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Transmission of Continuous-Alphabet Sources over MIMO Channels

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Abstract

We introduce three soft-decision decoding channel-optimized vector quantizers (COVQs) to transmit analog sources over space-time orthogonal block (STOB) coded flat Rayleigh fading channels with binary phase-shift keying (BPSK) modulation. One main objective is to judiciously utilize the soft information of the STOB-coded channel in the design of the vector quantizers while keeping a low system complexity. To meet this objective, we introduce a simple space-time decoding structure which consists of a space-time soft detector, followed by an optimized linear combiner and a scalar uniform quantizer with resolution q. We then note that the concatenation of the space-time modulator/encoder, fading channel, and space-time receiver can be described by a binary-input 2^q-output discrete memoryless channel (DMC). The scalar uniform quantizer is chosen so that the capacity of the equivalent DMC is maximized to fully exploit the system's soft information. We next determine the transition probabilities of the DMC in closed form and use them to design three COVQ schemes with various degrees of the knowledge of the channel noise power at the transmitter. Comparison to traditional coding schemes, which perform separate source and channel coding operations, is also provided.

1. INTRODUCTION

Space-time orthogonal block coding [1, 11] was recently developed to improve the performance of wireless communication systems. Like many other error protection schemes, space-time codes are usually designed to operate on uniform independent and identically distributed (i.i.d.) bit-streams. In many situations, for example when the bit-stream is the result of quantization, the bit-stream to be protected is not i.i.d. and joint design of source and channel codes may yield improved results.

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This paper considers the transmission of analog sources over multiple-input multiple-output (MIMO) channels. We use channel-optimized vector quantization (COVQ) which is a joint source-channel coding method for compressing the source while rendering it robust against channel errors. Our contribution is to show how to exploit the soft information at the channel output. We also introduce an on-line version of the COVQ with enhanced performance. We propose a simple space-time decoding structure with two blocks of very modest computational needs. Our design is based on the observation that the concatenation of some system blocks is equivalent to a discrete memoryless channel (DMC) for which a COVQ can be designed. Inspired by the work in [9], we use soft-decision decoding as opposed to the soft-decoding methods to exploit the soft information available at the channel output. We choose soft-decision decoding because it is easier to implement and, as discussed in [9], its performance can be close to soft-decoding with reduced computational complexity (at the expense of higher memory requirements).

In all, three COVQ systems are studied. The first (classical) COVQ assumes that the channel signal-to-noise ratio (CSNR) is known at both the transmitter and the receiver. As the CSNR could vary during transmission or may not be available to the transmitter at all, we have designed two fixed-encoder adaptive-decoder (FEAD) COVQs. The first FEAD COVQ uses only the knowledge of the CSNR at the receiver (as in [10]), while the other one, the On-line FEAD COVQ, employs the knowledge of the channel fading coefficients at the receiver. An important feature of the FEAD COVQs is that their decoder codebooks are *computed* in terms of the transmitter parameters, and not through a training process as for classical COVQs,

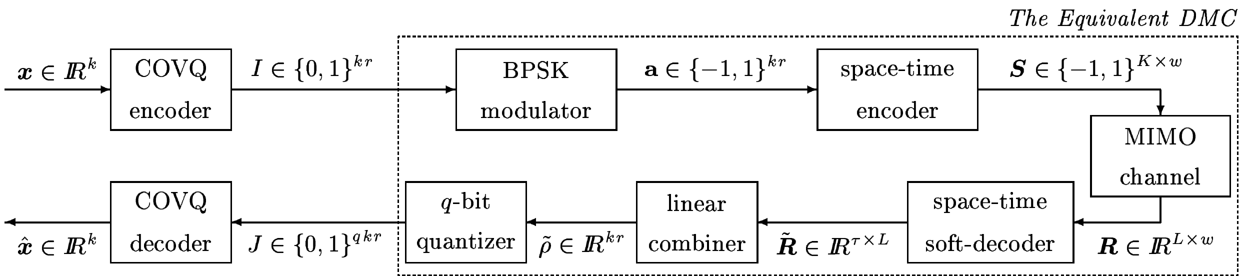

Figure 1: System block diagram, where every τ bits in a kr-bit index I is transmitted via a space-time codeword $\mathbf{S} = (\mathbf{s}_1, ..., \mathbf{s}_w)$ (with orthogonal columns $\mathbf{s}_i, i = 1, ..., w = g\tau$), is received as $\boldsymbol{R} = (\boldsymbol{r}_1, ..., \boldsymbol{r}_w)$, and space-time soft-decoded as $\tilde{\boldsymbol{R}} = (\tilde{\boldsymbol{r}}^1, ..., \tilde{\boldsymbol{r}}^L)$. For simplicity, we have assumed that $\tau = kr$.

thus reducing memory requirements at the receiver.

2. SYSTEM COMPONENTS

The system block diagram is depicted in Figure 1.

2.1. The STOB Coded MIMO Channel

The communication system considered here employs BPSK modulation, K transmit, and L receive antennas. The baseband constellation points are $c_{(1)} = 1$ and $c_{(2)} = -1$. The channel is assumed to be Rayleigh flat fading with unit-variance i.i.d. Rayleigh distribution. We assume that the receiver has perfect knowledge of the path gains which are assumed constant during a codeword transmission, but varying in an i.i.d. fashion among codeword intervals. The input-output relationship of STOB coded channels can be written as [2, 8]

$$\tilde{\boldsymbol{r}}^j = g\sqrt{\frac{\gamma_s}{K}}\bar{H}_j\mathbf{c} + \tilde{\boldsymbol{n}}^j,$$

where $\mathbf{c}$ indicates the $\tau \times 1$ vector of transmitted symbols, $\bar{H}_j = \sum_i H_{ji}^2$ (with H_{ji} being the path gain from transmit antenna i to receive antenna j), g is the inverse of the space-time code rate, and γ_s is the CSNR at each receive antenna. $\tilde{\boldsymbol{n}}^j = (\tilde{N}_1^j, ..., \tilde{N}_\tau^j)^T$ is an additive white Gaussian noise vector with i.i.d. components of distribution $\tilde{N}_i^j \sim \mathcal{N}(0, g\bar{H}_j)$. We assume that the MIMO channel input $\mathbf{c}$, the noise $\tilde{\boldsymbol{n}}$, and the fading coefficients are independent from each other.

Symbol i can be detected by only considering the i^{th} entry of the vectors $\tilde{\boldsymbol{r}}^j$, $1 \le j \le L$. For our application, this means that the bits corresponding to a COVQ index can be detected independently.

Letting $\delta^2 = 2g\gamma_s/K$, the pairwise error probability of ML decoded BPSK symbols equals [2]

$$\Lambda(\delta) \triangleq \frac{1}{2}\left(1 - \frac{\delta}{\sqrt{2+\delta^2}}\sum_{k=0}^{KL-1}\binom{2k}{k}\frac{1}{(2\delta^2+4)^k}\right). \quad (1)$$

2.2. Soft-Decision Decoding: Linear Combining and Uniform Quantization

To process the signals of the receive antennas and capture their soft-information, we employ space-time soft-decoding followed by linear combining because i) the linear combiner has a very simple structure, ii) its design criterion allows the DMC distribution to be determined in closed form, and iii) its output is continuous, making soft-decision decoding possible. This design problem is a variation of classical maximum ratio combining. Let us set $\tilde{\rho}_i^j \triangleq \sqrt{\frac{K}{\gamma_s}}\tilde{R}_i^j/(g\bar{H}_j)$, where $\tilde{R}_i^j$ is the i^{th} entry of $\tilde{\boldsymbol{r}}^j$. The output of the linear combiner is

$$\tilde{\rho}_i = \sum_{j=1}^{L}\alpha_j\tilde{\rho}_i^j = \sum_{j=1}^{L}\alpha_j(c_i + \tilde{\nu}_i^j),$$

where α_j's are the weights to be determined, so that the SNR at the output of the linear combiner is maximized under the constraint that $\sum_j \alpha_j^2 = 1$. Using Lagrange multipliers, the optimal solution can be found to be $\alpha_j = \bar{H}_j/\bar{H}$, where $\bar{H} = \sum_j \bar{H}_j$. The output of the linear combiner is hence given by

$$\tilde{\rho}_i = c_i + \tilde{\nu}_i, \quad (2)$$

where $\tilde{\nu}_i = \sum_{j=1}^{L}\alpha_j\tilde{\nu}_i^j$. It can be shown that

$$\tilde{\nu}_i \sim \mathcal{N}\left(0, \frac{K}{g\gamma_s\bar{H}}\right). \quad (3)$$

The linear combiner output, $\tilde{\rho}_i$, is next fed into a "uniform" scalar quantizer which acts as the soft-decision decoder. Let us denote the decision levels of this quantizer by $\{u_k\}_{k=-1}^{N-1}$ and its codepoints by $\{w_k\}_{k=0}^{N-1}$, where $N = 2^q$ is the number of the codewords. The decision regions of the uniform quantizer are given by

$$u_k = \begin{cases} -\infty, & \text{if } k = -1 \\ (k+1-N/2)\Delta, & \text{if } k = 0, \cdots, N-2 \\ +\infty, & \text{if } k = N-1, \end{cases}$$

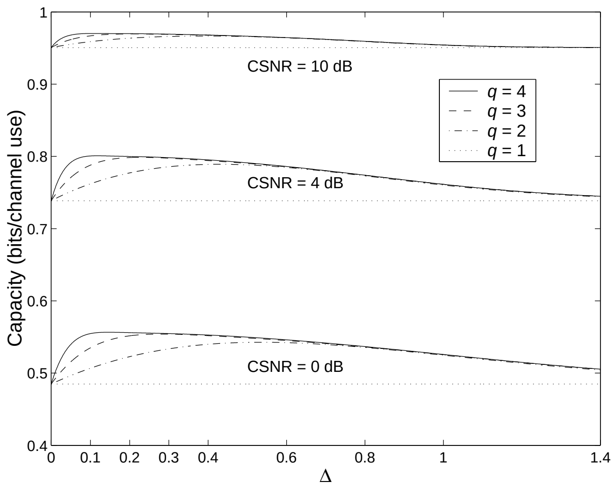

Figure 2: DMC capacity versus the step-size of the uniform quantizer, $K = 2$, $L = 1$.

and the quantization rule is simply

$$f(\tilde{\rho}) = k, \text{ if } \tilde{\rho} \in (u_{k-1}, u_k], k = 0, \cdots, N-1.$$

For each CSNR, we numerically select Δ, the step-size of the uniform quantizer, such that the capacity of the DMC equivalent to the blocks between the COVQ encoder and decoder (see Figure 1) is maximized. This is a sub-optimal criterion (as our goal is to minimize the overall mean squared error), but as observed from the simulations of [9], there is a large correlation between a high channel capacity and reduced distortion, which is our ultimate goal.

As shown in the next subsection, the channel transition probability matrix is symmetric. Therefore, to calculate channel capacity, we compute the mutual information between the channel input and output assuming that the input has a uniform distribution. Channel capacity is easy to compute in this case and the above search for the best step-size takes a fraction of a second to complete.

Channel capacity versus Δ is plotted in Figure 2 for three CSNR values. We observe that soft-decision decoding is specially beneficial at low CSNR and also that it is important to choose the right step-size: too small or too large a Δ will make little improvement in channel capacity (and hence in the overall distortion). One final observation is that the soft-decision decoding gain is minimal for $q > 3$.

2.3. Transition Probabilities of the Equivalent DMC

For COVQ design, we need to derive the transition probabilities of the 2^{kr}-input 2^{qkr}-output discrete channel represented by the concatenation of the blocks between the COVQ encoder and decoder. Since the detection of bits which correspond to each quantizer index is decoupled, the COVQ indices are decoded independently of one another, and the discrete channel is equivalent to a binary-input 2^q-output DMC used kr times. The required transition probabilities are $P(w_k|c_i)$, $i = 1, 2$, for all w_k. Using (1) and (2), we have

$$P(w_k|c_i) = \Lambda\left((u_{k-1} - c_i)\delta\right) - \Lambda\left((u_k - c_i)\delta\right), \quad (4)$$

where $\Lambda(\cdot)$ is defined in (1). Note that the DMC transition probability matrix is symmetric in the sense of [5], hence an input with a uniform distribution achieves channel capacity.

We denote the natural binary representation of the index of decision region S_i by $\{b_l\}_{l=1}^{kr}$ and that of codevector w_j by $\{B_l\}_{l=1}^{kr}$, where B_l is a binary q-tuple. The COVQ index transition probabilities can then be computed as

$$P_{J|I}(j|i) = \prod_{l=1}^{kr} P\left(w_{B_l}|c_{(2-b_l)}\right). \quad (5)$$

3. SOFT-DECISION DECODING QUANTIZATION

3.1. Soft-Decision Decoding COVQ

The transition probability in (5) can be used in one of the well-known COVQ training algorithms, such as the modified GLA algorithm [6, 7] to design a soft-decision decoding COVQ for STOB coded channels. Every input k-tuple is encoded at a rate of r bps. Hence, the input space is partitioned into $N_e = 2^{kr}$ subsets (denoted by $\{S_i\}_{i=0}^{N_e-1}$). The received vector of kr real-valued signals is soft-decision decoded at a rate of q bits per dimension resulting in $N_d = 2^{qkr}$ codevectors. The input space partitioning and the codebook are optimized based on the necessary conditions for optimality as explained in, for example, [9].

3.2. Soft-Decision Decoding FEAD COVQ

The COVQ requires that the CSNR be known at both the transmitter and the receiver. There are cases where the CSNR varies over time and it cannot be fed back to the transmitter, or where the memory constraints make it impossible to store many codebooks. In this section, we design a COVQ with a fixed encoder whose decoder adapts itself to the channel conditions. Let us denote the decoder codevectors by $\{\boldsymbol{y}_j\}_{j=1}^{N_d}$. The incoming samples are encoded assuming a design

CSNR based on which the encoder codebook is selected and the transition probabilities used for encoding are computed. Let us denote the encoder index by I and the decoder index by J. It can be shown that the optimal value of $\boldsymbol{y}_j$ in the minimum mean square sense is given by

$$\boldsymbol{y}_j^* = E\{\boldsymbol{x}|J=j\} = \sum_{i=0}^{N_e-1} \boldsymbol{m}_i P_{I|J}(i|j),$$

where $\boldsymbol{m}_i = E\{\boldsymbol{x}|I=i\}$ denotes the conditional mean of the samples in S_i. $P_{I|J}(i|j)$ is given by

$$P_{I|J}(i|j) = \frac{P_{I,J}(i,j)}{P_J(j)} = \frac{P_{J|I}(j|i)P_I(i)}{\sum_{k=0}^{N_e-1} P_{J|I}(j|k)P_I(k)},$$

where $P_I(i) = P(I=i) = P(\boldsymbol{x} \in S_i)$.

3.3. On-line FEAD Soft-Decision Decoding COVQ

The fading coefficients are available to the receiver for space-time decoding. In this section we show how to use them to enhance COVQ decoding. We observe in (2) that the output of the linear combiner has an identical form to the output of an additive white Gaussian noise (AWGN) channel. Hence $\tilde{\rho}_i$ can be soft-decision decoded using the step-size of the AWGN channels using the results of [9]. In particular, the channel transition probabilities are given by

$$\begin{aligned} P(w_k|c_j,\boldsymbol{H}) &= Q\left((u_{k-1}-c_j)\delta\sqrt{\bar{H}}\right) \\ &- Q\left((u_k-c_j)\delta\sqrt{\bar{H}}\right), \end{aligned}$$

where $Q(\cdot)$ is the Gaussian error function.

4. NUMERICAL RESULTS

We consider the transmission of zero-mean unit-variance i.i.d. Gaussian and Gauss-Markov sources over MIMO channels. 500,000 training vectors and 850,000 test vectors are employed. Each test is performed 5 times and the average signal-to-distortion ratio (SDR) in dB is reported. MIMO systems with K transmit and L receive antennas are referred to as (K-L) systems. Alamouti's code [1] is used for the dual transmit-antenna systems. The real (rate 1) code of [11] is employed for the quad-transmit system because our constellation is real.

Several training strategies were examined, and the best one in terms of having consistent results and high training SDR was used as follows. For any given COVQ rate r, dimension k, and number of soft-decision decoding bits q, we first train a k-dimensional rate-qr VQ with the Split algorithm [6]. We next use the Simulated Annealing algorithm [3] which aims to minimize the average end-to-end distortion for a given VQ codebook through optimizing the assignment of indices of the VQ codevectors. It can be shown that the cost function to be minimized equals

$$\sum_{i=0}^{N_e-1} P_I(i) \sum_{j=0}^{N_d-1} P_{J|I}(j|i)\langle \boldsymbol{y}_{\pi(j)}, (\boldsymbol{y}_{\pi(j)} - 2\boldsymbol{m}_i)\rangle.$$

where $\langle \boldsymbol{\zeta}, \boldsymbol{\eta}\rangle = \sum_i \zeta_i \eta_i$ is the standard inner product and $\pi : \{0,\cdots,N_d-1\} \rightarrow \{0,\cdots,N_d-1\}$ is the one-to-one mapping function to be optimized. Simulated Annealing is used only at the highest CSNR. We then use an approach similar to the one in [4]; namely, we use the modified generalized Lloyd algorithm to derive the COVQ codebooks starting from the highest CSNR to the lowest and vice-versa.

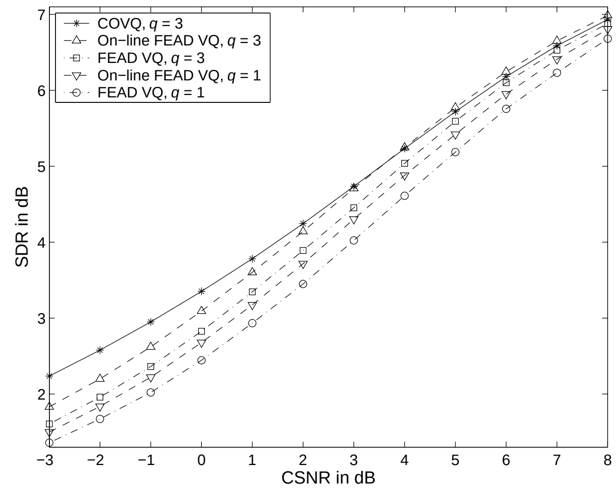

Figure 3: Comparison between the COVQ, FEAD VQ, and On-line FEAD VQ for $\mathcal{N}(0,1)$ Gauss-Markov source. COVQ rate and dimension are 1 bps and 2, respectively. $K=2$ and $L=1$.

4.1. COVQ, FEAD COVQ and On-Line FEAD COVQ

The three quantizers presented in this paper are compared in Figure 3, where a $\mathcal{N}(0,1)$ Gauss-Markov source (with correlation coefficient $\rho = 0.9$) is quantized with dimension 2 and rate 1 bps and sent over a system with $K=2$ and $L=1$ using Alamouti's code [1]. The FEAD VQs are designed assuming a noiseless channel and hence the three systems perform very close to each other as the CSNR grows because the channel mismatch of the VQs decreases. The On-line FEAD

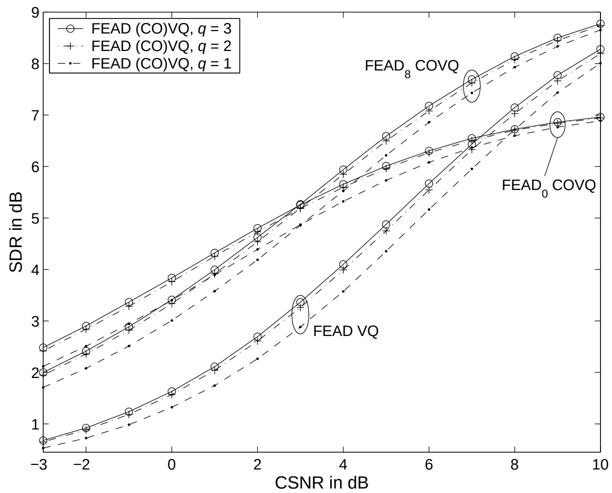

Figure 4: Performance of the FEAD COVQ with three different design CSNR of 0, 8 and ∞ dB. The source is i.i.d. $\mathcal{N}(0,1)$ and is vector quantized at rate 2.0 bps and soft-decoded with q bits. Quantization dimension is 2. $K=2$ and $L=1$.

VQ maintains its gain over the FEAD VQ when soft-decision decoding is employed. It is also observed that although the On-line FEAD VQ encoder assumes the channel is noiseless (CSNR $\to \infty$), at high CSNR values it outperforms the COVQ, which is designed for the exact CSNR.

Figure 4 shows that the degradation due to mismatch can be significantly reduced by proper selection of the design CSNR. This figure suggests that for a system with 2 transmit and one receive antennas, a good design CSNR is 8 dB (a FEAD$_\gamma$ COVQ is one which uses a design CSNR of γ).

4.2. COVQ versus Tandem Coding

We next compare our COVQ-based system with traditional tandem coding schemes which use separate source coding and channel coding with VQ and 64-state non-systematic convolutional coding (CC), respectively. We consider, in Figure 5, a system with $K=2$, $L=1$, and quantization with dimension $k=2$. The overall rate is 2 bps, hence the possible (VQ, CC) code rates are (0.5, 1/4), (1, 1/2), (1.5, 3/4), and (2, 0) (the channel code rate of each curve is stated in the figure). We observe in Figure 5 that at most CSNR values, the jointly designed COVQ outperforms the substantially more complex tandem systems. The results for i.i.d. sources are shown in Figure 6 and indicate an even better performance for the COVQ system.

We also observe that COVQ would beat unequal error protection (UEP)-based systems in most CSNR values, because UEP systems can only use discrete rates for the source and channel codes.

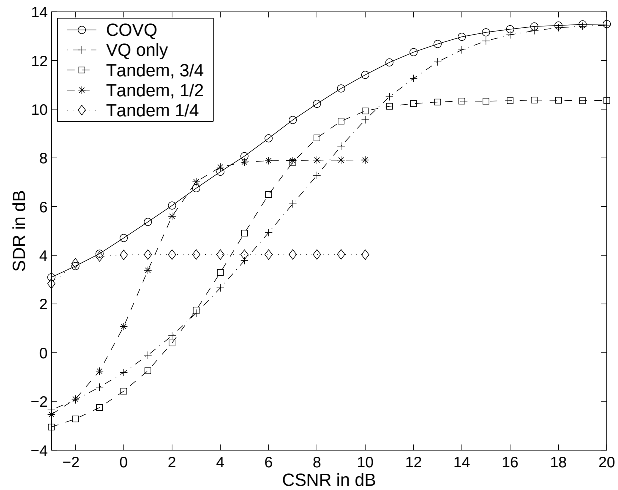

Figure 5: Performance of jointly designed versus tandem coding schemes for a $\mathcal{N}(0,1)$ Gauss-Markov source ($\rho = 0.9$). Quantization dimension is 2. $K=2$ and $L=1$.

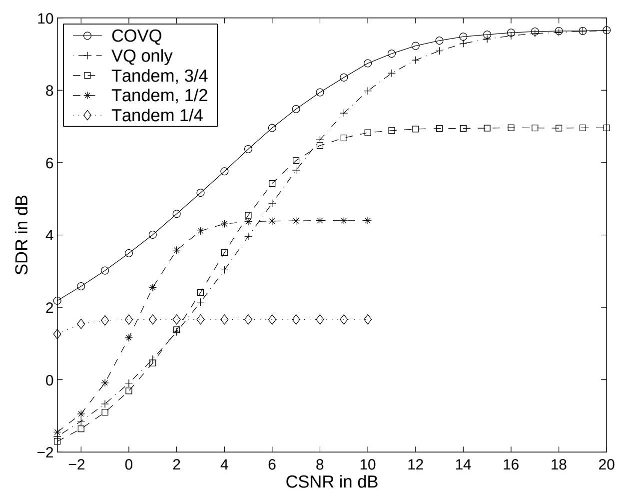

Figure 6: Performance of jointly designed versus tandem coding schemes for an i.i.d. $\mathcal{N}(0,1)$ source. Quantization dimension is 2. $K=2$ and $L=1$.

4.3. COVQ for Various MIMO Channels

Figure 7 plots the SDR curves of some COVQ-based systems versus the CSNR for various MIMO channels with $k=2$ and $r=1$ bps (the system with four transmit antennas uses the real code in [11]). The gain of using MIMO channels over the single-antenna channel

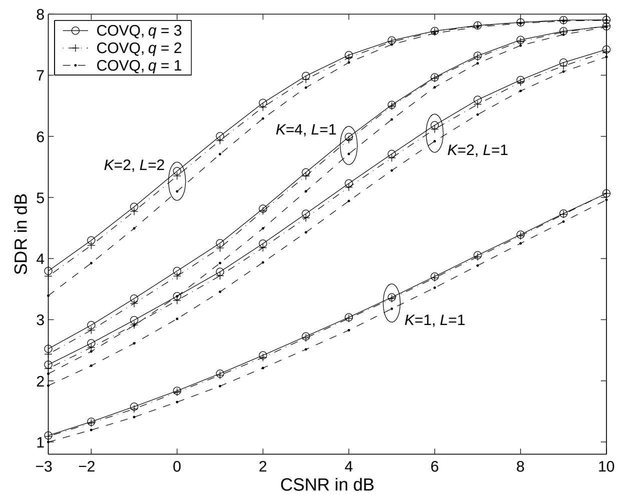

Figure 7: Performance of the COVQ-based system for various MIMO channels. The source is a zero-mean unit-variance Gauss-Markov source ($\rho = 0.9$) vector quantized at rate 1 bps and soft-decision decoded with q bits. Quantization dimension is 2.

is obvious. For example, at SDR = 5 dB, the dual transmit single receive system outperforms the single-antenna system by 6 dB (for hard-decoding) and is outperformed by the 2-transmit 2-receive system by 4.3 dB. The latter figure demonstrates the effectiveness of our linear combiner. For SDR = 5 dB, the CSNR gain due to soft-decision decoding with $q = 3$ bits is 0.8 dB for the MIMO systems considered.

5. CONCLUSION

We presented three soft-decision decoding COVQ-based systems for communicating analog sources over STOB coded MIMO channels The proposed systems depend on whether the actual CSNR is available to the transmitter and whether the COVQ decoder is aware of the fading coefficients. The soft information of the channel is utilized through space-time soft decoding, linear combining, and scalar uniform quantization. Simple design methods were proposed for the linear combiner and the uniform quantizer.

It was shown that using 3 soft-decision decoding bits can achieve almost all of the gain available through soft-decision decoding. This gain is very significant, specially when transmit diversity is employed and/or when the source is correlated. For a dual transmit antenna system and at SDR = 5 dB, using a second receive antenna results in 4.3 dB CSNR gain over a single-receive antenna system for a unit-variance Gauss-Markov source. For the COVQ dimension and rates considered here, the use of only 2 soft-decision decoding bits results in typically 0.9 dB gain in CSNR over hard decoding. The COVQ-based system was shown to outperform tandem systems which use separate source and channel coding.

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