77 < 1$ $C' = 1.43 > 1$		$C = 0.77 < 1$ $C' = 1.57 > 1$	
		8.57	8.57	11.46	11.46	7.13	7.13	3.05	3.05

Table 4.4 (d): First sub-channel (with $Cor = 5 \times 10^{-3}$) has higher SNR.

ρ_{av}	(q, q')	(SNR, SNR′) (dB)							
		(15,10)		(15,5)		(15,2)		(10,5)	
		MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)	MAP	(θ^*, θ'^*)
0.0	(1,2)	$C = 0.79 < 1$ $C' = 2.47 > 1$		$C = 0.84 < 1$ $C' = 2.50 > 1$		$C = 0.85 < 1$ $C' = 2.02 > 1$		$C = 0.84 < 1$ $C' = 2.50 > 1$	
		3.08	3.08	7.20	7.20	11.52	11.52	8.67	8.67
	(1,3)	$C = 0.77 < 1$ $C' = 1.57 > 1$		$C = 0.78 < 1$ $C' = 1.43 > 1$		$C = 0.79 < 1$ $C' = 1.35 > 1$		$C = 0.78 < 1$ $C' = 1.43 > 1$	
		3.08	3.08	7.21	7.21	11.72	11.72	8.45	8.45
	(2,3)	$C = 0.77 < 1$ $C' = 1.57 > 1$		$C = 0.78 < 1$ $C' = 1.43 > 1$		$C = 0.79 < 1$ $C' = 1.35 > 1$		$C = 0.78 < 1$ $C' = 1.43 > 1$	
		3.07	3.07	7.16	7.16	11.55	11.55	8.68	8.68
	(3,2)	$C = 0.79 < 1$ $C' = 1.57 > 1$		$C = 0.84 < 1$ $C' = 1.57 > 1$		$C = 0.85 < 1$ $C' = 1.57 > 1$		$C = 0.84 < 1$ $C' = 1.57 > 1$	
		3.11	3.11	7.15	7.15	11.48	11.48	8.52	8.52
	(3,1)	$C = 1.05 > 1$ $C' = 1.57 > 1$		$C = 1.05 > 1$ $C' = 1.57 > 1$		$C = 1.05 > 1$ $C' = 1.57 > 1$		$C = 1.21 > 1$ $C' = 1.57 > 1$	
		3.08	3.08	7.11	7.11	11.59	11.59	8.50	8.50
	(2,1)	$C = 1.48 > 1$ $C' = 2.74 > 1$		$C = 1.39 > 1$ $C' = 2.74 > 1$		$C = 1.31 > 1$ $C' = 2.74 > 1$		$C = 1.39 > 1$ $C' = 2.47 > 1$	
		3.05	3.05	7.11	7.11	11.70	11.70	8.61	8.61

Table 4.5: SQ-MAC-MAP simulation SDR results (in dB) for two correlated sources with Markovian correlation parameter sent over the orthogonal MAC channel with memoryless NBNDC-QB sub-channels ($Cor = 0.0$), moderately and highly correlated NBNDC-QB sub-channels ($Cor = 0.5$ and $Cor = 0.9$) with identical parameters (SNR, q) and $M = \alpha = 1$.

ρ_{av}	q	n	Fully interleaved (Cor=0)				Cor=0.5				Cor=0.9			
			SNR (dB)				SNR (dB)				SNR (dB)			
			15	10	5	2	15	10	5	2	15	10	5	2
0.25	1	1	4.17	3.75	2.77	1.94	4.17	3.74	2.77	1.93	4.17	3.72	2.77	1.94
		2	8.14	6.47	3.81	2.06	8.21	6.63	4.10	2.42	8.41	7.05	4.94	3.50
		3	11.06	7.75	3.92	2.00	11.13	7.83	4.11	2.09	11.68	9.06	5.72	3.75
	2	1	4.16	3.73	2.79	1.94	4.17	3.76	2.81	1.97	4.23	3.94	3.09	2.32
		2	8.23	6.65	4.06	2.48	8.41	7.01	4.53	2.88	8.88	8.05	6.28	4.83
		3	11.33	8.20	4.48	2.26	11.83	8.90	5.08	2.96	13.19	11.13	7.92	5.73
	3	1	4.17	3.76	2.83	1.97	4.18	3.77	2.87	2.04	4.25	3.97	3.22	2.47
		2	8.27	6.69	4.15	2.47	8.47	7.13	4.75	3.09	8.95	8.25	6.72	5.43
		3	11.45	8.34	4.65	2.63	11.92	9.11	5.44	3.27	13.46	11.66	8.60	6.59
0.61	1	1	4.16	3.75	2.78	1.94	4.15	3.75	2.79	1.93	4.19	3.73	2.74	1.78
		2	8.12	6.48	3.79	1.69	8.23	6.64	4.16	2.51	8.67	7.72	5.88	4.52
		3	10.94	7.74	3.87	1.86	11.06	7.89	4.37	2.44	12.42	10.18	6.97	5.13
	2	1	4.17	3.76	2.81	2.20	4.24	3.91	3.09	2.31	4.35	4.16	3.69	3.02
		2	8.39	7.03	4.67	3.04	8.63	7.49	5.23	3.55	9.06	8.60	7.38	6.05
		3	11.78	8.91	5.19	3.19	12.44	9.65	5.97	3.74	13.80	12.24	9.46	7.45
	3	1	4.22	3.88	3.07	2.28	4.27	3.97	3.24	2.48	4.35	4.24	3.89	3.33
		2	8.46	7.15	4.82	3.10	8.69	7.61	5.49	3.87	9.13	8.82	7.85	6.81
		3	11.94	9.09	5.48	3.36	12.54	10.06	6.42	4.23	13.98	12.83	10.39	8.63

Table 4.6: SQ with instantaneous decoder (θ^*, θ'^*) - simulation SDR results (in dB) for two correlated sources with Markovian correlation parameter sent over the orthogonal MAC channel with memoryless NBNDC-QB sub-channels ($Cor = 0.0$), moderately and highly correlated NBNDC-QB sub-channels ($Cor = 0.5$ and $Cor = 0.9$) with identical parameters (SNR, q) and $M = \alpha = 1$.

ρ_{av}	q	n	Fully interleaved (Cor=0) SNR (dB)				Cor=0.5 SNR (dB)				Cor=0.9 SNR (dB)			
			15	10	5	2	15	10	5	2	15	10	5	2
0.25	1	1	4.17	3.75	2.77	1.94	4.17	3.74	2.77	1.93	4.17	3.72	2.77	1.95
		2	8.14	6.47	3.85	2.11	8.21	6.63	4.10	2.41	8.31	6.76	4.30	2.71
		3	11.06	7.75	3.96	1.86	11.13	7.83	4.09	1.98	11.22	8.16	4.41	2.24
	2	1	4.16	3.73	2.79	1.94	4.17	3.74	2.79	1.94	4.17	3.76	2.77	1.95
		2	8.16	6.49	3.83	2.14	8.22	6.64	4.09	2.39	8.30	6.78	4.32	2.68
		3	11.03	7.77	3.97	1.87	11.09	7.85	4.08	1.97	11.22	8.18	4.38	2.22
	3	1	4.16	3.74	2.79	1.93	4.17	3.72	2.78	1.94	4.16	3.74	2.76	1.94
		2	8.15	6.47	3.84	2.13	8.25	6.66	4.09	2.39	8.34	6.79	4.34	2.65
		3	11.06	7.78	3.96	1.86	11.06	7.86	4.07	1.94	11.36	8.13	4.34	2.28
0.61	1	1	4.16	3.75	2.78	1.94	4.15	3.75	2.80	1.95	4.18	3.73	2.78	1.91
		2	8.12	6.48	3.85	2.13	8.23	6.63	4.11	2.45	8.31	6.80	4.28	2.74
		3	10.94	7.75	3.96	1.86	11.05	7.80	4.11	1.99	11.30	8.15	4.38	2.33
	2	1	4.17	3.75	2.79	1.95	4.18	3.75	2.78	1.93	4.17	3.75	2.82	1.96
		2	8.15	6.45	3.87	2.14	8.24	6.66	4.12	2.42	8.28	6.75	4.34	2.61
		3	11.03	7.73	3.96	1.84	11.14	7.80	4.06	1.97	11.37	8.08	4.27	2.23
	3	1	4.16	3.73	2.79	1.95	4.17	3.74	2.78	1.95	4.17	3.72	2.79	1.94
		2	8.15	6.45	3.83	2.14	8.22	6.64	4.09	2.42	8.30	6.78	4.30	2.65
		3	11.01	7.75	3.95	1.83	11.04	7.86	4.02	1.99	11.18	8.19	4.39	2.30

Table 4.7: SQ-MAC-MAP simulation SDR results (in dB) for two correlated sources with Markovian correlation parameter sent over the orthogonal MAC channel with memoryless NBNDC-QB sub-channels ($Cor = 0.0$), moderately and highly correlated NBNDC-QB sub-channels ($Cor = 0.5$ and $Cor = 0.9$) with $n = 2$, $M = \alpha = 1$ and various parameters set (q, Cor, SNR).

Part (a): Two sub-channels with identical parameters (SNR, q).

ρ_{av}	(q, q')	$Cor = Cor' = 0$ SNR (dB)				$Cor = Cor' = 0.5$ SNR (dB)				$Cor = Cor' = 0.9$ SNR (dB)			
		(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)
0.25	(1,1)	8.14	6.47	3.81	2.06	8.21	6.63	4.10	2.42	8.41	7.05	4.94	3.50
	(2,2)	8.23	6.65	4.06	2.48	8.41	7.01	4.53	2.88	8.88	8.05	6.28	4.83
	(3,3)	8.27	6.69	4.15	2.47	8.47	7.13	4.75	3.09	8.95	8.25	6.72	5.43
0.61	(1,1)	8.12	6.48	3.79	1.69	8.23	6.64	4.16	2.51	8.67	7.72	5.88	4.52
	(2,2)	8.39	7.03	4.67	3.04	8.63	7.49	5.23	3.55	9.06	8.60	7.38	6.05
	(3,3)	8.46	7.15	4.82	3.10	8.69	7.61	5.49	3.87	9.13	8.82	7.85	6.81

Table 4.7 (b): Two sub-channels with identical parameter SNR

ρ_{av}	(q, q')	$Cor = Cor' = 0$ SNR (dB)				$Cor = Cor' = 0.5$ SNR (dB)				$Cor = Cor' = 0.9$ SNR (dB)			
		(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)
0.25	(1,2)	8.19	6.56	3.92	2.26	8.31	6.83	4.30	2.61	8.61	7.54	5.51	4.07
	(1,3)	8.19	6.59	3.97	2.28	8.32	6.89	4.40	2.70	8.70	7.63	5.70	4.27
	(2,3)	8.20	6.66	4.12	2.46	8.43	7.07	4.67	3.01	8.91	8.19	6.48	5.15
0.61	(1,2)	8.27	6.76	4.16	2.43	8.41	7.05	4.63	2.95	8.89	8.16	6.61	5.26
	(1,3)	8.29	6.79	4.26	2.46	8.45	7.12	4.72	3.05	8.95	8.21	6.79	5.53
	(2,3)	8.44	7.10	4.78	3.11	8.66	7.53	5.40	3.69	9.10	8.70	7.66	6.48

Table 4.7 (c): Two sub-channels with identical parameter q

ρ_{av}	(q, q')	$Cor = Cor' = 0$ SNR (dB)				$Cor = Cor' = 0.5$ SNR (dB)				$Cor = Cor' = 0.9$ SNR (dB)			
		(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)
0.25	(1,1)	7.24	5.46	4.19	4.93	7.35	5.71	4.43	5.18	7.68	6.42	5.46	5.89
	(2,2)	7.38	5.72	4.58	5.18	7.67	6.09	4.94	5.64	8.45	7.46	6.57	7.15
	(3,3)	7.38	5.78	4.58	5.28	7.74	6.29	5.11	5.80	8.63	7.74	7.00	7.50
0.61	(1,1)	7.21	5.48	4.29	4.95	7.33	5.76	4.70	5.32	8.18	7.28	6.67	6.83
	(2,2)	7.70	6.29	5.36	5.75	8.04	6.74	5.70	6.26	8.86	8.31	7.77	7.96
	(3,3)	7.77	6.42	5.40	5.90	8.16	7.01	5.98	6.52	8.98	8.54	8.07	8.37

Table 4.7 (d): Two sub-channels with unequal parameters q and SNR

ρ_{av}	(q, q')	$Cor = Cor' = 0$ SNR (dB)				$Cor = Cor' = 0.5$ SNR (dB)				$Cor = Cor' = 0.9$ SNR (dB)			
		(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)
0.25	(1,2)	7.35	5.73	4.58	5.14	7.60	6.07	4.93	5.49	8.24	7.31	6.39	6.70
	(1,3)	7.38	5.77	4.58	5.18	7.64	6.23	5.09	5.63	8.33	7.52	6.81	6.94
	(2,3)	7.40	5.80	4.57	5.25	7.72	6.29	5.13	5.76	8.57	7.72	6.96	7.43
	(3,2)	7.39	5.77	4.59	5.23	7.68	6.14	5.00	5.69	8.47	7.53	6.61	7.25
	(3,1)	7.25	5.50	4.19	5.01	7.45	5.77	4.44	5.33	7.91	6.62	5.54	6.33
	(2,1)	7.25	5.50	4.20	5.00	7.43	5.73	4.44	5.31	7.91	6.54	5.57	6.24
0.61	(1,2)	7.59	6.24	5.31	5.59	7.86	6.60	5.66	5.98	8.65	8.05	7.53	7.51
	(1,3)	7.63	6.33	5.31	5.70	7.93	6.82	5.86	6.10	8.74	8.31	7.83	7.73
	(2,3)	7.78	6.43	5.37	5.87	8.12	6.97	5.96	6.46	8.96	8.50	8.03	8.26
	(3,2)	7.71	6.33	5.32	5.83	8.05	6.76	5.70	6.32	8.85	8.28	7.82	8.21
	(3,1)	7.36	5.57	4.31	5.10	7.54	5.93	4.78	5.51	8.44	7.60	6.95	7.35
	(2,1)	7.35	5.53	4.31	5.10	7.54	5.84	4.76	5.50	8.41	7.53	6.86	7.28

Table 4.8: SQ with instantaneous decoder (θ^*, θ'^*) - simulation SDR results (in dB) for two correlated sources with Markovian correlation parameter sent over the orthogonal MAC channel with memoryless NBNDC-QB sub-channels (Cor = 0.0), moderately and highly correlated NBNDC-QB sub-channels (Cor = 0.5 and Cor = 0.9) with $n = 2$, $M = \alpha = 1$ and various parameters set (q, Cor, SNR).

Part (a): Two sub-channels with identical parameters (SNR, q).

ρ_{av}	(q, q')	$Cor = Cor' = 0$ SNR (dB)				$Cor = Cor' = 0.5$ SNR (dB)				$Cor = Cor' = 0.9$ SNR (dB)			
		(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)
0.25	(1,1)	8.14	6.47	3.85	2.11	8.21	6.63	4.10	2.41	8.31	6.76	4.30	2.71
	(2,2)	8.16	6.49	3.83	2.14	8.22	6.64	4.09	2.39	8.30	6.78	4.32	2.68
	(3,3)	8.15	6.47	3.84	2.13	8.25	6.66	4.09	2.39	8.34	6.79	4.34	2.65
0.61	(1,1)	8.12	6.48	3.85	2.13	8.23	6.63	4.11	2.45	8.31	6.80	4.28	2.74
	(2,2)	8.15	6.45	3.87	2.14	8.24	6.66	4.12	2.42	8.28	6.75	4.34	2.61
	(3,3)	8.15	6.45	3.83	2.14	8.22	6.64	4.09	2.42	8.30	6.78	4.30	2.65

Table 4.8 (b): Two sub-channels with identical parameter SNR

ρ_{av}	(q, q')	$Cor = Cor' = 0$ SNR (dB)				$Cor = Cor' = 0.5$ SNR (dB)				$Cor = Cor' = 0.9$ SNR (dB)			
		(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)	(15,15)	(10,10)	(5,5)	(2,2)
0.25	(1,2)	8.15	6.49	3.84	2.13	8.22	6.65	4.09	2.41	8.29	6.82	4.28	2.66
	(1,3)	8.14	6.49	3.84	2.14	8.21	6.66	4.12	2.40	8.32	6.81	4.34	2.66
	(2,3)	8.12	6.46	3.86	2.12	8.24	6.66	4.11	2.43	8.27	6.83	4.27	2.70
0.61	(1,2)	8.13	6.47	3.84	2.10	8.23	6.66	4.10	2.38	8.31	6.73	4.27	2.69
	(1,3)	8.14	6.47	3.84	2.13	8.20	6.65	4.09	2.37	8.31	6.80	4.27	2.64
	(2,3)	8.15	6.51	3.85	2.12	8.23	6.64	4.13	2.40	8.28	6.83	4.40	2.65

Table 4.8 (c): Two sub-channels with identical parameter q

ρ_{av}	(q, q')	$Cor = Cor' = 0$ SNR (dB)				$Cor = Cor' = 0.5$ SNR (dB)				$Cor = Cor' = 0.9$ SNR (dB)			
		(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)
0.25	(1,1)	7.24	5.47	4.18	4.96	7.35	5.71	4.41	5.18	7.45	5.90	4.65	5.36
	(2,2)	7.24	5.48	4.17	4.95	7.37	5.67	4.41	5.20	7.48	5.87	4.62	5.39
	(3,3)	7.21	5.47	4.17	4.96	7.37	5.70	4.39	5.18	7.47	5.86	4.59	5.39
0.61	(1,1)	7.21	5.50	4.22	4.98	7.33	5.66	4.40	5.26	7.43	5.85	4.54	5.52
	(2,2)	7.22	5.48	4.17	4.94	7.37	5.69	4.39	5.20	7.48	5.94	4.64	5.39
	(3,3)	7.24	5.48	4.18	4.95	7.41	5.67	4.45	5.17	7.48	5.88	4.64	5.47

Table 4.8 (d): Two sub-channels with unequal parameters q and SNR

ρ	(q, q')	$Cor = Cor' = 0$ SNR (dB)				$Cor = Cor' = 0.5$ SNR (dB)				$Cor = Cor' = 0.9$ SNR (dB)			
		(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)	(15,10)	(15,5)	(15,2)	(10,5)
0.25	(1,2)	7.24	5.50	4.18	4.95	7.37	5.71	4.42	5.19	7.48	5.91	4.61	5.43
	(1,3)	7.25	5.47	4.19	4.96	7.36	5.69	4.39	5.19	7.46	5.83	4.57	5.44
	(2,3)	7.23	5.50	4.17	4.96	7.37	5.70	4.42	5.19	7.46	5.92	4.61	5.42
	(3,2)	7.25	5.51	4.16	4.97	7.36	5.71	4.43	5.21	7.49	5.82	4.63	5.40
	(3,1)	7.21	5.49	4.17	4.96	7.37	5.71	4.40	5.20	7.46	5.93	4.59	5.38
	(2,1)	7.22	5.50	4.19	4.96	7.36	5.68	4.40	5.21	7.46	5.88	4.67	5.44
0.61	(1,2)	7.22	5.50	4.21	4.99	7.34	5.69	4.47	5.20	7.52	5.79	4.72	5.42
	(1,3)	7.20	5.48	4.16	5.00	7.36	5.69	4.41	5.18	7.48	5.88	4.68	5.40
	(2,3)	7.25	5.48	4.18	4.97	7.38	5.70	4.40	5.18	7.43	5.92	4.62	5.42
	(3,2)	7.21	5.48	4.16	4.98	7.34	5.68	4.38	5.21	7.46	5.87	4.66	5.48
	(3,1)	7.23	5.51	4.16	4.94	7.36	5.74	4.42	5.17	7.46	5.89	4.57	5.41
	(2,1)	7.26	5.48	4.17	4.95	7.39	5.65	4.41	5.19	7.47	5.83	4.61	5.34

Chapter 5

Conclusions and Future Work

In this work, we studied joint MAP decoding for a system with two correlated Gaussian sources, scalar quantizers followed by properly chosen index assignments, and an orthogonal Rayleigh DFC MAC modeled with two independent NBNDC-QB subchannels. The NBNDC-QB is a mathematically tractable model (its transition probabilities, noise entropy rate and autocorrelation function are known in closed form) and is numerically shown to be an effective approximation, in terms of SDR, for an end-to-end Rayleigh DFC used with low coding rates. The system is called SQ-MAC-MAP, where the joint MAP decoder is implemented via a modified Viterbi algorithm. We investigated two scenarios for this system; first, the sources assumed to be memoryless and generate symbols according to a bivariate Gaussian distribution with a correlation parameter which is constant in time. In the second scenario, the correlation parameter is a first order Markov process with two state which causes change in the joint distribution over time and creates memory in the source symbols. Considering two-level quantizers in both scenarios, we derived a necessary and a sufficient condition under which our instantaneous symbol-by-symbol decoder can replace the joint MAP decoder without loss of optimality. Finally, numerical results illustrate our theoretical results and verify that the proposed system can make use of the sources' correlation and statistics, channel noise memory, and channel soft-decision information to improve SDR performance.

There are several extensions of the current work that can be investigated in the future studies. Finding necessary and sufficient conditions under which the MAP decoder becomes unnecessary is still an open problem for systems with channel memory order or coding rates greater than one. Also, it is interesting to study a system with more than two users and derive similar results for it. Furthermore, evaluating the system's effectiveness under joint MAP decoding of the Markovian sources by fitting it to the underlying fading channel is an interesting research direction. Many emerging topics such as data survivability in the distributed data storage can use the idea of harnessing the correlation (or any other shared information) between the users to jointly decode the data transmitted to a common node; this goal can be achieved by considering the mutual information and designing a more efficient (simpler and faster) channel coder for each node. On the other hand, studying the conditions under which a simple instantaneous decoder is enough to optimally decode the messages, can give guidelines on how to distribute the data and optimize the system.

Appendix A

Proof of Theorem 3.1

For the pair of mappings (θ^*, θ'^*) to be optimal sequence detection rules in the sense of minimizing the joint sequence error probability (i.e., they can replace the joint sequence MAP decoder), it is necessary and sufficient that for all input sequences $(x, x')^N \in (\mathcal{X} \times \mathcal{X}')^N$ and output sequences $(y, y')^N \in (\mathcal{Y} \times \mathcal{Y}')^N$, where $\mathcal{X} = \mathcal{X}' = \{0, 1\}$ and $\mathcal{Y} = \{0, 1, \ldots, 2^q - 1\}$ and $\mathcal{Y}' = \{0, 1, \ldots, 2^{q'} - 1\}$, the following holds

$$\gamma \triangleq \frac{\Pr\{(X, X')^N = (\tilde{y}, \tilde{y}')^N | (Y, Y')^N = (y, y')^N\}}{\Pr\{(X, X')^N = (x, x')^N | (Y, Y')^N = (y, y')^N\}} \geq 1; \tag{A.1}$$

where $(\tilde{y}, \tilde{y}')^N \triangleq (\theta^*(y), \theta'^*(y'))^N$ represents the sequence of simultaneously decoded pairs (i.e., $\tilde{y}_i = \theta^*(y_i)$ and $\tilde{y}'_i = \theta'^*(y'_i)$, $i = 1, 2, \ldots, N$).

A.1 Preliminaries

γ can be written as

$$\gamma = \frac{\Pr\{(Y, Y')^N = (y, y')^N | (X, X')^N = (\tilde{y}, \tilde{y}')^N\} \Pr\{(X, X')^N = (\tilde{y}, \tilde{y}')^N\}}{\Pr\{(Y, Y')^N = (y, y')^N | (X, X')^N = (x, x')^N\} \Pr\{(X, X')^N = (x, x')^N\}}. \tag{A.2}$$

Since the two sub-channels of the MAC are orthogonal and the input sequences are independent of the noise processes, we have

$$\begin{aligned}\gamma &= \frac{\Pr\{Y_1^N = y_1^N | X_1^N = \tilde{y}_1^N\}\Pr\{{Y'}_1^N = {y'}_1^N | {X'}_1^N = {\tilde{y}'}{}_1^N\}\Pr\{(X,X')^N = (\tilde{y},\tilde{y}')^N\}}{\Pr\{Y_1^N = y_1^N | X_1^N = x_1^N\}\Pr\{{Y'}_1^N = {y'}_1^N | {X'}_1^N = x_1^N\}\Pr\{(X,X')^N = (x,x')^N\}} \\ &= \frac{\Pr\{Z_1^N = a_1^N\}\Pr\{{Z'}_1^N = {a'}_1^N\}\Pr\{(X,X')^N = (\tilde{y},\tilde{y}')^N\}}{\Pr\{Z_1^N = z_1^N\}\Pr\{{Z'}_1^N = {z'}_1^N\}\Pr\{(X,X')^N = (x,x')^N\}} \\ &= \frac{\Pr\{Z_1 = a_1\}\Pr\{Z_1' = a_1'\}P(\tilde{y}_1,\tilde{y}_1')}{\Pr\{Z_1 = z_1\}\Pr\{Z_1' = z_1'\}P(x_1,{x'}_1)}\prod_{k=2}^{N}\frac{Q(a_k|a_{k-1})Q'(a_k'|a_{k-1}')P(\tilde{y}_k,\tilde{y}_k')}{Q(z_k|z_{k-1})Q'(z_k'|z_{k-1}')P(x_k,{x'}_k)},\end{aligned} \tag{A.3}$$

where, for $i = 1, 2, \ldots N$, $z_i \triangleq \frac{y_i-(2^q-1)x_i}{(-1)^{x_i}}$ and $a_i \triangleq \frac{y_i-(2^q-1)\tilde{y}_i}{(-1)^{\tilde{y}_i}}$; and $z_i' \triangleq \frac{y_i'-(2^{q'}-1){x'}_i}{(-1)^{{x'}_i}}$ and $a_i' \triangleq \frac{y_i'-(2^{q'}-1)\tilde{y}_i'}{(-1)^{\tilde{y}_i'}}$. Since θ and θ^* are in the form of (3.10), we have $a_i \leq 2^{q-1}-1$ and $a_i' \leq 2^{q'-1}-1$, for any $i \in \{1, 2, \ldots, N\}$. The last equation in (A.3) comes from the fact that for a queue noise model with $M = 1$, the noise process is a homogeneous first-order Markov process where

$$\Pr\{Z_k = z_k\} = \rho_{z_k}, \qquad z_k \in \mathcal{Y} \tag{A.4}$$

and

$$Q(z_k|z_{k-1}) = [\epsilon\delta_{z_k,z_{k-1}} + (1-\epsilon)\rho_{z_k}], \qquad z_k, z_{k-1} \in \mathcal{Y} \tag{A.5}$$

where $\delta_{z_k,z_{k-1}} = 1$ if $z_k = z_{k-1}$ and $\delta_{z_k,z_{k-1}} = 0$ otherwise.

Next, we review some properties regarding the NBNDC-QB sub-channels. Considering the first sub-channel, the following statements hold [29, Appendix C]:

$$z_k = \begin{cases} a_k, & \text{if} \quad z_k \leq 2^{q-1}-1 \\ 2^q-1-a_k, & \text{if} \quad \text{otherwise} \end{cases} \qquad ; k \in \{1, 2, \ldots, N\} \tag{A.6}$$

For any $k \in \{2, \ldots, N\}$,

- If $x_k = \tilde{y}_k$ and $x_{k-1} = \tilde{y}_{k-1}$,

$$\frac{Q(a_k|a_{k-1})}{Q(z_k|z_{k-1})} \frac{Q(a_k|a_{k-1})}{Q(a_k|a_{k-1})} = 1. \tag{A.7}$$

- If $x_k = \tilde{y}_k$ and $x_{k-1} \neq \tilde{y}_{k-1}$,

$$\min \frac{Q(a_k|a_{k-1})}{Q(z_k|z_{k-1})} = \min \frac{Q(a_k|a_{k-1})}{Q(a_k|2^q - 1 - a_{k-1})} = 1, \tag{A.8}$$

  where the equality can be achieved if and only if $a_k \neq a_{k-1}$.

- If $x_k \neq \tilde{y}_k$ and $x_{k-1} = \tilde{y}_{k-1}$,

$$\min \frac{Q(a_k|a_{k-1})}{Q(z_k|z_{k-1})} = \min \frac{Q(a_k|a_{k-1})}{Q(2^q - 1 - a_k|a_{k-1})} = \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}, \tag{A.9}$$

  where the equality can be achieved if $z_k = 2^{q-1}$ and $a_k \neq a_{k-1}$.

- If $x_k \neq \tilde{y}_k$ and $x_{k-1} \neq \tilde{y}_{k-1}$,

$$\min \frac{Q(a_k|a_{k-1})}{Q(z_k|z_{k-1})} = \min \frac{Q(a_k|a_{k-1})}{Q(2^q - 1 - a_k|2^q - 1 - a_{k-1})} = \frac{\epsilon + (1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon + (1-\epsilon)\rho_{2^{q-1}}}, \tag{A.10}$$

  where the equality can be achieved if $z_k = z_{k-1} = 2^{q-1}$.

The previous results are also applicable in the second sub-channel, using its corresponding parameters.

Furthermore, $\dfrac{P(\tilde{y}_k, \tilde{y}'_k)}{P(x_k, {x'}_k)} = 1$ if $x_k \oplus \tilde{y}_k = 0$ and ${x'}_k \oplus \tilde{y}'_k = 0$ or $x_k \oplus \tilde{y}_k = 1$ and ${x'}_k \oplus \tilde{y}'_k = 1$, where $\oplus$ is modulo-2 addition. Otherwise, due to the symmetry in the source distribution, we have

$$B \triangleq \min \frac{P(\tilde{y}_k, \tilde{y}'_k)}{P(x_k, x'_k)} = \min \left\{ \left(\frac{P(0,0)}{\frac{1}{2} - P(0,0)} \right), \left(\frac{\frac{1}{2} - P(0,0)}{P(0,0)} \right) \right\}. \tag{A.11}$$

We partition the index set $\mathcal{K} = \{2, 3, \ldots, N\}$ as follows:

$$\mathcal{K} = \bigcup_{i=0}^{15} \mathcal{A}_i,$$

where

$$\mathcal{A}_i \triangleq \{k \in \mathcal{K} : x_k \oplus \tilde{y}_k = i_3, x'_k \oplus \tilde{y}'_k = i_2, x_{k-1} \oplus \tilde{y}_{k-1} = i_1, x'_{k-1} \oplus \tilde{y}'_{k-1} = i_0\}, \tag{A.12}$$

and $(i_3 i_2 i_1 i_0)$ is the binary representation of i.

Hence

$$\gamma = \frac{\Pr\{Z_1 = a_1\} \Pr\{Z'_1 = a'_1\} P(\tilde{y}_1, \tilde{y}'_1)}{\Pr\{Z_1 = z_1\} \Pr\{Z'_1 = z'_1\} P(x_1, x'_1)} \prod_{i=0}^{15} \gamma_i \tag{A.13}$$

where

$$\gamma_i \triangleq \prod_{k \in \mathcal{A}_i} \frac{Q(a_k|a_{k-1}) Q'(a'_k|a'_{k-1}) P(\tilde{y}_k, \tilde{y}'_k)}{Q(z_k|z_{k-1}) Q'(z'_k|z'_{k-1}) P(x_k, x'_k)} \tag{A.14}$$

- In set $\mathcal{A}_0$, we have

$$x_k = \tilde{y}_k, \quad x'_k = \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad x'_{k-1} = \tilde{y}'_{k-1}$$

$$z_k = a_k, \quad z'_k = a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result

$$\gamma_0 = 1. \tag{A.15}$$

- In set $\mathcal{A}_1$, we have

$$x_k = \tilde{y}_k, \quad x'_k = \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad x'_{k-1} \neq \tilde{y}'_{k-1}$$
$$z_k = a_k, \quad z'_k = a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = 2^{q'} - 1 - a'_{k-1}.$$

As a result
$$\gamma_1 = \prod_{k \in \mathcal{A}_1} \frac{Q'(a'_k | a'_{k-1})}{Q'(a'_k | 2^{q'} - 1 - a'_{k-1})} \geq \prod_{k \in \mathcal{A}_1} \gamma_{\mathcal{A}_1} = 1, \tag{A.16}$$
where $\gamma_{\mathcal{A}_1} \triangleq \min \frac{Q'(a'_k | a'_{k-1})}{Q'(a'_k | 2^{q'} - 1 - a'_{k-1})} = 1$.

- In set $\mathcal{A}_2$, we have

$$x_k = \tilde{y}_k, \quad x'_k = \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad x'_{k-1} = \tilde{y}'_{k-1}$$
$$z_k = a_k, \quad z'_k = a'_k, \quad z_{k-1} = 2^q - 1 - a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result
$$\gamma_2 = \prod_{k \in \mathcal{A}_2} \frac{Q(a_k | a_{k-1})}{Q(a_k | 2^q - 1 - a_{k-1})} \geq \prod_{k \in \mathcal{A}_2} \gamma_{\mathcal{A}_2} = 1, \tag{A.17}$$
where $\gamma_{\mathcal{A}_2} \triangleq \min \frac{Q(a_k | a_{k-1})}{Q(a_k | 2^q - 1 - a_{k-1})} = 1$.

- In set $\mathcal{A}_3$, we have

$$x_k = \tilde{y}_k, \quad x'_k = \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad x'_{k-1} \neq \tilde{y}'_{k-1}$$
$$z_k = a_k, \quad z'_k = a'_k, \quad z_{k-1} = 2^q - 1 - a_{k-1}, \quad z'_{k-1} = 2^{q'} - 1 - a'_{k-1}.$$

As a result

$$\gamma_3 = \prod_{k \in \mathcal{A}_3} \frac{Q(a_k|a_{k-1})}{Q(a_k|2^q - 1 - a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(a'_k|2^{q'} - 1 - a'_{k-1})} \geq \prod_{k \in \mathcal{A}_3} \gamma_{\mathcal{A}_3} = 1, \quad \text{(A.18)}$$

where $\gamma_{\mathcal{A}_3} \triangleq \gamma_{\mathcal{A}_1}\gamma_{\mathcal{A}_2} = 1$.

- In set $\mathcal{A}_4$, we have

$$x_k = \tilde{y}_k, \quad {x'}_k \neq \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad {x'}_{k-1} = \tilde{y}'_{k-1}$$
$$z_k = a_k, \quad z'_k = 2^{q'} - 1 - a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result

$$\gamma_4 \geq \prod_{k \in \mathcal{A}_4} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^{q'} - 1 - a'_k|a'_{k-1})} \times B \geq \prod_{k \in \mathcal{A}_4} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \times B = \prod_{k \in \mathcal{A}_4} \gamma_{\mathcal{A}_4}, \quad \text{(A.19)}$$

where $\gamma_{\mathcal{A}_4} \triangleq \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \times B$.

- In set $\mathcal{A}_5$, we have

$$x_k = \tilde{y}_k, \quad {x'}_k \neq \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad {x'}_{k-1} \neq \tilde{y}'_{k-1}$$
$$z_k = a_k, \quad z'_k = 2^{q'} - 1 - a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = 2^{q'} - 1 - a'_{k-1}.$$

As a result

$$\begin{aligned} \gamma_5 &\geq \prod_{k \in \mathcal{A}_5} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^{q'} - 1 - a'_k|2^{q'} - 1 - a'_{k-1})} \times B \\ &\geq \prod_{k \in \mathcal{A}_5} \frac{\epsilon' + (1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon' + (1-\epsilon')\rho'_{2^{q'}-1}} \times B = \prod_{k \in \mathcal{A}_5} \gamma_{\mathcal{A}_5}, \end{aligned} \quad \text{(A.20)}$$

where $\gamma_{\mathcal{A}_5} \triangleq \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'}-1}} \times B$.

- In set $\mathcal{A}_6$, we have

$$x_k = \tilde{y}_k, \quad {x'}_k \neq \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad {x'}_{k-1} = \tilde{y}'_{k-1}$$
$$z_k = a_k, \quad z'_k = 2^{q'} - 1 - a'_k, \quad z_{k-1} = 2^q - 1 - a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result

$$\begin{aligned}\gamma_6 &= \prod_{k \in \mathcal{A}_6} \frac{Q(a_k|a_{k-1})}{Q(a_k|2^q-1-a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^{q'}-1-a'_k|a'_{k-1})} \times B \\ &\geq \prod_{k \in \mathcal{A}_6} \gamma_{\mathcal{A}_2} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \times B = \prod_{k \in \mathcal{A}_6} \gamma_{\mathcal{A}_6},\end{aligned} \tag{A.21}$$

where $\gamma_{\mathcal{A}_6} \triangleq \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \times B = \gamma_{\mathcal{A}_4}$.

- In set $\mathcal{A}_7$, we have

$$x_k = \tilde{y}_k, \quad {x'}_k \neq \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad {x'}_{k-1} \neq \tilde{y}'_{k-1}$$
$$z_k = a_k, \quad z'_k = 2^{q'} - 1 - a'_k, \quad z_{k-1} = 2^q - 1 - a_{k-1}, \quad z'_{k-1} = 2^{q'} - 1 - a'_{k-1}.$$

As a result

$$\begin{aligned}\gamma_7 &= \prod_{k \in \mathcal{A}_7} \frac{Q(a_k|a_{k-1})}{Q(a_k|2^q-1-a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^{q'}-1-a'_k|2^{q'}-1-a'_{k-1})} \times B \\ &\geq \prod_{k \in \mathcal{A}_7} \gamma_{\mathcal{A}_2} \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'}-1}} \times B = \prod_{k \in \mathcal{A}_7} \gamma_{\mathcal{A}_7},\end{aligned} \tag{A.22}$$

where $\gamma_{\mathcal{A}_7} \triangleq \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'}-1}} \times B = \gamma_{\mathcal{A}_5}$.

- In set $\mathcal{A}_8$,

$$x_k \neq \tilde{y}_k, \quad {x'}_k = \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad {x'}_{k-1} = \tilde{y}'_{k-1}$$
$$z_k = 2^q - 1 - a_k, \quad z'_k = a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result

$$\begin{aligned}\gamma_8 &= \prod_{k\in\mathcal{A}_8} \frac{Q(a_k|a_{k-1})}{Q(2^q-1-a_k|a_{k-1})} \times B \\ &\geq \prod_{k\in\mathcal{A}_8} \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \times B = \prod_{k\in\mathcal{A}_8} \gamma_{\mathcal{A}_8},\end{aligned} \tag{A.23}$$

where $\gamma_{\mathcal{A}_8} \triangleq \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \times B$.

- In set $\mathcal{A}_9$, we have

$$x_k \neq \tilde{y}_k, \quad {x'}_k = \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad {x'}_{k-1} \neq \tilde{y}'_{k-1}$$
$$z_k = 2^q - 1 - a_k, \quad z'_k = a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = 2^{q'} - 1 - a'_{k-1}.$$

As a result

$$\begin{aligned}\gamma_9 &= \prod_{k\in\mathcal{A}_9} \frac{Q(a_k|a_{k-1})}{Q(2^q-1-a_k|a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(a'_k|2^{q'}-1-a'_{k-1})} \times B \\ &\geq \prod_{k\in\mathcal{A}_9} \gamma_{\mathcal{A}_1} \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \times B = \prod_{k\in\mathcal{A}_9} \gamma_{\mathcal{A}_9},\end{aligned} \tag{A.24}$$

where $\gamma_{\mathcal{A}_9} \triangleq \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \times B = \gamma_{\mathcal{A}_8}$.

- In set $\mathcal{A}_{10}$, we have

$$x_k \neq \tilde{y}_k, \quad {x'}_k = \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad {x'}_{k-1} = \tilde{y}'_{k-1}$$
$$z_k = 2^q - 1 - a_k, \quad z'_k = a'_k, \quad z_{k-1} = 2^q - 1 - a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result

$$\begin{aligned}\gamma_{10} &\geq \prod_{k \in \mathcal{A}_{10}} \frac{Q(a_k|a_{k-1})}{Q(2^q - 1 - a_k|2^q - 1 - a_{k-1})} \times B \\ &\geq \prod_{k \in \mathcal{A}_{10}} \frac{\epsilon + (1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon + (1-\epsilon)\rho_{2^{q-1}}} \times B = \prod_{k \in \mathcal{A}_{10}} \gamma_{\mathcal{A}_{10}},\end{aligned} \tag{A.25}$$

where $\gamma_{\mathcal{A}_{10}} \triangleq \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}} \times B$.

- In set $\mathcal{A}_{11}$, we have

$$x_k \neq \tilde{y}_k, \quad {x'}_k = \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad {x'}_{k-1} \neq \tilde{y}'_{k-1}$$
$$z_k = 2^q - 1 - a_k, \quad z'_k = a'_k, \quad z_{k-1} = 2^q - 1 - a_{k-1}, \quad z'_{k-1} = 2^{q'} - 1 - a'_{k-1}.$$

As a result

$$\begin{aligned}\gamma_{11} &= \prod_{k \in \mathcal{A}_{11}} \frac{Q(a_k|a_{k-1})}{Q(2^q - 1 - a_k|2^q - 1 - a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(a'_k|2^{q'} - 1 - a'_{k-1})} \times B \\ &\geq \prod_{k \in \mathcal{A}_{11}} \gamma_{\mathcal{A}_1} \frac{\epsilon + (1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon + (1-\epsilon)\rho_{2^{q-1}}} \times B = \prod_{k \in \mathcal{A}_{11}} \gamma_{\mathcal{A}_{11}},\end{aligned} \tag{A.26}$$

where $\gamma_{\mathcal{A}_{11}} \triangleq \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}} \times B = \gamma_{\mathcal{A}_{10}}$.

- In set $\mathcal{A}_{12}$, we have

$$x_k \neq \tilde{y}_k, \quad {x'}_k \neq \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad {x'}_{k-1} = \tilde{y}'_{k-1}$$

$$z_k = 2^q - 1 - a_k, \quad z'_k = 2^{q'} - 1 - a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result

$$\begin{aligned} \gamma_{12} &= \prod_{k \in \mathcal{A}_{12}} \frac{Q(a_k|a_{k-1})}{Q(2^q - 1 - a_k|a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^{q'} - 1 - a'_k|a'_{k-1})} \\ &\geq \prod_{k \in \mathcal{A}_{12}} \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}} = \prod_{k \in \mathcal{A}_{12}} \gamma_{\mathcal{A}_{12}} \geq 1, \end{aligned} \tag{A.27}$$

where $\gamma_{\mathcal{A}_{12}} \triangleq \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}} \geq 1$.

- In set $\mathcal{A}_{13}$, we have

$$x_k \neq \tilde{y}_k, \quad {x'}_k \neq \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad {x'}_{k-1} \neq \tilde{y}'_{k-1}$$

$$z_k = 2^q - 1 - a_k, \quad z'_k = 2^{q'} - 1 - a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = 2^{q'} - 1 - a'_{k-1}.$$

As a result

$$\begin{aligned} \gamma_{13} &= \prod_{k \in \mathcal{A}_{13}} \frac{Q(a_k|a_{k-1})}{Q(2^q - 1 - a_k|a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^{q'} - 1 - a'_k|2^{q'} - 1 - a'_{k-1})} \\ &\geq \prod_{k \in \mathcal{A}_{13}} \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \frac{\epsilon' + (1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon' + (1-\epsilon')\rho'_{2^{q'-1}}} = \prod_{k \in \mathcal{A}_{13}} \gamma_{\mathcal{A}_{13}} \geq 1 \end{aligned} \tag{A.28}$$

where $\gamma_{\mathcal{A}_{13}} \triangleq \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \frac{\epsilon' + (1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon' + (1-\epsilon')\rho'_{2^{q'-1}}} \geq 1$.

- In set $\mathcal{A}_{14}$, we have

$$x_k \neq \tilde{y}_k, \quad {x'}_k \neq \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad {x'}_{k-1} = \tilde{y}'_{k-1}$$
$$z_k = 2^q - 1 - a_k, \quad z'_k = 2^{q'} - 1 - a'_k, \quad z_{k-1} = 2^q - 1 - a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result

$$\begin{aligned}\gamma_{14} &= \prod_{k \in \mathcal{A}_{14}} \frac{Q(a_k|a_{k-1})}{Q(2^q - 1 - a_k|2^q - 1 - a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^{q'} - 1 - a'_k|a'_{k-1})} \\ &\geq \prod_{k \in \mathcal{A}_{14}} \frac{\epsilon + (1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon + (1-\epsilon)\rho_{2^q-1}} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} = \prod_{k \in \mathcal{A}_{14}} \gamma_{\mathcal{A}_{14}} \geq 1\end{aligned} \tag{A.29}$$

where $\gamma_{\mathcal{A}_{14}} \triangleq \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^q-1}} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \geq 1$.

- In set $\mathcal{A}_{15}$, we have

$$x_k \neq \tilde{y}_k, \quad {x'}_k \neq \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad {x'}_{k-1} \neq \tilde{y}'_{k-1}$$
$$z_k = 2^q - 1 - a_k, \quad z'_k = 2^{q'} - 1 - a'_k, \quad z_{k-1} = 2^q - 1 - a_{k-1}, \quad z'_{k-1} = 2^{q'} - 1 - a'_{k-1}.$$

As a result

$$\begin{aligned}\gamma_{15} &= \prod_{k \in \mathcal{A}_{15}} \frac{Q(a_k|a_{k-1})}{Q(2^q - 1 - a_k|2^q - 1 - a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^{q'} - 1 - a'_k|2^{q'} - 1 - a'_{k-1})} \\ &\geq \prod_{k \in \mathcal{A}_{15}} \frac{\epsilon + (1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon + (1-\epsilon)\rho_{2^q-1}} \frac{\epsilon' + (1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon' + (1-\epsilon')\rho'_{2^{q'}-1}} = \prod_{k \in \mathcal{A}_{15}} \gamma_{\mathcal{A}_{15}} \geq 1\end{aligned} \tag{A.30}$$

where $\gamma_{\mathcal{A}_{15}} \triangleq \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^q-1}} \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'}-1}} \geq 1$.

A.2 Sufficient Condition

In order to derive a sufficient condition for optimal detection, we need to find a lower bound for γ which is greater than or equal to one for all input and output sequences; so a comparison between γ_i's is required. $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_{12}, \mathcal{A}_{13}, \mathcal{A}_{14}$ and $\mathcal{A}_{15}$ are the only cases with $\gamma_i \geq 1$; so we need to evaluate the other remaining cases. Since the number of elements in each $\mathcal{A}_i$ (denoted by $|\mathcal{A}_i|$), $i \in \{0, \ldots 15\}$ is not deterministic, we compare $\gamma_{\mathcal{A}_i}$ which is the minimum value of γ_i when $|\mathcal{A}_i| = 1$, for $i = 0, \ldots, 15$. Cases $\mathcal{A}_8, \mathcal{A}_9, \mathcal{A}_{10}$, and $\mathcal{A}_{11}$ can be compared with each other as follows,

$$\gamma_{\mathcal{A}_8} = \gamma_{\mathcal{A}_9} \geq \gamma_{\mathcal{A}_{10}} = \gamma_{\mathcal{A}_{11}} \tag{A.31}$$

Cases $\mathcal{A}_4, \mathcal{A}_5, \mathcal{A}_6$, and $\mathcal{A}_7$ can be compared with each other as follows,

$$\gamma_{\mathcal{A}_4} = \gamma_{\mathcal{A}_6} \geq \gamma_{\mathcal{A}_5} = \gamma_{\mathcal{A}_7} \tag{A.32}$$

In (A.31) and (A.32) equality holds when $\epsilon = 0$ and $\epsilon' = 0$, respectively.
Finally, we must compare $\gamma_{\mathcal{A}_5}$ and $\gamma_{\mathcal{A}_{10}}$ which will lead into comparing $\frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}}}$ and $\frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}}$.
As a result,

$$\min\{\gamma_{\mathcal{A}_4}, \gamma_{\mathcal{A}_5}, \gamma_{\mathcal{A}_6}, \gamma_{\mathcal{A}_7}, \gamma_{\mathcal{A}_8}, \gamma_{\mathcal{A}_9}, \gamma_{\mathcal{A}_{10}}, \gamma_{\mathcal{A}_{11}}\} = \min\left\{\left(\frac{P(0,0)}{\frac{1}{2} - P(0,0)}\right), \left(\frac{\frac{1}{2} - P(0,0)}{P(0,0)}\right)\right\} A, \tag{A.33}$$

where

$$A = \min\left\{\frac{\epsilon' + (1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon' + (1-\epsilon')\rho'_{2^{q'-1}}}, \frac{\epsilon + (1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon + (1-\epsilon)\rho_{2^{q-1}}}\right\}. \tag{A.34}$$

If condition (3.21) holds, by looking at (A.33), we can conclude that $\min\{\gamma_{\mathcal{A}_i}, i \in \{1, 2, \ldots, 15\}\} \geq 1$ which results in $\gamma_i \geq 1$, for any $i \in \{1, 2, \ldots, 15\}$. Hence

$$\prod_{i=0}^{i=15} \gamma_i \geq 1. \tag{A.35}$$

Thus, we only have to consider value of the initial term in (A.13), denoted by $C \triangleq \frac{\Pr\{Z_1=a_1\}\Pr\{Z_1'=a_1'\}P(\tilde{y}_1,\tilde{y}_1')}{\Pr\{Z_1=z_1\}\Pr\{Z_1'=z_1'\}P(x_1,x'_1)}$. Hence, we must compare four possible cases that can occur for this term.

Case 1) $x_1 = \tilde{y}_1, x'_1 = \tilde{y}_1' \Rightarrow z_1 = a_1, z_1' = a_1'$:

$$C = \frac{\Pr\{Z_1 = z_1\}\Pr\{Z_1' = z_1'\}P(x_1, x'_1)}{\Pr\{Z_1 = z_1\}\Pr\{Z_1' = z_1'\}P(x_1, x'_1)} = 1. \tag{A.36}$$

Case 2) $x_1 = \tilde{y}_1, x'_1 \neq \tilde{y}_1' \Rightarrow z_1 = a_1, z_1' = 2^{q'} - 1 - a_1'$:

$$C = \frac{\Pr\{Z_1 = a_1\}\Pr\{Z_1' = a_1'\}P(x_1, 1 - x'_1)}{\Pr\{Z_1 = a_1\}\Pr\{Z_1' = 2^{q'} - 1 - a_1'\}P(x_1, x'_1)} \geq \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \times B. \tag{A.37}$$

Case 3) $x_1 \neq \tilde{y}_1, x'_1 = \tilde{y}_1' \Rightarrow z_1 = 2^q - 1 - a_1, z_1' = a_1'$:

$$C = \frac{\Pr\{Z_1 = a_1\}\Pr\{Z_1' = a_1'\}P(1 - x_1, x'_1)}{\Pr\{Z_1 = 2^q - 1 - a_1\}\Pr\{Z_1' = a_1'\}P(x_1, x'_1)} \geq \frac{\rho_{2^{q-1}-1}}{\rho_{2^q-1}} \times B. \tag{A.38}$$

Case 4) $x_1 \neq \tilde{y}_1, x'_1 \neq \tilde{y}_1' \Rightarrow z_1 = 2^q - 1 - a_1, z_1' = 2^{q'} - 1 - a_1'$:

$$C = \frac{\Pr\{Z_1 = a_1\}\Pr\{Z_1' = a_1'\}P(1 - x_1, 1 - x'_1)}{\Pr\{Z_1 = 2^q - 1 - a_1\}\Pr\{Z_1' = 2^{q'} - 1 - a_1'\}P(x_1, x'_1)} \geq \frac{\rho_{2^{q-1}-1}}{\rho_{2^q-1}} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \geq 1. \tag{A.39}$$

We have the following inequalities

$$\begin{aligned}\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \geq 1 \quad &\Rightarrow \quad \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \geq \frac{\epsilon + (1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon + (1-\epsilon)\rho_{2^{q-1}}}, \\ \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}} \geq 1 \quad &\Rightarrow \quad \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}} \geq \frac{\epsilon' + (1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon' + (1-\epsilon')\rho'_{2^{q'-1}}}.\end{aligned} \tag{A.40}$$

Therefore,

$$\min\left\{\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}, \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\right\} \geq A, \tag{A.41}$$

which implies that

$$C \geq \min\left\{\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}, \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\right\} \times B \geq A \times B. \tag{A.42}$$

If condition (3.21) holds, $C \geq 1$ in all the possible cases. Taking result (A.35) into account, we have

$$\gamma = C \prod_{i=0}^{i=15} \gamma_i \geq 1, \tag{A.43}$$

which means that the pair of mapping (θ^*, θ'^*) is an optimal sequence joint MAP decoding rule.

A.3 Necessary Condition

To prove the converse, assume that (3.21) does not hold; i.e., $B \times A < 1$, where

$$B = \min\left\{\left(\frac{P(0,0)}{\frac{1}{2} - P(0,0)}\right), \left(\frac{\frac{1}{2} - P(0,0)}{P(0,0)}\right)\right\}. \tag{A.44}$$

Without loss of generality, assume that $B = \dfrac{\frac{1}{2} - P(0,0)}{P(0,0)}$. Then, we present an example such that the corresponding γ will be less than one.

If $A = \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}}$, for the input and output sequences

$$(x, x')^N = (\ (0,0), (0,0), \ldots, (0,0)\ ),$$
$$(y, y')^N = (\ (0,0), (0,0), \ldots, (0,0), \underbrace{(2^{q-1},0), (2^{q-1},0), \ldots, (2^{q-1},0)}_{k \text{ times}}\ ),$$

we have

$$\gamma = 1 \times 1 \times \cdots \times 1 \times \left(\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \times B \right) \left(\frac{\epsilon + (1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon + (1-\epsilon)\rho_{2^{q-1}}} \times B \right)^{k-1}. \tag{A.45}$$

Note that $\left(\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \times B \right)$ may be greater than one; but $\lim_{k\to\infty} \gamma = 0$ which means that for N large enough the corresponding γ will become less than one. To be more precise, for the worst case when $\epsilon = 1$, $k > \frac{\log \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}}{\log B}$ will result in $\gamma < 1$.

If $A = \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}}}$, the sequences

$$(x, x')^N = (\ (0,0), (0,0), \ldots, (0,0)\ ),$$
$$(y, y')^N = (\ (0,0), (0,0), \ldots, (0,0), \underbrace{(0,2^{q'-1}), (0,2^{q'-1}), \ldots, (0,2^{q'-1})}_{k \text{ times}}\ ),$$

can be given as an example input-output pair resulting in $\gamma < 1$.

Hence, if (3.21) does not hold, there exists some $(x, x')^N$ and $(y, y')^N$ for large enough N such that the mapping functions (θ^*, θ'^*) do not decode as well as the joint MAP detector.

For the necessary condition with general N, if (3.23) does not hold, we have

$$A\left(\frac{\frac{1}{2}-P(0,0)}{P(0,0)}\right) \le \min\{\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}, \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\}\left(\frac{\frac{1}{2}-P(0,0)}{P(0,0)}\right) < 1. \tag{A.46}$$

Now, if $A = \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}}$, for the input and output sequences

$$(x,x')^N = (\ (0,0),(0,0),\ldots,(0,0)\ ),$$
$$(y,y')^N = (\ (2^{q-1},0),(2^{q-1},0),\ldots,(2^{q-1},0)\ ),$$

we have

$$\gamma = \left(\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \times B\right)\left(\frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}} \times B\right)^{N-1} < 1. \tag{A.47}$$

If $A = \frac{\epsilon'+(1-\epsilon')\rho_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho_{2^{q'-1}}}$, the sequences

$$(x,x')^N = (\ (0,0),(0,0),\ldots,(0,0)\ ),$$
$$(y,y')^N = (\ (0,2^{q'-1}),(0,2^{q'-1}),\ldots,(0,2^{q'-1})\ ),$$

can be given as the example input-output pair of sequences which results in

$$\gamma = \left(\frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}} \times B\right)\left(\frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho_{2^{q'-1}}} \times B\right)^{N-1} < 1. \tag{A.48}$$

Hence, if (3.23) does not hold, there exists some $(x,x')^N$ and $(y,y')^N$ for any $N \ge 1$ such that the instantaneous decoders (θ^*, θ'^*) are not optimal.

Appendix B

Proof of Theorem 4.1

For the pair of mappings (θ^*, θ'^*) to be optimal sequence detection rules in the sense of minimizing the joint sequence error probability (i.e., can replace the joint sequence MAP decoder), it is necessary and sufficient that for all input sequences $(x, x')^N \in (\mathcal{X} \times \mathcal{X}')^N$ and output sequences $(y, y')^N \in (\mathcal{Y} \times \mathcal{Y}')^N$, where $\mathcal{X} = \mathcal{X}' = \{0, 1\}$ and $\mathcal{Y} = \{0, 1, \ldots, 2^q - 1\}$ and $\mathcal{Y}' = \left\{0, 1, \ldots, 2^{q'} - 1\right\}$, the following holds

$$\gamma \triangleq \frac{\Pr\left\{(X, X')^N = (\tilde{y}, \tilde{y}')^N | (Y, Y')^N = (y, y')^N\right\}}{\Pr\left\{(X, X')^N = (x, x')^N | (Y, Y')^N = (y, y')^N\right\}} \geq 1; \tag{B.1}$$

$(\tilde{y}, \tilde{y}')^N \triangleq (\theta^*(y), \theta'^*(y'))^N$ represents the sequence of simultaneously decoded pairs (i.e., $\tilde{y}_i = \theta^*(y_i)$ and $\tilde{y}'_i = \theta'^*(y'_i)$, $i = 1, 2, \ldots, N$).

B.1 Preliminaries

γ can be written as

$$\gamma = \frac{\Pr\left\{(Y, Y')^N = (y, y')^N | (X, X')^N = (\tilde{y}, \tilde{y}')^N\right\} \Pr\left\{(X, X')^N = (\tilde{y}, \tilde{y}')^N\right\}}{\Pr\left\{(Y, Y')^N = (y, y')^N | (X, X')^N = (x, x')^N\right\} \Pr\left\{(X, X')^N = (x, x')^N\right\}}. \tag{B.2}$$

Since the two sub-channels of the MAC are orthogonal and the input sequences are independent of the noise processes, we have

$$\begin{aligned}
\gamma &= \frac{\Pr\left\{Y_1^N = y_1^N | X_1^N = \tilde{y}_1^N\right\} \Pr\left\{{Y'}_1^N = {y'}_1^N | {X'}_1^N = \tilde{y}'^N_1\right\} \Pr\left\{(X,X')^N = (\tilde{y},\tilde{y}')^N\right\}}{\Pr\left\{Y_1^N = y_1^N | X_1^N = x_1^N\right\} \Pr\left\{{Y'}_1^N = {y'}_1^N | {X'}_1^N = x_1^N\right\} \Pr\left\{(X,X')^N = (x,x')^N\right\}} \\
&= \frac{\Pr\left\{Z_1^N = a_1^N\right\} \Pr\left\{{Z'}_1^N = {a'}_1^N\right\} \Pr\left\{(X,X')^N = (\tilde{y},\tilde{y}')^N\right\}}{\Pr\left\{Z_1^N = z_1^N\right\} \Pr\left\{{Z'}_1^N = {z'}_1^N\right\} \Pr\left\{(X,X')^N = (x,x')^N\right\}} \\
&= \frac{\Pr\left\{Z_1 = a_1\right\} \Pr\left\{Z'_1 = a'_1\right\} P(\tilde{y}_1,\tilde{y}'_1)}{\Pr\left\{Z_1 = z_1\right\} \Pr\left\{Z'_1 = z'_1\right\} P(x_1,x'_1)} \prod_{k=2}^{N} \frac{Q(a_k|a_{k-1})Q'(a'_k|a'_{k-1})P((\tilde{y}_k,\tilde{y}'_k)|(\tilde{y}_{k-1},\tilde{y}'_{k-1}))}{Q(z_k|z_{k-1})Q'(z'_k|z'_{k-1})P((x_k,x'_k)|(x_{k-1},x'_{k-1}))} \\
&= \prod_{k=2}^{N} \frac{Q(a_k|a_{k-1})Q'(a'_k|a'_{k-1})P((\tilde{y}_k,\tilde{y}'_k)|(\tilde{y}_{k-1},\tilde{y}'_{k-1}))}{Q(z_k|z_{k-1})Q'(z'_k|z'_{k-1})P((x_k,x'_k)|(x_{k-1},x'_{k-1}))}.
\end{aligned} \tag{B.3}$$

Here the last equality follows from the assumption $X_1 = \tilde{Y}_1$ and $X'_1 = \tilde{Y}'_1$ in Theorem 4.1 which results in $z_1 = a_1$ and $z'_1 = a'_1$.

Similar to A, We partition the index set $\mathcal{K} = \{2, 3, \ldots, N\}$ as follows:

$$\mathcal{K} = \bigcup_{i=0}^{15} \mathcal{A}_i,$$

where

$$\mathcal{A}_i \triangleq \left\{k \in \mathcal{K} : x_k \oplus \tilde{y}_k = i_3, x'_k \oplus \tilde{y}'_k = i_2, x_{k-1} \oplus \tilde{y}_{k-1} = i_1, x'_{k-1} \oplus \tilde{y}'_{k-1} = i_0\right\} \tag{B.4}$$

$(i_3 i_2 i_1 i_0)$ is the binary representation of i and $\oplus$ shows the addition in mod 2.

Hence

$$\gamma = \prod_{i=0}^{15} \gamma_i \tag{B.5}$$

where

$$\gamma_i = \prod_{k \in \mathcal{A}_i} \frac{Q(a_k|a_{k-1})Q'(a'_k|a'_{k-1})P((\tilde{y}_k, \tilde{y}'_k)|(\tilde{y}_{k-1}, \tilde{y}'_{k-1}))}{Q(z_k|z_{k-1})Q'(z'_k|z'_{k-1})P((x_k, x'_k)|(x_{k-1}, x'_{k-1}))}. \tag{B.6}$$

Using the transition matrix T and the results in (A.7) to (A.10), we find a lower bound for $\gamma_i, i = 0, \ldots, 15$.

- In set $\mathcal{A}_0$, we have

$$x_k = \tilde{y}_k, \quad x'_k = \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad x'_{k-1} = \tilde{y}'_{k-1}$$
$$z_k = a_k, \quad z'_k = a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result

$$\gamma_0 = 1. \tag{B.7}$$

- In set $\mathcal{A}_1$, we have

$$x_k = \tilde{y}_k, \quad x'_k = \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad x'_{k-1} \neq \tilde{y}'_{k-1}$$
$$z_k = a_k, \quad z'_k = a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = 2^{q'} - 1 - a'_{k-1}.$$

As a result

$$\begin{aligned} \gamma_1 &= \prod_{k \in \mathcal{A}_1} \frac{Q'(a'_k|a'_{k-1})}{Q'(a'_k|2^{q'} - 1 - a'_{k-1})} \frac{P((x_k, x'_k)|(x_{k-1}, \bar{x}'_{k-1}))}{P((x_k, x'_k)|(x_{k-1}, x'_{k-1}))} \\ &\geq \prod_{k \in \mathcal{A}_1} \min\left\{\frac{P_{00}}{P_{10}}, \frac{P_{01}}{P_{11}}, \frac{P_{02}}{P_{12}}, \frac{P_{03}}{P_{13}}, \frac{P_{10}}{P_{00}}, \frac{P_{11}}{P_{01}}, \frac{P_{12}}{P_{02}}, \frac{P_{13}}{P_{03}}\right\} \\ &= \prod_{k \in \mathcal{A}_1} \min\left\{\frac{a}{c}, \frac{b}{d}, \frac{d}{b}, \frac{c}{a}\right\} = \prod_{k \in \mathcal{A}_1} \gamma_{\mathcal{A}_1}, \end{aligned} \tag{B.8}$$

where $\gamma_{\mathcal{A}_1} = \min\left\{\frac{a}{c}, \frac{b}{d}, \frac{d}{b}, \frac{c}{a}\right\}$ and $\bar{x} \triangleq 1 - x$.

- In set $\mathcal{A}_2$, we have

$$x_k = \tilde{y}_k, \quad x'_k = \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad x'_{k-1} = \tilde{y}'_{k-1}$$
$$z_k = a_k, \quad z'_k = a'_k, \quad z_{k-1} = 2^q - 1 - a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result

$$\begin{aligned}
\gamma_2 &= \prod_{k \in \mathcal{A}_2} \frac{Q(a_k|a_{k-1})}{Q(a_k|2^q - 1 - a_{k-1})} \frac{P((x_k, x'_k)|(\bar{x}_{k-1}, x'_{k-1}))}{P((x_k, x'_k)|(x_{k-1}, x'_{k-1}))} \\
&\geq \prod_{k \in \mathcal{A}_2} \min\left\{\frac{P_{00}}{P_{20}}, \frac{P_{01}}{P_{21}}, \frac{P_{02}}{P_{22}}, \frac{P_{03}}{P_{23}}, \frac{P_{20}}{P_{00}}, \frac{P_{21}}{P_{01}}, \frac{P_{22}}{P_{02}}, \frac{P_{23}}{P_{03}}\right\} \\
&= \prod_{k \in \mathcal{A}_2} \min\left\{\frac{P_{00}}{P_{13}}, \frac{P_{01}}{P_{12}}, \frac{P_{02}}{P_{11}}, \frac{P_{03}}{P_{10}}, \frac{P_{13}}{P_{00}}, \frac{P_{12}}{P_{01}}, \frac{P_{11}}{P_{02}}, \frac{P_{10}}{P_{03}}\right\} \\
&= \prod_{k \in \mathcal{A}_2} \min\left\{\frac{a}{c}, \frac{b}{d}, \frac{d}{b}, \frac{c}{a}\right\} = \prod_{k \in \mathcal{A}_2} \gamma_{\mathcal{A}_2},
\end{aligned} \tag{B.9}$$

where $\gamma_{\mathcal{A}_2} = \gamma_{\mathcal{A}_1}$.

- In set $\mathcal{A}_3$, we have

$$x_k = \tilde{y}_k, \quad x'_k = \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad x'_{k-1} \neq \tilde{y}'_{k-1}$$
$$z_k = a_k, \quad z'_k = a'_k, \quad z_{k-1} = 2^q - 1 - a_{k-1}, \quad z'_{k-1} = 2^{q'} - 1 - a'_{k-1}.$$

As a result

$$\begin{aligned}\gamma_3 &= \prod_{k\in\mathcal{A}_3} \frac{Q(a_k|a_{k-1})}{Q(a_k|2^q-1-a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(a'_k|2^{q'}-1-a'_{k-1})} \frac{P((x_k,x'_k)|(\bar{x}_{k-1},\bar{x'}_{k-1}))}{P((x_k,x'_k)|(x_{k-1},x'_{k-1}))} \\ &\geq \prod_{k\in\mathcal{A}_3} \min\left\{\frac{P_{00}}{P_{30}},\frac{P_{01}}{P_{31}},\frac{P_{10}}{P_{20}},\frac{P_{11}}{P_{21}},\frac{P_{30}}{P_{00}},\frac{P_{31}}{P_{01}},\frac{P_{20}}{P_{10}},\frac{P_{21}}{P_{11}}\right\} \\ &= \prod_{k\in\mathcal{A}_3} \min\left\{\frac{P_{00}}{P_{03}},\frac{P_{01}}{P_{02}},\frac{P_{10}}{P_{13}},\frac{P_{11}}{P_{12}},\frac{P_{03}}{P_{00}},\frac{P_{02}}{P_{01}},\frac{P_{13}}{P_{10}},\frac{P_{12}}{P_{11}}\right\} = \prod_{k\in\mathcal{A}_3}\gamma_{\mathcal{A}_3} = 1,\end{aligned} \tag{B.10}$$

where $\gamma_{\mathcal{A}_3} = 1$.

- In set $\mathcal{A}_4$, we have

$$x_k = \tilde{y}_k, \quad {x'}_k \neq \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad {x'}_{k-1} = \tilde{y}'_{k-1}$$

$$z_k = a_k, \quad z'_k = 2^{q'}-1-a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result

$$\begin{aligned}\gamma_4 &= \prod_{k\in\mathcal{A}_4} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^{q'}-1-a'_k|a'_{k-1})} \frac{P((x_k,\bar{x'}_k)|(x_{k-1},x'_{k-1}))}{P((x_k,x'_k)|(x_{k-1},x'_{k-1}))} \\ &\geq \prod_{k\in\mathcal{A}_4} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \min\left\{\frac{P_{00}}{P_{01}},\frac{P_{02}}{P_{03}},\frac{P_{11}}{P_{10}},\frac{P_{12}}{P_{13}},\frac{P_{01}}{P_{00}},\frac{P_{03}}{P_{02}},\frac{P_{10}}{P_{11}},\frac{P_{13}}{P_{12}}\right\} \\ &= \prod_{k\in\mathcal{A}_4} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \min\left\{\frac{a}{b},\frac{b}{a},\frac{c}{d},\frac{d}{c}\right\} = \prod_{k\in\mathcal{A}_4}\gamma_{\mathcal{A}_4}.\end{aligned} \tag{B.11}$$

where $\gamma_{\mathcal{A}_4} = \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \min\left\{\frac{a}{b},\frac{b}{a},\frac{c}{d},\frac{d}{c}\right\}$.

- In set $\mathcal{A}_5$, we have

$$x_k = \tilde{y}_k, \quad {x'}_k \neq \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad {x'}_{k-1} \neq \tilde{y}'_{k-1}$$

$$z_k = a_k, \quad z'_k = 2^{q'} - 1 - a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = 2^{q'} - 1 - a'_{k-1}.$$

As a result

$$\begin{aligned}\gamma_5 &= \prod_{k\in\mathcal{A}_5} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^{q'}-1-a'_k|2^{q'}-1-a'_{k-1})} \frac{P((x_k,\bar{x'}_k)|(x_{k-1},\bar{x'}_{k-1}))}{P((x_k,x'_k)|(x_{k-1},x'_{k-1}))} \\ &\geq \prod_{k\in\mathcal{A}_5} \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'}-1}} \min\left\{\frac{P_{00}}{P_{11}},\frac{P_{01}}{P_{10}},\frac{P_{02}}{P_{13}},\frac{P_{03}}{P_{12}},\frac{P_{11}}{P_{00}},\frac{P_{10}}{P_{01}},\frac{P_{13}}{P_{02}},\frac{P_{12}}{P_{03}}\right\} \\ &= \prod_{k\in\mathcal{A}_5} \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'}-1}} \min\left\{\frac{a}{d},\frac{b}{c},\frac{c}{b},\frac{d}{a}\right\} = \prod_{k\in\mathcal{A}_5}\gamma_{\mathcal{A}_5},\end{aligned} \tag{B.12}$$

where $\gamma_{\mathcal{A}_5} = \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'}-1}} \min\left\{\frac{a}{d},\frac{b}{c},\frac{c}{b},\frac{d}{a}\right\}$.

- In set $\mathcal{A}_6$, we have

$$x_k = \tilde{y}_k, \quad {x'}_k \neq \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad {x'}_{k-1} = \tilde{y}'_{k-1}$$

$$z_k = a_k, \quad z'_k = 2^{q'} - 1 - a'_k, \quad z_{k-1} = 2^{q} - 1 - a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result

$$\begin{aligned}
\gamma_6 &= \prod_{k\in\mathcal{A}_6} \frac{Q(a_k|a_{k-1})}{Q(a_k|2^q-1-a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^{q'}-1-a'_k|a'_{k-1})} \frac{P((x_k,\bar{x}'_k)|(\bar{x}_{k-1},x'_{k-1}))}{P((x_k,x'_k)|(x_{k-1},x'_{k-1}))} \\
&\geq \prod_{k\in\mathcal{A}_6} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \min\left\{\frac{P_{00}}{P_{21}},\frac{P_{01}}{P_{20}},\frac{P_{02}}{P_{23}},\frac{P_{03}}{P_{22}},\frac{P_{21}}{P_{00}},\frac{P_{20}}{P_{01}},\frac{P_{23}}{P_{02}},\frac{P_{22}}{P_{03}}\right\} \\
&= \prod_{k\in\mathcal{A}_6} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \min\left\{\frac{P_{00}}{P_{12}},\frac{P_{01}}{P_{13}},\frac{P_{02}}{P_{10}},\frac{P_{03}}{P_{11}},\frac{P_{12}}{P_{00}},\frac{P_{13}}{P_{01}},\frac{P_{10}}{P_{02}},\frac{P_{11}}{P_{03}}\right\}. \\
&= \prod_{k\in\mathcal{A}_6} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \min\left\{\frac{a}{d},\frac{b}{c},\frac{c}{b},\frac{d}{a}\right\} = \prod_{k\in\mathcal{A}_6} \gamma_{\mathcal{A}_6},
\end{aligned} \tag{B.13}$$

where $\gamma_{\mathcal{A}_6} = \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \min\left\{\frac{a}{d},\frac{b}{c},\frac{c}{b},\frac{d}{a}\right\}$.

- In set $\mathcal{A}_7$, we have

$$x_k = \tilde{y}_k, \quad x'_k \neq \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad x'_{k-1} \neq \tilde{y}'_{k-1}$$

$$z_k = a_k, \quad z'_k = 2^{q'}-1-a'_k, \quad z_{k-1} = 2^q-1-a_{k-1}, \quad z'_{k-1} = 2^{q'}-1-a'_{k-1}.$$

As a result

$$\begin{aligned}
\gamma_7 &= \prod_{k\in\mathcal{A}_7} \frac{Q(a_k|a_{k-1})}{Q(a_k|2^q-1-a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^q-1-a'_k|2^q-1-a'_{k-1})} \frac{P((x_k,\bar{x'}_k)|(\bar{x}_{k-1},\bar{x'}_{k-1}))}{P((x_k,x'_k)|(x_{k-1},x'_{k-1}))} \\
&\geq \prod_{k\in\mathcal{A}_7} \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'}-1}} \min\left\{\frac{P_{00}}{P_{31}}, \frac{P_{01}}{P_{30}}, \frac{P_{10}}{P_{21}}, \frac{P_{11}}{P_{20}}, \frac{P_{31}}{P_{00}}, \frac{P_{30}}{P_{01}}, \frac{P_{21}}{P_{10}}, \frac{P_{20}}{P_{11}}\right\} \\
&= \prod_{k\in\mathcal{A}_7} \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'}-1}} \min\left\{\frac{P_{00}}{P_{02}}, \frac{P_{01}}{P_{03}}, \frac{P_{10}}{P_{12}}, \frac{P_{11}}{P_{13}}, \frac{P_{02}}{P_{00}}, \frac{P_{03}}{P_{01}}, \frac{P_{12}}{P_{10}}, \frac{P_{13}}{P_{11}}\right\} \\
&= \prod_{k\in\mathcal{A}_7} \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'}-1}} \min\left\{\frac{a}{b}, \frac{b}{a}, \frac{c}{d}, \frac{d}{c}\right\} = \prod_{k\in\mathcal{A}_7} \gamma_{\mathcal{A}_7}.
\end{aligned} \tag{B.14}$$

where $\gamma_{\mathcal{A}_7} = \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'}-1}} \min\left\{\frac{a}{b}, \frac{b}{a}, \frac{c}{d}, \frac{d}{c}\right\}$.

- In set $\mathcal{A}_8$,

$$x_k \neq \tilde{y}_k, \quad {x'}_k = \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad {x'}_{k-1} = \tilde{y}'_{k-1}$$

$$z_k = 2^q - 1 - a_k, \quad z'_k = a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result

$$\begin{aligned}
\gamma_8 &= \prod_{k\in\mathcal{A}_8} \frac{Q(a_k|a_{k-1})}{Q(2^q-1-a_k|a_{k-1})} \frac{P((\bar{x}_k,x'_k)|(x_{k-1},x'_{k-1}))}{P((x_k,x'_k)|(x_{k-1},x'_{k-1}))} \\
&\geq \prod_{k\in\mathcal{A}_8} \frac{\rho_{2^{q-1}-1}}{\rho_{2^q-1}} \min\left\{\frac{P_{00}}{P_{02}}, \frac{P_{01}}{P_{03}}, \frac{P_{10}}{P_{12}}, \frac{P_{11}}{P_{13}}, \frac{P_{02}}{P_{00}}, \frac{P_{03}}{P_{01}}, \frac{P_{12}}{P_{10}}, \frac{P_{13}}{P_{11}}\right\} \\
&\geq \prod_{k\in\mathcal{A}_8} \frac{\rho_{2^{q-1}-1}}{\rho_{2^q-1}} \min\left\{\frac{a}{b}, \frac{b}{a}, \frac{c}{d}, \frac{d}{c}\right\} = \prod_{k\in\mathcal{A}_8} \gamma_{\mathcal{A}_8},
\end{aligned} \tag{B.15}$$

where $\gamma_{\mathcal{A}_8} = \frac{\rho_{2^{q-1}-1}}{\rho_{2^q-1}} \min\left\{\frac{a}{b}, \frac{b}{a}, \frac{c}{d}, \frac{d}{c}\right\}$.

- In set $\mathcal{A}_9$, we have

$$x_k \neq \tilde{y}_k, \quad {x'}_k = \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad {x'}_{k-1} \neq \tilde{y}'_{k-1}$$
$$z_k = 2^q - 1 - a_k, \quad z'_k = a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = 2^{q'} - 1 - a'_{k-1}.$$

As a result

$$\begin{aligned}
\gamma_9 &= \prod_{k \in \mathcal{A}_9} \frac{Q(a_k|a_{k-1})}{Q(2^q - 1 - a_k|a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(a'_k|2^{q'} - 1 - a'_{k-1})} \frac{P((\bar{x}_k, x'_k)|(x_{k-1}, \bar{x'}_{k-1}))}{P((x_k, x'_k)|(x_{k-1}, x'_{k-1}))} \\
&\geq \prod_{k \in \mathcal{A}_9} \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \min\left\{\frac{P_{00}}{P_{12}}, \frac{P_{01}}{P_{13}}, \frac{P_{02}}{P_{10}}, \frac{P_{03}}{P_{11}}, \frac{P_{12}}{P_{00}}, \frac{P_{13}}{P_{01}}, \frac{P_{10}}{P_{02}}, \frac{P_{11}}{P_{03}}\right\} \\
&= \prod_{k \in \mathcal{A}_9} \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \min\left\{\frac{a}{d}, \frac{b}{c}, \frac{c}{b}, \frac{d}{a}\right\} = \prod_{k \in \mathcal{A}_9} \gamma_{\mathcal{A}_9},
\end{aligned} \tag{B.16}$$

where $\gamma_{\mathcal{A}_9} = \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \min\left\{\frac{a}{d}, \frac{b}{c}, \frac{c}{b}, \frac{d}{a}\right\}$.

- In set $\mathcal{A}_{10}$, we have

$$x_k \neq \tilde{y}_k, \quad {x'}_k = \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad {x'}_{k-1} = \tilde{y}'_{k-1}$$
$$z_k = 2^q - 1 - a_k, \quad z'_k = a'_k, \quad z_{k-1} = 2^q - 1 - a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result

$$
\begin{aligned}
\gamma_{10} &\geq \prod_{k\in\mathcal{A}_{10}} \frac{Q(a_k|a_{k-1})}{Q(2^q-1-a_k|2^q-1-a_{k-1})} \frac{P((\bar{x}_k,x'_k)|(\bar{x}_{k-1},x'_{k-1}))}{P((x_k,x'_k)|(x_{k-1},x'_{k-1}))} \\
&\geq \prod_{k\in\mathcal{A}_{10}} \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}} \min\left\{\frac{P_{00}}{P_{22}},\frac{P_{01}}{P_{23}},\frac{P_{02}}{P_{20}},\frac{P_{03}}{P_{21}},\frac{P_{22}}{P_{00}},\frac{P_{23}}{P_{01}},\frac{P_{20}}{P_{02}},\frac{P_{21}}{P_{03}}\right\} \\
&= \prod_{k\in\mathcal{A}_{10}} \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}} \min\left\{\frac{P_{00}}{P_{11}},\frac{P_{01}}{P_{10}},\frac{P_{02}}{P_{13}},\frac{P_{03}}{P_{12}},\frac{P_{11}}{P_{00}},\frac{P_{10}}{P_{01}},\frac{P_{13}}{P_{02}},\frac{P_{12}}{P_{03}}\right\} \\
&= \prod_{k\in\mathcal{A}_{10}} \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}} \min\left\{\frac{a}{d},\frac{b}{c},\frac{c}{b},\frac{d}{a}\right\} = \prod_{k\in\mathcal{A}_{10}} \gamma_{\mathcal{A}_{10}},
\end{aligned}
\tag{B.17}
$$

where $\gamma_{\mathcal{A}_{10}} = \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}} \min\left\{\frac{a}{d},\frac{b}{c},\frac{c}{b},\frac{d}{a}\right\}$.

- In set $\mathcal{A}_{11}$, we have

$$
x_k \neq \tilde{y}_k, \quad {x'}_k = \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad {x'}_{k-1} \neq \tilde{y}'_{k-1}
$$

$$
z_k = 2^q-1-a_k, \quad z'_k = a'_k, \quad z_{k-1} = 2^q-1-a_{k-1}, \quad z'_{k-1} = 2^{q'}-1-a'_{k-1}.
$$

As a result

$$
\begin{aligned}
\gamma_{11} &= \prod_{k\in\mathcal{A}_{11}} \frac{Q(a_k|a_{k-1})}{Q(2^q-1-a_k|2^q-1-a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(a'_k|2^{q'}-1-a'_{k-1})} \frac{P((\bar{x}_k,x'_k)|(\bar{x}_{k-1},\bar{x}'_{k-1}))}{P((x_k,x'_k)|(x_{k-1},x'_{k-1}))} \\
&\geq \prod_{k\in\mathcal{A}_{11}} \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}} \min\left\{\frac{P_{00}}{P_{32}},\frac{P_{02}}{P_{30}},\frac{P_{10}}{P_{22}},\frac{P_{12}}{P_{20}},\frac{P_{32}}{P_{00}},\frac{P_{30}}{P_{02}},\frac{P_{22}}{P_{10}},\frac{P_{20}}{P_{12}}\right\} \\
&= \prod_{k\in\mathcal{A}_{11}} \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}} \min\left\{\frac{P_{00}}{P_{01}},\frac{P_{02}}{P_{03}},\frac{P_{10}}{P_{11}},\frac{P_{12}}{P_{13}},\frac{P_{01}}{P_{00}},\frac{P_{03}}{P_{02}},\frac{P_{11}}{P_{10}},\frac{P_{13}}{P_{12}}\right\} \\
&= \prod_{k\in\mathcal{A}_{11}} \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}} \min\left\{\frac{a}{b},\frac{b}{a},\frac{c}{d},\frac{d}{c}\right\} = \prod_{k\in\mathcal{A}_{11}} \gamma_{\mathcal{A}_{11}},
\end{aligned}
\tag{B.18}
$$

where $\gamma_{\mathcal{A}_{11}} = \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^q-1}} \min\left\{\frac{a}{b}, \frac{b}{a}, \frac{c}{d}, \frac{d}{c}\right\}$.

- In set $\mathcal{A}_{12}$, we have

$$x_k \neq \tilde{y}_k, \quad {x'}_k \neq \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad {x'}_{k-1} = \tilde{y}'_{k-1}$$
$$z_k = 2^q - 1 - a_k, \quad z'_k = 2^{q'} - 1 - a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result

$$\begin{aligned}
\gamma_{12} &= \prod_{k \in \mathcal{A}_{12}} \frac{Q(a_k|a_{k-1})}{Q(2^q-1-a_k|a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^{q'}-1-a'_k|a'_{k-1})} \frac{P((\bar{x}_k, \bar{x'}_k)|(x_{k-1}, x'_{k-1}))}{P((x_k, x'_k)|(x_{k-1}, x'_{k-1}))} \\
&\geq \prod_{k \in \mathcal{A}_{12}} \frac{\rho_{2^{q-1}-1}}{\rho_{2^q-1}} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \min\left\{\frac{P_{00}}{P_{03}}, \frac{P_{01}}{P_{02}}, \frac{P_{10}}{P_{13}}, \frac{P_{11}}{P_{12}}, \frac{P_{03}}{P_{00}}, \frac{P_{02}}{P_{01}}, \frac{P_{13}}{P_{10}}, \frac{P_{12}}{P_{11}}\right\} \\
&= \prod_{k \in \mathcal{A}_{12}} \frac{\rho_{2^{q-1}-1}}{\rho_{2^q-1}} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} = \prod_{k \in \mathcal{A}_{12}} \gamma_{\mathcal{A}_{12}},
\end{aligned} \tag{B.19}$$

where $\gamma_{\mathcal{A}_{12}} = \frac{\rho_{2^{q-1}-1}}{\rho_{2^q-1}} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}}$.

- In set $\mathcal{A}_{13}$, we have

$$x_k \neq \tilde{y}_k, \quad {x'}_k \neq \tilde{y}'_k, \quad x_{k-1} = \tilde{y}_{k-1}, \quad {x'}_{k-1} \neq \tilde{y}'_{k-1}$$
$$z_k = 2^q - 1 - a_k, \quad z'_k = 2^{q'} - 1 - a'_k, \quad z_{k-1} = a_{k-1}, \quad z'_{k-1} = 2^{q'} - 1 - a'_{k-1}.$$

As a result

$$\begin{aligned}
\gamma_{13} &= \prod_{k\in\mathcal{A}_{13}} \frac{Q(a_k|a_{k-1})}{Q(2^q-1-a_k|a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^{q'}-1-a'_k|2^{q'}-1-a'_{k-1})} \frac{P((\bar{x}_k,\bar{x'}_k)|(x_{k-1},\bar{x'}_{k-1}))}{P((x_k,x'_k)|(x_{k-1},x'_{k-1}))} \\
&\geq \prod_{k\in\mathcal{A}_{13}} \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}}} \min\left\{\frac{P_{00}}{P_{13}},\frac{P_{01}}{P_{12}},\frac{P_{02}}{P_{11}},\frac{P_{03}}{P_{10}},\frac{P_{13}}{P_{00}},\frac{P_{12}}{P_{01}},\frac{P_{11}}{P_{02}},\frac{P_{10}}{P_{03}}\right\} \\
&= \prod_{k\in\mathcal{A}_{13}} \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}}} \min\left\{\frac{a}{c},\frac{b}{d},\frac{d}{b},\frac{c}{a}\right\} = \prod_{k\in\mathcal{A}_{13}} \gamma_{\mathcal{A}_{13}},
\end{aligned} \tag{B.20}$$

where $\gamma_{\mathcal{A}_{13}} = \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}}} \min\left\{\frac{a}{c},\frac{b}{d},\frac{d}{b},\frac{c}{a}\right\}$.

- In set $\mathcal{A}_{14}$, we have

$$x_k \neq \tilde{y}_k, \quad x'_k \neq \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad x'_{k-1} = \tilde{y}'_{k-1}$$

$$z_k = 2^q-1-a_k, \quad z'_k = 2^{q'}-1-a'_k, \quad z_{k-1} = 2^q-1-a_{k-1}, \quad z'_{k-1} = a'_{k-1}.$$

As a result

$$\begin{aligned}
\gamma_{14} &= \prod_{k\in\mathcal{A}_{14}} \frac{Q(a_k|a_{k-1})}{Q(2^q-1-a_k|2^q-1-a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^{q'}-1-a'_k|a'_{k-1})} \frac{P((\bar{x}_k,\bar{x'}_k)|(\bar{x}_{k-1},x'_{k-1}))}{P((x_k,x'_k)|(x_{k-1},x'_{k-1}))} \\
&\geq \prod_{k\in\mathcal{A}_{14}} \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}} \min\left\{\frac{P_{00}}{P_{23}},\frac{P_{01}}{P_{22}},\frac{P_{02}}{P_{21}},\frac{P_{03}}{P_{20}},\frac{P_{23}}{P_{00}},\frac{P_{22}}{P_{01}},\frac{P_{21}}{P_{02}},\frac{P_{20}}{P_{03}}\right\} \\
&= \prod_{k\in\mathcal{A}_{14}} \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}} \min\left\{\frac{P_{00}}{P_{10}},\frac{P_{01}}{P_{11}},\frac{P_{02}}{P_{12}},\frac{P_{03}}{P_{13}},\frac{P_{10}}{P_{00}},\frac{P_{11}}{P_{01}},\frac{P_{12}}{P_{02}},\frac{P_{13}}{P_{03}}\right\} \\
&= \prod_{k\in\mathcal{A}_{14}} \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}} \min\left\{\frac{a}{c},\frac{b}{d},\frac{d}{b},\frac{c}{a}\right\} = \prod_{k\in\mathcal{A}_{14}} \gamma_{\mathcal{A}_{14}},
\end{aligned} \tag{B.21}$$

where $\gamma_{\mathcal{A}_{14}} = \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q}-1}} \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}} \min\left\{\frac{a}{c}, \frac{b}{d}, \frac{d}{b}, \frac{c}{a}\right\}$.

- In set $\mathcal{A}_{15}$, we have

$$x_k \neq \tilde{y}_k, \quad {x'}_k \neq \tilde{y}'_k, \quad x_{k-1} \neq \tilde{y}_{k-1}, \quad {x'}_{k-1} \neq \tilde{y}'_{k-1}$$

$$z_k = 2^q - 1 - a_k, \quad z'_k = 2^{q'} - 1 - a'_k, \quad z_{k-1} = 2^q - 1 - a_{k-1}, \quad z'_{k-1} = 2^{q'} - 1 - a'_{k-1}.$$

As a result

$$\begin{aligned}\gamma_{15} = \prod_{k\in\mathcal{A}_{15}} &\frac{Q(a_k|a_{k-1})}{Q(2^q-1-a_k|2^q-1-a_{k-1})} \frac{Q'(a'_k|a'_{k-1})}{Q'(2^q-1-a'_k|2^q-1-a'_{k-1})} \times \\ &\frac{P((\bar{x}_k, \bar{x'}_k)|(\bar{x}_{k-1}, \bar{x'}_{k-1}))}{P((x_k, x'_k)|(x_{k-1}, x'_{k-1}))} \geq \prod_{k\in\mathcal{A}_{15}} \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q}-1}} \frac{\epsilon+(1-\epsilon)\rho'_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho'_{2^{q}-1}} \\ &= \prod_{k\in\mathcal{A}_{15}} \gamma_{\mathcal{A}_{15}} \geq 1,\end{aligned} \tag{B.22}$$

where $\gamma_{\mathcal{A}_{15}} = \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q}-1}} \frac{\epsilon+(1-\epsilon)\rho'_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho'_{2^{q}-1}}$.

In order to achieve a sufficient/necessary condition for optimal detection, we need to find a lower bound for γ; so a comparison between γ_i's is required. For cases $\mathcal{A}_0, \mathcal{A}_3, \mathcal{A}_{12}$, and $\mathcal{A}_{15}$, we will have that $\gamma_0, \gamma_3, \gamma_{12}$, and γ_{15} are greater than or equal to one; so we evaluate the other cases by comparing their $\gamma_{\mathcal{A}_i}$s.

Cases $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_{13}, \mathcal{A}_{14}$ can be compared with each other as follows,

$$\begin{aligned}
&\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}\frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}}}\min\left\{\frac{a}{c},\frac{b}{d},\frac{d}{b},\frac{c}{a}\right\}\geq\min\left\{\frac{a}{c},\frac{b}{d},\frac{d}{b},\frac{c}{a}\right\}\Longrightarrow\gamma_{\mathcal{A}_{13}}\geq\gamma_{\mathcal{A}_1}\\
&\frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}}\frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\min\left\{\frac{a}{c},\frac{b}{d},\frac{d}{b},\frac{c}{a}\right\}\geq\min\left\{\frac{a}{c},\frac{b}{d},\frac{d}{b},\frac{c}{a}\right\}\Longrightarrow\gamma_{\mathcal{A}_{14}}\geq\gamma_{\mathcal{A}_2}.
\end{aligned}\tag{B.23}$$

Similarly, we can look at cases $\mathcal{A}_4,\mathcal{A}_7,\mathcal{A}_8,\mathcal{A}_{11}$

$$\begin{aligned}
&\frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\min\left\{\frac{a}{b},\frac{b}{a},\frac{c}{d},\frac{d}{c}\right\}\geq\frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}}}\min\left\{\frac{a}{b},\frac{b}{a},\frac{c}{d},\frac{d}{c}\right\}\Longrightarrow\gamma_{\mathcal{A}_4}\geq\gamma_{\mathcal{A}_7}\\
&\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}\min\left\{\frac{a}{b},\frac{b}{a},\frac{c}{d},\frac{d}{c}\right\}\geq\frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}}\min\left\{\frac{a}{b},\frac{b}{a},\frac{c}{d},\frac{d}{c}\right\}\Longrightarrow\gamma_{\mathcal{A}_8}\geq\gamma_{\mathcal{A}_{11}}.
\end{aligned}\tag{B.24}$$

In the same way, we can compare the cases $\mathcal{A}_5,\mathcal{A}_6,\mathcal{A}_9,\mathcal{A}_{10}$

$$\begin{aligned}
&\frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\min\left\{\frac{a}{d},\frac{b}{c},\frac{c}{b},\frac{d}{a}\right\}\geq\frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}}}\min\left\{\frac{a}{d},\frac{b}{c},\frac{c}{b},\frac{d}{a}\right\}\Longrightarrow\gamma_{\mathcal{A}_6}\geq\gamma_{\mathcal{A}_5}\\
&\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}\min\left\{\frac{a}{d},\frac{b}{c},\frac{c}{b},\frac{d}{a}\right\}\geq\frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q-1}}}\min\left\{\frac{a}{d},\frac{b}{c},\frac{c}{b},\frac{d}{a}\right\}\Longrightarrow\gamma_{\mathcal{A}_9}\geq\gamma_{\mathcal{A}_{10}}.
\end{aligned}\tag{B.25}$$

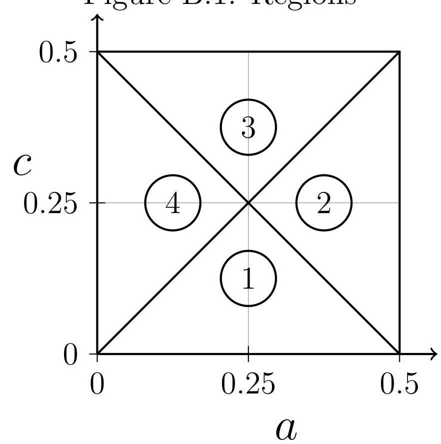
We define the following quantities:

$$\begin{aligned}
A_1&=\min\left\{\frac{a}{c},\frac{b}{d},\frac{d}{b},\frac{c}{a}\right\},\\
A_2&=\min\left\{\frac{a}{b},\frac{b}{a},\frac{c}{d},\frac{d}{c}\right\},\\
A_3&=\min\left\{\frac{a}{d},\frac{b}{c},\frac{c}{b},\frac{d}{a}\right\}.
\end{aligned}\tag{B.26}$$

We need to find $\min\{A_1, A_2, A_3\}$ for further progress. For this purpose, we divide the space of (a, c), $0 \leq a, c \leq 1$ into four regions:

- Region 1: $0 \leq c < 0.25$ and $0 \leq c < a \leq 0.5$ and $0 < a + c \leq 0.5$
- Region 2: $0.25 < a \leq 0.5$ and $0 < c \leq a \leq 0.5$ and $0.5 < a + c \leq 1$
- Region 3: $0.25 < c \leq 0.5$ and $0 \leq a < c \leq 0.5$ and $0.5 \leq a + c < 1$
- Region 4: $0 \leq a < 0.25$ and $0 \leq a \leq c < 0.5$ and $0 \leq a + c < 0.5$

Figure B.1: Regions



In each region, using the fact that $a + b = c + d = 1/2$ and performing algebraic manipulation on the descriptions of that area, required results to make a comparison between A_1, A_2, and A_3 can be acquired. For the Region 1, the following results can be acquired:

$$\left.\begin{aligned} a > c &\Longrightarrow \frac{a}{c} > \frac{c}{a} \\ d > b &\Longrightarrow \frac{d}{b} > \frac{b}{d} \\ a + c < 1/2 &\Longrightarrow \frac{b}{d} > \frac{c}{a} \end{aligned}\right\} \Longrightarrow A_1 = \frac{c}{a}\,. \tag{B.27}$$

Furthermore,

$$\left.\begin{aligned} c < 0.25 &\Longrightarrow \frac{d}{c} > \frac{c}{d} \\ a > c &\Longrightarrow \frac{a}{b} > \frac{c}{d} \\ a + c < 1/2 &\Longrightarrow \frac{b}{a} > \frac{c}{d} \end{aligned}\right\} \Longrightarrow A_2 = \frac{c}{d}\ . \tag{B.28}$$

Finally,

$$\left.\begin{aligned} a + c < 1/2 &\Longrightarrow c < b &&\Longrightarrow \frac{b}{c} > \frac{c}{b} && \text{(I)} \\ a + c < 1/2 &\Longrightarrow a < d &&\Longrightarrow \frac{d}{a} > \frac{a}{d} && \text{(II)} \\ a + c < 1/2 & &&\Longrightarrow \frac{a}{d} > \frac{c}{b} && \text{(III)} \end{aligned}\right\} \Longrightarrow A_3 = \frac{c}{b}\ . \tag{B.29}$$

Since $d > b$ and $d > a$, we will have $A_2 < A_3$ and $A_2 < A_1$.

Using the symmetry in the definition of the regions and the A_1, A_2, and A_3 expressions, similar results can be derived.

For Region 2, we have

$$A_1 = \frac{b}{d}, \quad A_2 = \frac{b}{a}, \quad A_3 = \frac{b}{c} \tag{B.30}$$

$$\min\left\{\frac{b}{d}, \frac{b}{a}, \frac{b}{c}\right\} = \frac{b}{a} = A_2 \tag{B.31}$$

For Region 3, we have

$$A_1 = \frac{d}{b}, \quad A_2 = \frac{d}{c}, \quad A_3 = \frac{d}{a} \tag{B.32}$$

$$\min\left\{\frac{d}{b}, \frac{d}{c}, \frac{d}{a}\right\} = \frac{d}{c} = A_2 \tag{B.33}$$

For Region 4, we have

$$A_1 = \frac{a}{c}, \quad A_2 = \frac{a}{b}, \quad A_3 = \frac{a}{d} \tag{B.34}$$

$$\min\left\{\frac{a}{c}, \frac{a}{b}, \frac{a}{d}\right\} = \frac{a}{b} = A_2 \tag{B.35}$$

Considering the previous discussions, we can conclude that for all regions

$$\min\{A_1, A_2, A_3\} = A_2. \tag{B.36}$$

As a result,

$$\begin{array}{ccc} \gamma_{\mathcal{A}_7} < \gamma_{\mathcal{A}_5} & \text{and} & \gamma_{\mathcal{A}_4} < \gamma_{\mathcal{A}_6} \\ \gamma_{\mathcal{A}_8} < \gamma_{\mathcal{A}_9} & \text{and} & \gamma_{\mathcal{A}_{11}} < \gamma_{\mathcal{A}_{10}} \end{array} \tag{B.37}$$

Further comparison requires more knowledge about sub-channel parameters (SNR, q, ϵ) and (SNR', q', ϵ').

For any pair of input and output sequences $\{(X_i, Y_i)\}_{i=1}^{\infty}$, we define a sequence of states $\{\mathcal{S}_i\}_{i=2}^{\infty}$; where $\mathcal{S}_i = \mathcal{A}_j, j \in \{0, \ldots, 15\}$, if $i \in \mathcal{A}_j$ by the definition of partitions. Since each state $\mathcal{S}_i$ depends on the (x_i, y_i) and (x'_i, y'_i) as well as (x_{i-1}, y_{i-1}) and (x'_{i-1}, y'_{i-1}), any state can only be followed by certain number of states which are specified with one in Table (B.1).

B.2 Necessary Condition

Considering Table (B.1) and inequalities in (B.23) to (B.25) and (B.37), we prove that the necessary condition in (4.26) must hold if the mapping functions (θ, θ') given

Table B.1: Transition table for the states

Current State \ Next State	$\mathcal{A}_0$	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$	$\mathcal{A}_5$	$\mathcal{A}_6$	$\mathcal{A}_7$	$\mathcal{A}_8$	$\mathcal{A}_9$	$\mathcal{A}_{10}$	$\mathcal{A}_{11}$	$\mathcal{A}_{12}$	$\mathcal{A}_{13}$	$\mathcal{A}_{14}$	$\mathcal{A}_{15}$
$\mathcal{A}_0$	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
$\mathcal{A}_1$	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
$\mathcal{A}_2$	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
$\mathcal{A}_3$	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
$\mathcal{A}_4$	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0
$\mathcal{A}_5$	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0
$\mathcal{A}_6$	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0
$\mathcal{A}_7$	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0
$\mathcal{A}_8$	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0
$\mathcal{A}_9$	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0
$\mathcal{A}_{10}$	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0
$\mathcal{A}_{11}$	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0
$\mathcal{A}_{12}$	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1
$\mathcal{A}_{13}$	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1
$\mathcal{A}_{14}$	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1
$\mathcal{A}_{15}$	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1

in (3.10) are optimal sequence MAP detection rules. We present a pair of input and output sequences which results in $\gamma < 1$ if the necessary condition does not hold. This condition can be written as

$$\min\{\gamma_{\mathcal{A}_4}, \gamma_{\mathcal{A}_8}\} = \min\left\{\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}, \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\right\} A_2 < 1 \tag{B.38}$$

Without loss of generality, assume that $\min\left\{\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}, \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\right\} = \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}$.

- If $A_2 = \dfrac{c}{d}$,
  the input and output sequences $(x, x')^N = (\ (0,1), (0,1), \ldots, (0,1)\ )$ and $(y, y')^N = (\ (0,1), (0, 2^{q-1}-1), (0,1), (0, 2^{q-1}-1), \ldots\ )$ result in

  $$\gamma = \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\frac{c}{d} \times \frac{b}{d} \times \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\frac{c}{d} \times \frac{b}{d} \times \ldots. \tag{B.39}$$

  Since $A_2 = \frac{c}{d}$, we realize that (a, c) is in Region 1; hence, $\frac{b}{d} < 1$. We can conclude that the proposed input and output sequences result in $\gamma < 1$.

- If $A_2 = \dfrac{b}{a}$,
  the input and output sequences $(x, x')^N = (\ (0,0), (0,0), \ldots, (0,0)\ )$ and $(y, y')^N = (\ (0,0), (0, 2^{q-1}), (0,0), (0, 2^{q-1}), \ldots\ )$ result in

  $$\gamma = \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\frac{b}{a} \times \frac{c}{a} \times \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\frac{b}{a} \times \frac{c}{a} \times \ldots. \tag{B.40}$$

  Since $A_2 = \frac{b}{a}$, we realize that (a, c) is in Region 2; hence, $\frac{c}{a} < 1$. We can conclude that the proposed input and output sequences result in $\gamma < 1$.

- If $A_2 = \dfrac{d}{c}$,

the input and output sequences $(x,x')^N = (\ (0,1),(0,0),(0,1),(0,0),\dots\ )$ and $(y,y')^N = (\ (0,1),(0,2^{q-1}),(0,1),(0,2^{q-1}),\dots\ )$ result in

$$\gamma = \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\frac{d}{c} \times \frac{d}{b} \times \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\frac{d}{c} \times \frac{d}{b} \times \dots. \tag{B.41}$$

Since $A_2 = \frac{d}{c}$, we realize that (a,c) is in Region 3; hence, $\frac{d}{b} < 1$. We can conclude that the proposed input and output sequences result in $\gamma < 1$.

- If $A_2 = \dfrac{a}{b}$,

  the input and output sequences $(x,x')^N = (\ (0,0),(0,1),(0,0),(0,1),\dots\ )$ and $(y,y')^N = (\ (0,0),(0,2^{q-1}-1),(0,0),(0,2^{q-1}-1),\dots\ )$ result in

$$\gamma = \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\frac{a}{b} \times \frac{a}{c} \times \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\frac{a}{b} \times \frac{a}{c} \times \dots. \tag{B.42}$$

  Since $A_2 = \frac{a}{b}$, we realize that (a,c) is in Region 4; hence, $\frac{a}{c} < 1$. We can conclude that the proposed input and output sequences result in $\gamma < 1$.

Similarly, if $\min\left\{\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}, \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\right\} = \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}$, switching X with X' and Y with Y' in the above input and output sequence examples will result in $\gamma < 1$.

For the case when N is large enough, if the necessary condition 4.27 does not hold, we will have

$$\min\left\{A\min\left\{\frac{a}{d},\frac{d}{a}\right\}, \min\left\{\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}, \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\right\}\min\left\{\frac{a^2}{bc},\frac{bc}{a^2},\frac{bc}{d^2},\frac{d^2}{bc}\right\}\right\} < 1, \tag{B.43}$$

where

$$A = \min\left\{\frac{\epsilon' + (1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon' + (1-\epsilon')\rho'_{2^{q'-1}}}, \frac{\epsilon + (1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon + (1-\epsilon)\rho_{2^{q-1}}}\right\}.$$

There are two possible cases:

- First case, assume that $A \min\left\{\frac{a}{d}, \frac{d}{a}\right\} < 1$.

  Without loss of generality, let's $A = \frac{\epsilon+(1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon+(1-\epsilon)\rho_{2^{q}-1}}$.

  If $\min\left\{\frac{a}{d}, \frac{d}{a}\right\} = \frac{a}{d}$, the input and output sequences $(x, x')^N = (\ (0,1), (0,1), \ldots, (0,1)\ )$ and

  $(y, y')^N = (\ (0,1), (2^{q-1}, 1), (2^{q-1}, 1), \ldots, (2^{q-1}, 1)\ )$ result in

$$\gamma = \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q}-1}} \times \frac{a}{d} \times \left(\frac{\epsilon + (1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon + (1-\epsilon)\rho_{2^{q}-1}} \times \frac{a}{d} \right)^{N-1},$$

  which goes to zero for a large enough N.

  If $\min\left\{\frac{a}{d}, \frac{d}{a}\right\} = \frac{d}{a}$, the input and output sequences $(x, x')^N = (\ (0,0), (0,0), \ldots, (0,0)\ )$ and $(y, y')^N = (\ (0,0), (2^{q-1}, 0), (2^{q-1}, 0), \ldots, (2^{q-1}, 0)\ )$ result in

$$\gamma = \frac{\rho_{2^{q-1}-1}}{\rho_{2^{q}-1}} \times \frac{d}{a} \times \left(\frac{\epsilon + (1-\epsilon)\rho_{2^{q-1}-1}}{\epsilon + (1-\epsilon)\rho_{2^{q}-1}} \times \frac{d}{a} \right)^{N-1},$$

  which goes to zero for a large enough N.

  Similarly, if $A = \frac{\epsilon'+(1-\epsilon')\rho'_{2^{q'-1}-1}}{\epsilon'+(1-\epsilon')\rho'_{2^{q'}-1}}$, switching X with X' and Y with Y' in the above input and output sequence examples will result in $\gamma = 0$ for a large enough N.

- Second case, assume that $\min\left\{\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q}-1}}, \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'}-1}}\right\} \min\left\{\frac{a^2}{bc}, \frac{bc}{a^2}, \frac{bc}{d^2}, \frac{d^2}{bc}\right\} < 1$.

  One can verify that the input and output sequences given as examples in the proof of the necessary condition (4.26) for general N will result in the $\gamma < 1$ for a large enough N.

As an example, if $\min\left\{\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}}, \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\right\}\min\left\{\frac{a^2}{bc}, \frac{bc}{a^2}, \frac{bc}{d^2}, \frac{d^2}{bc}\right\} = \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\frac{bc}{d^2}$, the input and output sequences $(x,x')^N = (\ (0,1),(0,1),\dots,(0,1)\ )$ and $(y,y')^N = (\ (0,1),(0,2^{q-1}-1),(0,1),(0,2^{q-1}-1),\dots\ )$ result in

$$\gamma = \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\frac{c}{d} \times \frac{b}{d} \times \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\frac{c}{d} \times \frac{b}{d} \times \dots \leq \frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\left\{\frac{\rho'_{2^{q'-1}-1}}{\rho'_{2^{q'-1}}}\frac{bc}{d^2}\right\}^{\lfloor\frac{N-1}{2}\rfloor},$$

which goes to zero as N grows. Thus, there exists N large enough for which $\gamma < 1$.

B.3 Sufficient condition

In order to prove the sufficient condition (4.24) for the optimal detection, we show that γ computed via (B.3) for any input and output sequences is greater than or equal to one under the sufficient condition.

First, we present a lower bound for the γ. Assume that $\{\mathcal{S}_i\}_{i=2}^N$ is the state sequence assigned to an arbitrary input and output sequences $\{(x_i,x'_i)\}_{i=1}^N$ and $\{(y_i,y'_i)\}_{i=1}^N$. We can write the following lower bound for the corresponding γ

$$\gamma \geq \prod_{i=1}^{N}\gamma_{\mathcal{S}_i}, \tag{B.44}$$

where $\gamma_{\mathcal{S}_i} \in \{\gamma_{\mathcal{A}_1},\dots\gamma_{\mathcal{A}_{15}}\}, i = 2,3,\dots,N$, is a lower bound for the contribution to the γ of the whole sequence by the input and output at time i. In fact,

$$\frac{Q(a_i|a_{i-1})Q'(a'_i|a'_{i-1})P((\tilde{y}_i,\tilde{y}'_i)|(\tilde{y}_{i-1},\tilde{y}'_{i-1}))}{Q(z_i|z_{i-1})Q'(z'_i|z'_{i-1})P((x_i,x'_i)|(x_{i-1},x'_{i-1}))} \geq \gamma_{\mathcal{S}_i}.$$

Notice that the proposed lower bound only depends on the corresponding state sequence not the exact values of input and output sequences.

Furthermore, the sufficient condition (4.24) can be written as

$$\min\{\gamma_{\mathcal{A}_7}, \gamma_{\mathcal{A}_{11}}\}\,\gamma_{\mathcal{A}_1} \geq 1. \tag{B.45}$$

This inequality along with those in (B.23) to (B.25) and (B.37) result in

$$\begin{aligned}
&\gamma_{\mathcal{A}_0} = \gamma_{\mathcal{A}_3} = 1,\\
&\gamma_{\mathcal{A}_1} = \gamma_{\mathcal{A}_2} \leq 1 \quad \xRightarrow{(B.45)} \quad \min\{\gamma_{\mathcal{A}_7}, \gamma_{\mathcal{A}_{11}}\} \geq 1,\\
&\min\{\gamma_{\mathcal{A}_4}, \gamma_{\mathcal{A}_5}, \gamma_{\mathcal{A}_6}, \gamma_{\mathcal{A}_7}\} = \gamma_{\mathcal{A}_7} \geq 1,\\
&\min\{\gamma_{\mathcal{A}_8}, \gamma_{\mathcal{A}_9}, \gamma_{\mathcal{A}_{10}}, \gamma_{\mathcal{A}_{11}}\} = \gamma_{\mathcal{A}_{11}} \geq 1,\\
&\min\{\gamma_{\mathcal{A}_{13}}, \gamma_{\mathcal{A}_{14}}\} \geq \min\{\gamma_{\mathcal{A}_7}, \gamma_{\mathcal{A}_{11}}\}\,\gamma_{\mathcal{A}_1} \geq 1,\\
&\min\{\gamma_{\mathcal{A}_{12}}, \gamma_{\mathcal{A}_{15}}\} \geq 1.
\end{aligned} \tag{B.46}$$

which implies that for all $i \in \{0, 3, 4, \ldots, 15\}$

$$\gamma_{\mathcal{A}_i} \geq 1; \tag{B.47}$$

and only $\gamma_{\mathcal{A}_1}$ and $\gamma_{\mathcal{A}_2}$ are less than one.

Lemma B.1. *γ computed for any input and output sequences $\{(x_i, x'_i)\}_{i=1}^N$ and $\{(y_i, y'_i)\}_{i=1}^N$ with $N \geq 2$ is greater than or equal to one under the sufficient condition (B.45).*

Proof of Lemma B.1. We use strong induction to prove the lemma.

First step: for $N = 2$, we have

$$\gamma \geq \gamma_{\mathcal{S}_2}$$

Noting (B.46) and the fact that $\mathcal{S}_2$ cannot be $\mathcal{A}_1$ or $\mathcal{A}_2$ because of the assumption $(\tilde{Y}_1, \tilde{Y}_1') = (X_1, X_1')$ in the theorem, $\gamma_{\mathcal{S}_2} \geq 1$ which results in $\gamma \geq 1$.

The inductive step: We assume that the statement in Lemma B.1 holds for all $N \in \{2, 3, \ldots, M\}$, and show that it is true for $N = M + 1$.

Consider arbitrary input and output sequences $\{(x_i, x_i')\}_{i=1}^N$ and $\{(y_i, y_i')\}_{i=1}^N$ with the corresponding state sequence $\{\mathcal{S}_i\}_{i=2}^N$. Looking at $\mathcal{S}_N$, we have three cases.

- If $\mathcal{S}_N \notin \{\mathcal{A}_1, \mathcal{A}_2\}$, we write γ as

$$\begin{aligned}\gamma = \prod_{k=2}^{N-1} \frac{Q(a_k|a_{k-1})Q'(a_k'|a_{k-1}')P((\tilde{y}_k, \tilde{y}_k')|(\tilde{y}_{k-1}, \tilde{y}_{k-1}'))}{Q(z_k|z_{k-1})Q'(z_k'|z_{k-1}')P((x_k, x_k')|(x_{k-1}, x_{k-1}'))} \times \\ \frac{Q(a_N|a_{N-1})Q'(a_N'|a_{N-1}')P((\tilde{y}_N, \tilde{y}_N')|(\tilde{y}_{N-1}, \tilde{y}_{N-1}'))}{Q(z_N|z_{N-1})Q'(z_N'|z_{N-1}')P((x_N, x_N')|(x_{N-1}, x_{N-1}'))}.\end{aligned} \tag{B.48}$$

  The first term is the γ computed for the sequences; hence, it is greater than or equal to one due to the induction hypothesis. As a result,

$$\gamma \geq \gamma_{\mathcal{S}_N} \geq 1. \tag{B.49}$$

- If $\mathcal{S}_N \in \{\mathcal{A}_1, \mathcal{A}_2\}$, we write γ as

$$\begin{aligned}\gamma = \prod_{k=2}^{N-2} \frac{Q(a_k|a_{k-1})Q'(a_k'|a_{k-1}')P((\tilde{y}_k, \tilde{y}_k')|(\tilde{y}_{k-1}, \tilde{y}_{k-1}'))}{Q(z_k|z_{k-1})Q'(z_k'|z_{k-1}')P((x_k, x_k')|(x_{k-1}, x_{k-1}'))} \times \\ \prod_{k=N-1}^{N} \frac{Q(a_k|a_{k-1})Q'(a_k'|a_{k-1}')P((\tilde{y}_k, \tilde{y}_k')|(\tilde{y}_{k-1}, \tilde{y}_{k-1}'))}{Q(z_k|z_{k-1})Q'(z_k'|z_{k-1}')P((x_k, x_k')|(x_{k-1}, x_{k-1}'))}\end{aligned} \tag{B.50}$$

Based on the induction hypothesis, the first product term in (B.50) is greater than or equal to one. Hence, we can write

$$\gamma \geq \gamma_{\mathcal{S}_{N-1}} \gamma_{\mathcal{S}_N}. \tag{B.51}$$

Looking at Table (B.1), if $\mathcal{S}_N = \mathcal{A}_1$, $\mathcal{S}_{N-1} \in \{\mathcal{A}_4, \mathcal{A}_5, \mathcal{A}_6, \mathcal{A}_7\}$. From (B.46) and (B.45), we know that

$$\min\{\gamma_{\mathcal{A}_4}\gamma_{\mathcal{A}_1}, \gamma_{\mathcal{A}_5}\gamma_{\mathcal{A}_1}, \gamma_{\mathcal{A}_6}\gamma_{\mathcal{A}_1}, \gamma_{\mathcal{A}_7}\gamma_{\mathcal{A}_1}\} = \gamma_{\mathcal{A}_7}\gamma_{\mathcal{A}_1} \geq 1.$$

Similarly, if $\mathcal{S}_N = \mathcal{A}_1$, $\mathcal{S}_{N-1} \in \{\mathcal{A}_4, \mathcal{A}_5, \mathcal{A}_6, \mathcal{A}_7\}$. We have

$$\min\{\gamma_{\mathcal{A}_8}\gamma_{\mathcal{A}_2}, \gamma_{\mathcal{A}_9}\gamma_{\mathcal{A}_2}, \gamma_{\mathcal{A}_{10}}\gamma_{\mathcal{A}_2}, \gamma_{\mathcal{A}_{11}}\gamma_{\mathcal{A}_2}\} = \gamma_{\mathcal{A}_{11}}\gamma_{\mathcal{A}_2} \geq 1.$$

As the result, we can conclude

$$\gamma \geq \gamma_{\mathcal{S}_{N-1}} \gamma_{\mathcal{S}_N} \geq 1. \tag{B.52}$$

This completes the proof.

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