Abstract

The branch of statistics known as *functional data analysis* seeks to analyze data which presents itself in the form of functions. A unique characteristic of functional data is that it displays two very different types of variation. Variation in range is variation in the value of a function over different realizations of that function. Variation in domain is variation in the point in the domain of the function at which a particular value, or event, occurs.

Functional registration, a technique for separating range variation from domain variation, involves continuously deforming the domain of each function so as to make certain known events, called *landmarks*, occur at the same points in the domain of each observation. The observed functions of the deformed domains are called *registered functions* and can be thought of as containing all of the observed range variation without any of the domain variation. The domain deformation functions used to register the observations are called *registration functions* and can be thought of as containing all of the observed domain variation without any of the range variation.

In order to estimate registration functions for a set of functional observations containing landmarks, it is necessary to estimate smooth continuous functions from discrete observations at arbitrarily spaced points in the domain. In one dimension, this estimation can be performed using various *spline functions* and good functional registration techniques based on spline functions are available. The existing techniques for arbitrarily scattered points in the plane, on the other hand, are less convenient than spline techniques for estimating functions of $\mathbb{R}$.

This thesis presents a method for constructing finite-element approximations of a class of bivariate smoothing splines analogous to one-dimensional L-splines. These functions are applied to a sample bivariate registration problem and the quality of the registration is analyzed through a variety of simulations. It is concluded that the functional estimates are an effective tool for bivariate registration. The methods presented can be generalized to estimate functions of 3 variables.