Optimized Distributed Detection over a Two Sensor Binary Gaussian MAC Network

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***Abstract*—We consider a distributed detection system where two noisy sensors transmit a binary source over a Gaussian multiple access channel (MAC). We derive the constellation designs for the sensors which minimizes the system error probability under individual power constraints. Three distinct cases arise from this optimization, where in the most notable case (Case III), one of the sensors should not necessarily use all of its allocated power, hence enabling savings that can prolong the sensor's battery life.**

I. INTRODUCTION

Wireless sensor networks are a useful tool for reliably monitoring physical properties of the environment from a centralized location. This includes both the estimation of a real valued parameter (such as temperature or rain fall measurements) and the detection of an event occurring (such as the occurrence of forest fires or a security breach). These types of problems are referred to as distributed estimation, and distributed detection, respectively. In this work, we are interested in minimizing error probability for a distributed detection system.

Previous work on distributed detection usually fixes a certain variety of constellation design, while optimizing some related proxy metrics for error probability. For example, [1] uses orthogonal channels, and optimizes power allocation under the Jeffreys-divergence metric. Alternatively, [2] assumes a single multiple access channel (MAC) for the entire network, and uses the deflection coefficient as the metric for optimization.

We propose a generalized problem of finding an optimized constellation design to minimize error probability under a given source and channel model. This is similar in principle to works such as [3]–[5], where general constellation design is optimized for a chosen criterion. In [3], the optimal joint binary constellation design for two correlated sources was derived. In [4] and [5], a minimum inter-constellation distance criterion is used to optimize constellations for multiple sources.

To apply this concept to distributed detection, we simplify the problem to a two sensor network so that an analytical optimization of the error probability can be performed. The event of interest is modelled as a non-uniformly distributed binary source, and the sensors introduce errors by passing the source through independent stationary binary symmetric channels. The sensors choose binary constellations to send their signals over a Gaussian MAC to the fusion center, which then performs optimal maximum-a-posteriori (MAP) detection to recover the transmitted source bits. With this setup, we analytically derive the optimal constellation design to minimize the system error probability under individual power constraints for each sensor. The optimization is split into three cases, based on the parameters describing the source distribution and sensors' noise. The derived theoretical optimality results are supported by numerical examples and simulations. Our most notable result is that in certain cases (which are dominant when the source is nearly uniformly distributed), the noisier sensor should use a portion but not all of its allocated power, hence realizing energy savings for its battery.

A summary of the main results is shown in Table I, and a numerical example reinforcing the optimization of Case III is given in Fig. 1. The complete proofs of the optimization results as well as more detailed numerical examples are provided in [6].

II. SYSTEM MODEL

Let X be a binary event that is to be observed by a sensor network. Without loss of generality, it is assumed that the source is distributed such that $p_1 \triangleq \Pr(X=1) \leq 0.5$. We also define $p_0 \triangleq \Pr(X=0) = 1 - p_1$. There are two sensors, X_1 and X_2 observing the source X, which are modelled as passing X through two independent memoryless binary symmetric channels. This is expressed as $X_s = X \oplus Z_s,\ s = 1, 2$, where $\oplus$ denotes addition modulo-2, with Z_1 and Z_2 being independent Bernoulli noise variables with means (or channel crossover probabilities) ϵ_1 and ϵ_2, respectively. It is also assumed that X is independent from (Z_1, Z_2). Without loss of generality, Sensor 1 is assumed to have stronger correlation to the original source X than Sensor 2 (e.g., Sensor 1 is located in closer proximity to the source than Sensor 2). This is numerically expressed as $0 < \epsilon_1 \leq \epsilon_2 < 0.5$. The sensors are assumed to be unable to communicate with each other, so they must transmit their data independently using binary signaling constellations. The constellations for the sensors are represented as follows: $\mathcal{C}_s = \{c_{0,s}, c_{1,s}\}$, $s \in \{1, 2\}$, where for $i \in \{0, 1\}$, $c_{i,s} \in \mathbb{R}$ denotes the constellation point for sensor s assigned to $X_s = i$. Let $S_1 \in \mathcal{C}_1$, $S_2 \in \mathcal{C}_2$ be the random variables associated to each sensor's chosen constellation point. Also let $P_1^{\max}$ and $P_2^{\max}$ be the power constraints of sensors, i.e., $E[S_i^2] \leq P_i^{\max}$, $i \in \{1, 2\}$.

The sensors' signals are sent through a Gaussian MAC. The received signal R is described by the relation $R = S_1 + S_2 + Z$, where Z is a Gaussian noise variable with zero mean and variance $\frac{N_0}{2}$. It is assumed that Z is independent of the sensor signals S_1 and S_2. The overall signal $S_1 + S_2$ sent over the

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channel can be represented as a point in the combined constellation of $\mathcal{C}_1$ and $\mathcal{C}_2$, given by $\mathcal{C} = \{c_1 + c_2 \,|\, c_1 \in \mathcal{C}_1,\, c_2 \in \mathcal{C}_2\}$.

The event X is estimated from the received signal R using MAP detection. For $r \in \mathbb{R}$, the detected bit is determined as follows:

$$\begin{aligned}\hat{x}(r) &= \arg\max_{i\in\{0,1\}} \Pr(X = i \mid R = r)\\ &= \arg\max_{i\in\{0,1\}} p_i \sum_{(l,m)\in\{0,1\}^2} p_{lm|i} f_Z(r - a_{lm}),\end{aligned}$$

where f_Z is the probability density function of the channel noise variable Z, $p_{lm|i} \triangleq \Pr(X_1 = l, X_2 = m | X = i)$, and $a_{lm} \in \mathcal{C}$ denotes the superimposed constellation symbol associated with $X_1 = l$ and $X_2 = m$. In the case of a tie, we detect a 0. This is an arbitrary decision since the noise is a continuous random variable.

III. Summary of Main Results

For fixed system parameters p_1, ϵ_1, ϵ_2, N_0, $P_1^{\max}$ and $P_2^{\max}$, we show that the optimal constellation design for $\mathcal{C}_1$ and $\mathcal{C}_2$ which minimizes the system error probability are expressed as $\mathcal{C}_i = \{c_{0,i}, c_{1,i}\} = \left\{-\sqrt{\frac{p_1}{p_0}P_i^*},\ \sqrt{\frac{p_0}{p_1}P_i^*}\right\}$, for $i \in \{1,2\}$, where the optimal power allocations, P_i^*, are separated into three cases summarized in Table I, where

$$k_1 \triangleq \frac{\epsilon_1\epsilon_2}{1-\epsilon_1-\epsilon_2+2\epsilon_1\epsilon_2}, \quad k_2 \triangleq \frac{\epsilon_1-\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2-2\epsilon_1\epsilon_2},$$

and

$$\tilde{P}_2 \triangleq \frac{N_0 p_1 p_0}{2\sqrt{P_1^{\max}}} \ln \frac{(1-\epsilon_1-\epsilon_2)^2 - \Lambda}{(\epsilon_2-\epsilon_1)^2 - \Lambda},$$

with

$$\Lambda \triangleq \frac{(p_0-p_1)^2}{p_0 p_1}(1-\epsilon_1)(1-\epsilon_2)\epsilon_1\epsilon_2.$$

TABLE I: Optimal Power Allocation

Case	Condition	P_1^*	P_2^*
I	$0 \le p_1 \le k_1$	0	0
II	$k_1 < p_1 \le k_2$	$\sqrt{P_1^{\max}}$	$\sqrt{P_2^{\max}}$
III	$k_2 < p_1 \le 0.5$	$\sqrt{P_1^{\max}}$	$\min(\sqrt{P_2^{\max}}, \tilde{P}_2)$

IV. Numerical and Simulation Results

We illustrate the derived results numerically and verify that they are supported by simulated experiments. We show that the minimization problem is solved at the correct value of P_2 in Case III by fixing all parameters except P_2. We use the optimal asymmetric constellation design $\mathcal{C}_i = \{c_{0,i}, c_{1,i}\} = \left\{-\sqrt{\frac{p_1}{p_0}P_i},\ \sqrt{\frac{p_0}{p_1}P_i}\right\}$, for $i \in \{1,2\}$ and vary P_2 to compare error probabilities. For each value of P_2 the experimental data point is produced by sending 1,000,000 independent source bits via two simulated sensors and MAC, then using the MAP detection rule, the detected bit is compared to the true source. The error probability is calculated as the number of errors divided by the total number of bits sent in the experiment.

The error probability as a function of P_2 is shown in Fig. 1. This plot shows that the simulated and theoretical error performance closely match, while also observing that the simulated minimum power allocation $\tilde{P}_2$ for Sensor 2 coincides with its analytical expression obtained in Section III. Additionally, the error function is decreasing until reaching its minimum value, which illustrates why the optimal power allocation in Case III is $\min(\sqrt{P_2^{\max}}, \tilde{P}_2)$. If $\sqrt{P_2^{\max}} < \tilde{P}_2$, then the smallest admissible error value occurs at $P_2 = \sqrt{P_2^{\max}}$.

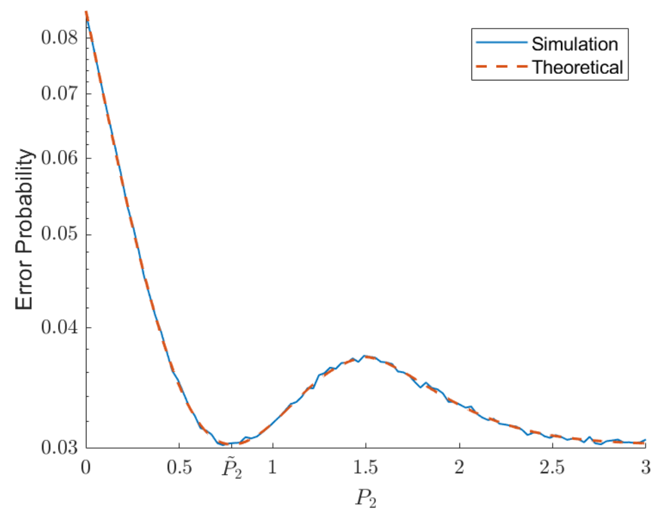

Fig. 1: Theoretical and simulated error probability in Case III ($p_1 = 0.45, \epsilon_1 = 0.01, \epsilon_2 = 0.05, P_1 = 1, N_0 = 1$).

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