

Exploiting Memory and Soft-Decision Information in Channel Optimized Quantization for Correlated Fading Channels

Shervin Shahidi
shervin.shahidi@queensu.ca

Fady Alajaji
fady@mast.queensu.ca

Tamás Linder
linder@mast.queensu.ca

Abstract—A channel optimized vector quantizer (COVQ) scheme is studied and evaluated for a recently introduced discrete binary-input 2^q -ary-output channel with Markovian ergodic noise based on a finite queue. This channel can effectively model binary-modulated correlated Rayleigh fading channels with output quantization of resolution q . It is shown that the system can successfully exploit the channel’s memory and soft-decision information. Signal-to-distortion gains of up to 2.3 dB are obtained for only 2 bits of soft-decision quantization over COVQ schemes designed for a hard-decision ($q = 1$) demodulated channel. Furthermore, gains as high as 4.6 dB can be achieved for a highly correlated channel, in comparison with systems designed for the ideally interleaved (memoryless) channel. Finally, the queue-based noise model is validated as an effective approximation of correlated fading channels by testing a COVQ trained using this model over the Rayleigh fading channel.

I. INTRODUCTION

In the presence of complexity and delay constraints, Shannon’s separate treatment of source and channel coding [1], [2] is no longer optimal and the need arises for more efficient joint source-channel coding (JSCC) schemes. Channel optimized vector quantization (COVQ) is a well-known low-delay robust lossy JSCC scheme which incorporates the channel’s statistics in the vector quantization design without the use of explicit (algebraic) channel coding [3]. It is also known that memory increases capacity for a well-behaved (ergodic) channel in the sense that the capacity of such a channel is strictly larger than that of the corresponding memoryless channel (with identical one-dimensional transition distribution) realized under ideal (infinite) interleaving [4], [5]. Indeed, it has been observed in [6] that incorporating the channel’s memory into the COVQ design for the case of binary (hard-demodulated) channels can significantly improve performance over the case where the channel’s memory is ignored and the COVQ is designed for the interleaved memoryless channel. On the other hand, it has been shown that using the channel’s soft-decision information improves capacity (and potentially the

performance of the coding system) relative to hard-decision decoding for several channel models [7]–[9]. Furthermore, it is known that for uncoded Gaussian channels (with or without fading), increasing the channel’s mutual information as a function of signal-to-distortion ratio (SNR) decreases the system’s minimum mean-square-error (MSE) distortion (e.g., cf. [10]). Although the channel model employed here is not identical (as it is coded and its output is quantized), we observe numerically that a similar relationship holds in the sense that increasing the channel’s capacity using the channel’s memory and soft-decision information, improves the system’s signal-to-distortion ratio (SDR) performance.

In this work, we design and implement a COVQ for the recently introduced channel model in [11] to exploit both the channel’s memory and soft-decision information. This channel model is called the non-binary noise discrete channel (NBNDNC). We use the queue-based noise introduced in [11] as the noise process in the NBNDNC model to provide closed form expressions for the channel transition distribution, and then use the obtained model as an alternative representation of a Rayleigh discrete fading channel (DFC). Note that in contrast to the NBNDNC with queue-based noise model (which we refer to as NBNDNC-QB), for the Rayleigh DFC no closed form transition distribution expression can be provided for block lengths of greater than 3, so that it can only be determined via numerical methods. We test the system designed for the NBNDNC model over the equivalent correlated Rayleigh DFC to simulate its performance in wireless communications. To design the COVQ, we adapt the algorithm introduced in [12], [13] and improved in [14].

The rest of the paper is organized as follows. In Section II, the channel models are explained. In Section III, the problem is defined and the details of the system implementation are provided. Section IV is devoted to numerical results. Conclusions are given in Section V.

II. NBNDNC-QB AND DFC CHANNELS

In this section we review the NBNDNC-QB and the Rayleigh DFC channel models.

This work was supported in part by NSERC of Canada. Sh. Shahidi is with the Department of Electrical and Computer Engineering, Queen’s University, Kingston, ON, Canada

F. Alajaji and T. Linder are with the Department of Mathematics and Statistics and the Department of Electrical and Computer Engineering, Queen’s University, Kingston, ON, Canada

A. NBNDC with queue-based noise

The NBNDC-QB model has binary-input and 2^q -ary-output. Its noise is modeled via a 2^q -ary M^{th} -order Markovian stationary ergodic process with $2^q + 2$ independent parameters. Note that the number of model parameters is independent of the memory order M , which is key to keep the complexity of the model low for arbitrarily large memory (typical values for the soft-decision resolution q are $q = 2$ and $q = 3$).

Specifically, the input data bits are affected by noise via the relation

$$Y_j = (2^q - 1)X_j + (-1)^{X_j}Z_j, \quad (1)$$

$Y_j, Z_j \in \{0, 1, \dots, 2^q - 1\}$ for $j = 1, 2, \dots$, where $\{Y_j\}$ is the channel output process, $\{X_j\}$ denotes the channel input binary process, and $\{Z_j\}$ is the noise process assumed to be independent of $\{X_j\}$. To calculate the n -fold transition probability distribution, note that since the noise process is independent of $\{X_j\}$,

$$P(Y^n = y^n | X^n = x^n) = P(Z^n = z^n) \quad (2)$$

where, according to (1),

$$z_k = \frac{y_k - (2^q - 1)x_k}{(-1)^{x_k}}, \quad k = 1, 2, \dots, n.$$

Therefore, one only needs to calculate the n -fold noise distribution: $P_{\text{NBNDC}}^{(n)}(z^n) = P_{\text{NBNDC}}\{Z_1 = z_1, Z_2 = z_2, \dots, Z_n = z_n\}$. The noise process is a non-binary generalization of the queue-based (QB) noise in [5], where the noise symbol is either selected from an urn with 2^q different colors of balls (representing different error symbols) and according to the probability distribution $(\rho_0, \rho_1, \dots, \rho_{2^q-1})$, or it is selected from a finite queue of length M , which is updated every time a noise symbol is generated (see [5], [11] for a detailed description of the procedure). The resulting QB noise process is a stationary M^{th} order Markov process described by only $2^q + 2$ independent parameters: the size of the queue, M , the probability distribution of the balls in the urn, and correlation parameters $0 \leq \varepsilon < 1$ and $\alpha \geq 0$. The channel transition probability is given in (17) of [11] and the channel correlation is given by (9) of [15].

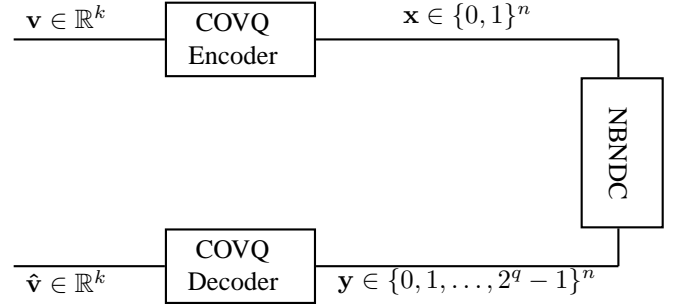
B. DFC.

Consider a discrete fading channel composed of a binary phase-shift keying (BPSK) modulator, a time-correlated flat Rayleigh fading channel with AWGN, and a q -bit soft-quantized demodulator. Let the input and output alphabets be $\mathcal{X} = \{0, 1\}$ and $\mathcal{Y} = \{1, 2, \dots, 2^q - 1\}$, respectively. Denoting the DFC binary input as $\{X_k\}$, $k = 1, 2, \dots$, the received channel symbols are given by

$$R_k = \sqrt{E_s}A_kS_k + N_k, \quad k = 1, 2, \dots$$

where E_s is the energy of signal sent over the channel, $S_k = 2X_k - 1 \in \{-1, 1\}$ is the BPSK modulated signal, and N_k is a white Gaussian noise with variance $N_0/2$ and independent of the input process. $\{A_k\}$ is the channel's fading process with $A_k = |G_k|$, where G_k is a time-correlated

Fig. 1. Block Diagram of a COVQ system



complex wide-sense stationary Rayleigh process with auto-correlation function given by $R[k] = J_0(2\pi f_D T |k|)$ from Clarke's model [16], where $f_D T$ is the normalized maximum doppler frequency and $J_0(\cdot)$ is the zeroth-order Bessel function of first kind. Therefore, A_k is Rayleigh distributed, with unit second moment. The fading process $\{A_k\}$ is assumed to be independent of the noise and input processes. The channel SNR is given by $\text{SNR} = E_s/N_0$.

The output R_k is then fed to a uniform soft-decision quantizer of resolution q -bits with step-size Δ to yield the discrete channel output

$$Y_k = j, \quad \text{if } R_k \in (T'_{j-1}, T'_j),$$

where T'_j are uniformly spaced thresholds with step-size Δ , such that

$$T'_j = \begin{cases} -\infty, & \text{if } j = -1 \\ (j + 1 - 2^{q-1})\Delta, & \text{if } j = 0, 1, \dots, 2^q - 2 \\ \infty, & \text{if } j = 2^q - 1. \end{cases}$$

Let $\delta \triangleq \Delta/\sqrt{E_s}$ and $T_j \triangleq T'_j/\sqrt{E_s}$. The channel block conditional probability for the DFC,

$$P_{\text{DFC}}^{(n)}(y^n | x^n) \triangleq \text{Pr}(Y^n = y^n | X^n = x^n), \quad (3)$$

can be calculated via (2) in [11]. For $n \leq 3$, $P_{\text{DFC}}^{(n)}(y^n | x^n)$ can be calculated in closed form. For $n > 3$, since the joint probability density function of arbitrarily correlated Rayleigh and Rician random variables is not known in closed form, it can only be determined via numerical methods. It can be shown that the DFC is actually an NBNDC as given by (1) with a stationary ergodic noise process [11]. To model a given Rayleigh DFC via the NBNDC-QB, we match the noise one-dimensional probability distributions by setting $\rho_j = P_{\text{DFC}}^{(1)}(j)$, where $j = \frac{y - (2^q - 1)x}{(-1)^x} \in \mathcal{Y}$ and $P_{\text{DFC}}^{(1)}(j)$ is given by (3) in [11], in terms of δ , q , and SNR, and match the noise correlation coefficients. The remaining QB parameters (M, ε) are estimated by minimizing the Kullback-Leibler divergence rate between the two (2^q -ary) noise processes.

III. COVQ FOR THE NBNDC-QB MODEL

Consider the communication system depicted in Fig 1. The input source to the COVQ encoder is a real-valued stationary and ergodic process $\{V_i\}_{i=1}^{\infty}$. The encoder mapping γ takes a vector of k source symbols $\mathbf{v} \in \mathbb{R}^k$ and outputs a binary

vector $\mathbf{x} \in \{0, 1\}^n$, such that $\gamma(\mathbf{v}) = \mathbf{x}$ if $\mathbf{v} \in \mathbf{S}_{\mathbf{x}}$, where $\{\mathbf{S}_{\mathbf{x}} : \mathbf{x} \in \{0, 1\}^n\}$ is a partition of \mathbb{R}^k . Then \mathbf{x} is sent over the NBND-C-QB.

The decoder is a mapping β that maps the received n -tuple 2^q -ary blocks \mathbf{y} to code-levels of the quantizer codebook:

$$\beta(\mathbf{y}) = \mathbf{c}_{\mathbf{y}}, \mathbf{c}_{\mathbf{y}} \in \mathbb{R}^k, \mathbf{y} \in \{0, 1, \dots, 2^q - 1\}^n.$$

The COVQ training algorithm aims to select the codebook $\mathcal{C} = \{\mathbf{c}_{\mathbf{i}}, \mathbf{i} \in \{0, 1, \dots, 2^q - 1\}^n\}$ and the partition set $\mathcal{P} = \{\mathbf{S}_{\mathbf{i}}, \mathbf{i} \in \{0, 1\}^n\}$ so that to minimize the following distortion-per-sample measure:

$$D(\mathcal{C}, \mathcal{P}) = \frac{1}{k} \sum_{\mathbf{x}} \sum_{\mathbf{y}} P(\mathbf{y} | \mathbf{x}) \int_{\mathbf{S}_{\mathbf{x}}} p(\mathbf{v}) \|\mathbf{v} - \mathbf{c}_{\mathbf{y}}\|^2 d\mathbf{v}, \quad (4)$$

where $P(\mathbf{y} | \mathbf{x})$ is calculated via (17) in [11] and $p(\mathbf{v})$ is the source probability density function. Letting $\mathcal{P}^* = \{\mathbf{S}_{\mathbf{x}}^*\}$ be the optimal partition for a given \mathcal{C} , and \mathcal{C}^* the optimal partition for a given \mathcal{P} , the optimal distortion is minimized by satisfying the following two (necessary) optimality conditions iteratively:

$$\mathbf{S}_{\mathbf{x}}^* = \left\{ \mathbf{v} : \sum_{\mathbf{y}} P(\mathbf{y} | \mathbf{x}) \|\mathbf{v} - \mathbf{c}_{\mathbf{y}}\|^2 \leq \sum_{\mathbf{y}} P(\mathbf{y} | \tilde{\mathbf{x}}) \|\mathbf{v} - \mathbf{c}_{\mathbf{y}}\|^2 \right\} \quad (5)$$

which can be directly obtained from (4), and

$$\mathbf{c}_{\mathbf{y}}^* = \frac{\sum_{\mathbf{x}} P(\mathbf{y} | \mathbf{x}) \int_{\mathbf{S}_{\mathbf{x}}} \mathbf{v} f(\mathbf{v}) d\mathbf{v}}{\sum_{\mathbf{x}} P(\mathbf{y} | \mathbf{x}) \int_{\mathbf{S}_{\mathbf{x}}} f(\mathbf{v}) d\mathbf{v}} \quad (6)$$

from [14]. It can be shown that the algorithm will always converge to a local optimum. To select the initial codebook and assigning indices to code levels, we have used the splitting and the simulated annealing algorithms respectively, as suggested in [3]. To be more specific, at first we consider the error free channel and train the COVQ with the initial codebook obtained from the splitting algorithm, followed by simulated annealing for a locally optimum index assignment. Then we use the resulting codebook as the initial state for a channel with high SNR. Afterwards, we gradually decrease the channel SNR while each time we set the previously found codebook (for higher SNR) as the current initial state, until we eventually reach the desired channel SNR.

Note that the training is off-line and after finding the optimal mappings γ and β , the system will perform with the only delay of receiving k symbols from source, mapping it to n binary digits, and then mapping each 2^q -ary n -tuple received at the output of the NBND-C-QB onto k real-valued symbols to yield the reconstruction vector.

IV. RESULTS AND COMPARISON

We herein present the numerical results obtained using the training algorithm and channel model described in the previous sections.

Several source distributions were tested, including independent and identically distributed (i.i.d.) Gaussian and Laplacian sources and correlated Gauss-Markov sources. All of the source models had zero mean and unit variance. For a given

DFC (with fixed SNR and f_{DT}) and q , we choose the value of δ that maximizes the DFC's capacity. We also choose the parameters of the NBND-C-QB, $(\rho_0, \rho_1, \dots, \rho_{2^q-1}), M, \epsilon$ and α , so that the two channel models are as close to each other as possible. We have used the values given in [15] in which the Kullback-Leibler divergence rate between the two channel (2^q -ary) noise processes is minimized over M, ϵ, α for $f_{DT} \in \{0.005, 0.01\}$, $\text{SNR}_{(dB)} \in \{2.0, 5.0, 10.0, 15.0\}$, $q = 2$, $\rho_j = P_{DFC}^{(1)}(j)$ from (3), and the δ value which maximizes the capacity.

For each source model, the COVQ was trained using 500,000 source vectors. The resulting channel optimized quantizer's performance was then tested over the aforementioned DFC channel. For generating the fading coefficients, we used the modified Clarke's method introduced in [17]. Training and simulation results (over the NBND-C-QB and Rayleigh DFC channels) in terms of SDR are shown in Tables I and II for an i.i.d. Gaussian source and in Table III for an i.i.d Laplace source. The channel parameters used for training/simulation are given in Table II of [15].

Table I depicts COVQ training results for a memoryless and highly correlated NBND-C-QB. Note that for the memoryless case (with Cor=0), the NBND-C-QB is identical to the DFC. Comparing Tables I with Cor = 0 and II (where Cor = 0.35, 0.32, 0.29, 0.22 for SNR= 15, 10, 5, 2 respectively), one can see that interleaving (as a means to realize the memoryless channel) may outperform the low correlated channels, especially when the channel SNR is low and block length n is high. Since the capacity of the correlated channel is strictly higher than that of the memoryless channel, this degradation may be due to poor selection of initial points for the vector quantizer. Nevertheless, it can be seen in Table I that for a highly correlated NBND-C-QB, except for rate $R = 1$ and $k = 1, 2$, the resulting COVQ consistently outperforms the memoryless case, with the maximum gains obtained for the case of $q = 1, R = 3, k = 3$. Note that since the COVQ only makes use of intra-block memory, for rate $R = 1$ and low dimensions k , the block length is so small that there is not much channel memory to be harnessed. Additionally, it is observed in Table I that the system considerably outperforms hard-quantization ($q = 1$), by as much as 2.3 dB for $R = 3, k = 1, \text{Cor} = 0$ when using only a 2-bit soft-decision quantizer ($q = 2$).

Comparing the training and simulation performance of the COVQ (see Tables II and III), we observe that there is a good conformity between the results of the two channel models, where the NBND-C-QB is used for training and the Rayleigh DFC for testing. However, for higher rates, some degradation between the simulation and training results is observed. Similar matching results were also observed for Gauss-Markov sources.

V. CONCLUSION

The performance results show that the COVQ system can successfully exploit the channel's memory and soft-decision

TABLE I
COVQ TRAINING SDR RESULTS (IN DB) FOR MEMORYLESS
NBNDQ-QB AND HIGHLY CORRELATED NBNDQ-QB WITH PARAMETERS
 $\alpha = 1$, $M = 1$, $\varepsilon = 0.9$; MEMORYLESS GAUSSIAN SOURCE.

q	R = n/k	k	SNR (dB)							
			Memoryless (Cor=0)				Cor=0.9			
			15	10	5	2	15	10	5	2
1	1	1	4.18	3.77	2.88	2.16	4.18	3.77	2.88	2.16
		2	4.16	3.75	2.87	2.15	4.16	3.75	2.87	2.15
		3	4.23	3.78	2.87	2.15	4.27	3.88	3.64	3.26
	2	1	8.16	6.58	4.23	2.84	8.35	7.05	5.24	5.70
		2	8.33	6.73	4.82	3.66	8.55	7.33	6.82	6.18
		3	8.57	7.13	5.12	3.79	8.81	8.29	7.37	6.76
	3	1	11.12	8.10	4.83	4.45	11.71	9.68	9.45	8.04
		2	11.64	9.28	6.64	4.92	12.09	11.50	10.00	8.83
		3	11.99	9.77	6.90	5.09	12.54	12.43	10.76	9.68
2	1	1	4.21	3.84	3.04	2.36	4.21	3.84	3.04	2.36
		2	4.19	3.82	3.03	2.35	4.19	3.83	3.03	2.35
		3	4.26	3.86	3.04	2.35	4.30	3.95	3.74	3.38
	2	1	8.29	6.84	4.61	3.21	8.47	7.27	5.51	6.00
		2	8.46	6.98	5.35	4.18	8.68	7.54	7.31	6.40
		3	8.70	7.47	5.69	4.30	8.93	7.67	7.52	6.97
	3	1	11.45	8.54	7.13	5.25	11.98	11.10	9.44	8.27
		2	12.36	10.31	7.61	5.82	12.28	11.37	10.00	8.91
		3	12.52	10.69	7.91	5.95	13.02	12.11	11.02	10.16

TABLE II
COVQ TRAINING RESULTS (FOR DFC-FITTED NBNDQ-QB) AND
SIMULATION RESULTS (FOR RAYLEIGH DFC) IN TERMS OF SDR (DB);
MEMORYLESS GAUSSIAN SOURCE. $f_D T = 0.005$

q	R = n/k	k	SNR (dB)							
			Training				Simulation			
			15	10	5	2	15	10	5	2
1	1	1	4.18	3.77	2.88	2.16	4.18	3.76	2.88	2.16
		2	4.16	3.75	2.87	2.15	4.16	3.75	2.87	2.14
		3	4.24	3.81	2.89	2.15	4.23	3.78	2.85	2.14
	2	1	8.23	6.71	4.44	3.02	8.23	6.71	4.36	2.95
		2	8.43	6.88	4.72	3.40	8.39	6.83	4.72	3.42
		3	8.67	7.08	4.94	3.59	8.65	7.05	4.96	3.61
	3	1	11.26	8.36	5.17	3.42	11.25	8.29	5.00	3.32
		2	11.66	8.85	5.98	4.42	11.62	8.88	6.04	4.53
		3	11.71	9.13	6.27	4.61	11.46	9.06	6.34	4.72
2	1	1	4.23	3.88	3.10	2.43	4.23	3.88	3.10	2.43
		2	4.21	3.87	3.09	2.42	4.21	3.87	3.08	2.42
		3	4.29	3.93	3.11	2.43	4.28	3.90	3.08	2.41
	2	1	8.44	7.12	4.93	3.50	8.47	7.18	4.93	3.50
		2	8.65	7.29	5.34	4.00	8.63	7.29	5.19	3.89
		3	8.90	7.49	5.54	4.27	8.87	7.48	5.34	4.07
	3	1	11.86	9.06	6.20	5.03	11.97	9.21	6.05	4.82
		2	12.25	9.94	7.20	5.54	12.25	9.65	6.58	5.06
		3	12.43	10.30	7.68	5.82	11.96	9.23	6.57	5.11

information to combat channel errors while having the advantage of low encoding/decoding delay. Furthermore, the NBNDQ-QB model, which (unlike the DFC) is mathematically tractable by virtue of having closed-form statistical expressions, was experimentally shown to be a practical model for the DFC in terms of COVQ performance.

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TABLE III
COVQ TRAINING RESULTS (FOR DFC-FITTED NBNDQ-QB) AND
SIMULATION RESULTS (FOR RAYLEIGH DFC) IN TERMS OF SDR (DB);
MEMORYLESS LAPLACIAN SOURCE. $f_D T = 0.01$

q	R = n/k	k	SNR (dB)							
			Training				Simulation			
			15	10	5	2	15	10	5	2
1	1	1	2.87	2.63	2.07	1.59	2.87	2.63	2.07	1.59
		2	3.47	3.12	2.42	1.84	3.46	3.12	2.41	1.84
		3	4.15	3.63	2.64	1.88	4.06	3.45	2.39	1.70
	2	1	6.67	5.46	3.62	2.46	6.71	5.45	3.58	2.42
		2	7.69	6.22	4.14	2.90	7.71	6.20	4.10	2.89
		3	8.21	6.62	4.56	3.21	8.17	6.53	4.50	3.17
	3	1	9.65	7.14	4.40	2.91	9.73	7.15	4.30	2.83
		2	10.78	8.20	5.31	3.93	10.79	8.19	5.26	3.94
		3	11.16	8.69	5.92	4.24	10.95	8.60	5.89	4.27
2	1	1	2.90	2.70	2.21	1.77	2.90	2.70	2.21	1.77
		2	3.51	3.23	2.60	2.06	3.50	3.22	2.59	2.06
		3	4.21	3.78	2.86	2.13	4.15	3.62	2.62	1.94
	2	1	6.89	5.88	4.09	2.89	6.96	5.98	4.21	2.97
		2	7.97	6.69	4.75	3.57	8.01	6.76	4.76	3.53
		3	8.50	7.08	5.15	3.89	8.46	7.04	4.95	3.66
	3	1	10.45	8.00	5.01	4.19	10.64	8.28	5.18	4.03
		2	11.50	9.16	6.60	4.97	11.48	8.95	6.08	4.55
		3	11.95	9.87	7.25	5.41	11.49	8.87	6.18	4.67

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