Optimized Signaling of Binary Correlated Sources over Gaussian Multiple Access Channels

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***Abstract*—This work focuses on the construction of optimized binary signaling schemes for two-sender uncoded transmission of correlated non-uniform sources over non-orthogonal Gaussian multiple access channels. Based on an error-rate analysis under joint maximum-a-posteriori decoding, optimized binary-pulsed-amplitude modulation constellations for two senders are derived to minimize the system's error rate. The joint probability distribution of the two-senders' source is observed to induce a special layout of optimized constellations which can effectively control the interference due to non-orthogonal transmission. Numerical results further confirm that significant gains are achievable by the proposed design.**

***Index Terms*—Joint source-channel coding, binary signaling, MAP detection, Gaussian multiple access channels, interference, correlated non-uniform sources, error analysis.**

I. Introduction

For delay-sensitive transmission systems and systems where sophisticated data compression and error control coding are unaffordable, a source-matched modulation scheme which directly maps each source message into a modulated symbol is the simplest alternative. In this paper, we consider a joint source-channel-modulation problem for such systems over non-orthogonal Gaussian multiple access channels (GMACs) [1]. Specifically, two senders (or users) coherently send correlated and non-uniformly distributed data to a receiver using (one-dimensional) binary signaling subject to requisite power constraints, and the receiver recovers the transmitted messages via joint maximum-a-posteriori (MAP) decoding in a real-time fashion. We consider the worse-case scenario where the signals are sent without multiplexing due to stringent limits on processing capabilities.

The considered problem setting is relevant to several applications. For example, in distributed storage systems [2], a file is often encoded in different ways and stored in separate locations. It is then simultaneously retrieved by a client from those locations through a shared data link to ensure its integrity and to reduce read latency. Moreover, in a two-way relay network [3], users desire to transmit their source messages to a relay node over the same time slot and frequency band to improve spectral efficiency. Other applications include uplink non-orthogonal multiple-access (NOMA) in cellular networks [4], gathering information in wireless sensor networks [5], and cooperative relaying in wireless communication [6].

Under the above setup, using an identical binary signaling scheme such as binary-pulse-amplitude modulation (BPAM) at both senders is inadequate because it results in a combined constellation for which the receiver cannot decode the received signal without error even when the channel is noise-free. To resolve this ambiguity, [7] suggested rotating one of two conventional antipodal constellations while [8] proposed to scale the two constellations with distinct amplitudes. Those ideas were employed to design higher-order modulations for independent and uniformly distributed sources over non-orthogonal GMACs with fading [9]-[11]. However, it is not clear what constellations are suitable for transmitting correlated and non-uniformly distributed sources under joint MAP decoding, even for non-orthogonal GMACs without fading.

Note that, rotation schemes for BPAM can orthogonalize the considered channels at the expense of receiver complexity (e.g., the received signals are two-dimensional), hence eliminating any ambiguity about the senders' signals. BPAM design for non-uniform sources over orthogonal GMACs without fading was studied in [12]. Here, we consider a more restricted situation where the receiver design is to be kept as simple as possible (without the use of rotation schemes). Furthermore, symmetric constellations are often not optimal for non-uniform sources as reported in [13]-[16] and thus power allocation schemes may not be sufficient. In this paper, we propose to design signals to match the source statistics and optimize the system performance.

The main contribution of this paper is the derivation of a closed form solution for optimized BPAM constellations in the high signal-to-noise (SNR) regime for non-orthogonal GMACs without fading. The optimized binary signaling schemes for both senders can be now determined via simple formulas, and the resulting system error rate is nearly optimal for a wide range of SNRs. We note that this design problem is different from the 4-PAM constellation design for single-sender systems with a non-uniformly distributed source, e.g., [13]-[16], since in our case the two senders are not allowed to cooperate and each sender has its own power constraint. In fact, these restrictions make our design problem challenging with no straightforward solution.

The rest of this paper is organized as follows. In Section II, we describe the transmission system and analyze its error rate performance under joint MAP decoding. Optimized constellations are derived in Section III and the performance of the proposed signaling schemes is systematically assessed via simulations in Section IV. Conclusions are drawn in Section V.

II. System Description and Error Rate Analysis

The transmission system we study is depicted in Fig. 1. In each time slot, the two senders separately and simultaneously

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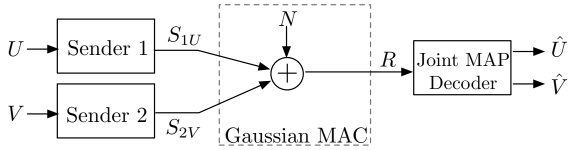

Fig. 1: The block diagram of the GMAC transmission system.

transmit their binary source messages over a MAC with additive white Gaussian (AWGN) noise. The source messages are assumed to be correlated, and hence a joint MAP decoder is employed at the receiver to minimize the joint symbol error rate. The binary messages of senders 1 and 2 are denoted by U and V, respectively, and they are governed by the joint probability distribution $p_{uv} \triangleq \Pr(U = u, V = v)$, for $u, v \in \{0, 1\}$. Let $p_1 \triangleq \Pr(U = 0)$ and $p_2 \triangleq \Pr(V = 0)$. To avoid uninteresting cases, we assume that $p_{uv} > 0$ for all u and v.

To transmit data over the GMAC system, sender j uses a BPAM constellation $\mathcal{S}_j = \{S_{j0}, S_{j1}\}$ subject to the average energy constraint $p_j S_{j0}^2 + (1 - p_j) S_{j1}^2 = E_j$, where $S_{jb} \in \mathbb{R}$ denotes the signal point representing the binary message b of sender j and E_j denotes the average energy for transmitting sender j's signal. When the source messages (U, V) are sent over the GMAC, the received signal is given by $R = S_{1U} + S_{2V} + N \triangleq A_{UV} + N$, where N is a real-valued zero-mean Gaussian noise random variable with variance σ^2 which is independent of (U, V). Let $\mathcal{A} \triangleq \{S_{1u} + S_{2v} : u, v \in \{0, 1\}\}$ denote the combined constellation. Clearly, multiple access interference occurs because the senders' transmitted signals are superposed without any multiplexing. An extreme case is when both senders employ identical BPAM constellations, i.e., $S_{10} = S_{20}$ and $S_{11} = S_{21}$, which results in $A_{10} = A_{01}$. To avoid this harmful interference, the mapping from $\{0, 1\} \times \{0, 1\}$ to $\mathcal{A}$ given by $A_{UV} = S_{1U} + S_{2V}$ must be bijective.

Suppose that $(U, V) = (u, v)$ is sent and received as signal $R = r$. The (optimal) joint MAP decoder generates an estimate of (U, V) via the decision rule

$$(\hat{u}, \hat{v}) = \operatorname*{arg\,max}_{(l,m)\in\{0,1\}^2} \Pr\big(U = l, V = m | R = r\big) = \operatorname*{arg\,max}_{(l,m)\in\{0,1\}^2} \ln p_{lm} + \frac{2rA_{lm} - A_{lm}^2}{2\sigma^2}.$$

Consider a new random variable $H_{lm} \triangleq \ln p_{lm} + (2RA_{lm} - A_{lm}^2)/2\sigma^2$. Given $R = r$, the realization of H_{lm}, denoted by h_{lm}, can be viewed as a decision score for A_{lm}. Define the scaled difference score as

$$\Delta_{uv,lm} \triangleq -\sigma^2 \cdot (H_{uv} - H_{lm}) = N(A_{lm} - A_{uv}) - \frac{(A_{lm} - A_{uv})^2}{2} - \sigma^2 \ln \frac{p_{uv}}{p_{lm}}.$$

The system's error rate $P_{\text{err}}^{\text{MAP}}$ can be written as

$$P_{\text{err}}^{\text{MAP}} = \sum_{(u,v)} p_{uv} \Pr\left((u, v) \neq \operatorname*{arg\,min}_{(l,m)} \Delta_{uv,lm}\right) = 1 - \sum_{(u,v)} p_{uv} \Pr\left((u, v) = \operatorname*{arg\,min}_{(l,m)} \Delta_{uv,lm}\right) = 1 - \sum_{(u,v)} p_{uv} \Pr\big(\Delta_{uv,lm} < 0 \text{ for all } (l, m) \neq (u, v)\big).$$

Since $\Delta_{uv,lm}$ is a Gaussian random variable whose mean and variance are respectively given by $-((A_{lm} - A_{uv})^2)/2 - \sigma^2 \ln\big(p_{uv}/p_{lm}\big)$ and $\sigma^2 (A_{lm} - A_{uv})^2$ and the decision regions are intervals, $P_{\text{err}}^{\text{MAP}}$ can be easily computed.

III. Optimized Design of Binary Constellations for Two-Sender GMAC

In this section, we derive the optimized constellations based on minimizing the above error rate under joint MAP decoding in the high SNR regime. According to the relative position of S_{1u} and S_{2v}, $u, v \in \{0, 1\}$, and the Euclidean distance between S_{j0} and S_{j1}, $j = 1, 2$, there are eight possible layouts for the combined constellation that need to be considered. One can derive the MAP decoding performance for each case, find optimal constellations for each case, and choose the one that achieves the minimum error rate as the optimal design. However, we note that tedious numerical computations and comparisons are required to obtain the optimal constellations, and the process will have to be repeated even if the p_{uv}'s are slightly changed.

To avoid designing signal constellations numerically, we design constellations under the high SNR assumption. Specifically, let $P_{\text{err}}^{\text{MAP}}(\mathcal{S}_1, \mathcal{S}_2, \sigma^2)$ denote the system's error rate when the constellations $\mathcal{S}_1$ and $\mathcal{S}_2$ are employed and the noise variance is σ^2. We design constellations $\mathcal{S}_1$ and $\mathcal{S}_2$ such that

$$\lim_{\sigma^2 \to 0} \frac{P_{\text{err}}^{\text{MAP}}(\mathcal{S}_1, \mathcal{S}_2, \sigma^2)}{P_{\text{err}}^{\text{MAP}}(\tilde{\mathcal{S}}_1, \tilde{\mathcal{S}}_2, \sigma^2)} \leq 1 \tag{1}$$

for any other constellations $\tilde{\mathcal{S}}_1$ and $\tilde{\mathcal{S}}_2$. In this way, an explicit construction of the constellations is obtained, and as we will see later, such a construction results in an negligible performance degradation relative to the truly optimal construction. We first need the following lemma.

Lemma 1. *Given average energy E, the maximally separated BPAM constellation for a binary source W with $\Pr(W = 0) = p$ is given by $S_0 = -\sqrt{(1 - p)E/p}$ and $S_1 = \sqrt{pE/(1 - p)}$, and the Euclidean distance between S_0 and S_1 is $S_1 - S_0 = \sqrt{E/(p(1 - p))}$.*

We remark that the binary constellation given in Lemma 1 is in fact the optimal constellation for single sender systems with a non-uniformly distributed binary source over AWGN channels [13][17]. Based on the optimality criterion given in (1), we next present our main result. For $j = 1, 2$, let $d_j \triangleq S_{j1} - S_{j0}$ and $d_{j,\max} \triangleq \sqrt{E_j/(p_j(1 - p_j))}$. Without loss of generality, we assume that p_1, p_2, E_1, and E_2 are such that $d_{2,\max} \leq d_{1,\max}$. For the case of $d_{2,\max} > d_{1,\max}$, we only need to swap the role of the two senders in the main result.

Theorem 2. *Suppose that $d_{1,\max} \geq d_{2,\max}$. The optimized constellation (in the sense of (1)) for sender 1 is given by*

$$S_{10} = -\sqrt{(1 - p_1)E_1/p_1}, \; S_{11} = \sqrt{p_1 E_1/(1 - p_1)}. \tag{2}$$

The optimized constellation for sender 2 is given by

$$S_{20} = -\sqrt{(1 - p_2)E_2/p_2}, \; S_{21} = \sqrt{p_2 E_2/(1 - p_2)} \tag{3}$$

if $|d_2^*| \geq d_{2,\max}$*, and otherwise we have*

$$S_{20} = S_{21} - d_2^*,\ S_{21} = d_2^* p_2 \pm \sqrt{(d_2^*)^2 p_2(p_2-1) + E_2} \quad (4)$$

where

$$d_2^* = \begin{cases} -4\sigma^2 \ln(p_{10}+p_{01})/d_{1,\max} + d_{1,\max}/2, \\ \qquad\qquad \textit{if } (p_{00}+p_{11}) \geq (p_{10}+p_{01}), \\ 4\sigma^2 \ln(p_{11}+p_{00})/d_{1,\max} - d_{1,\max}/2, \\ \qquad\qquad \textit{if } (p_{00}+p_{11}) < (p_{10}+p_{01}). \end{cases}$$

In Theorem 2, if $|d_2^*| \geq d_{2,\max}$, the optimal constellation for single sender systems can be employed for the GMAC systems without losing optimality (in the sense of (1)). However, we note that this case occurs rarely. For example, suppose that $E_1 = E_2$ and the SNR is high. In this case, the condition $|d_2^*| \geq d_{2,\max}$ is equivalent to $p_2(1-p_2) \geq 4p_1(1-p_1)$. Assume further that the source probabilities $[p_{00}, p_{01}, p_{10}, p_{11}]$ are uniformly drawn from the standard 3-simplex. Numerical calculation shows that in this case, the inequality will hold with a probability of approximately 0.0141. In other words, employing the optimal constellation of single sender systems is not sufficient to achieve the optimal performance for the GMAC systems in most cases.

To prove this theorem, for each case we first derive a closed form expression of the system's correct decoding probability in the high SNR regime, which is denoted by $\tilde{P}_\mathrm{c}$. The $\tilde{P}_\mathrm{c}$'s for all cases are listed in Table I. It is observed that by symmetry some cases can be disregarded without degrading our design. Let $Q(x) \triangleq \int_x^\infty (1/\sqrt{2\pi}) \exp(t^2/2)\, \mathrm{d}t$ denote the Gaussian Q-function. We summarize the observation in Lemmas 3 and 4.

Lemma 3. *The maximum of* $\tilde{P}_c$*(Case VII) is the same as the maximum of* $\tilde{P}_c$*(Case I).*

Proof: For Case VII, by defining $\bar{d}_j = -d_j$ for $j = 1, 2$, the correct decoding probability at high SNRs can be rewritten as

$$\begin{aligned}\tilde{P}_\mathrm{c}(\text{Case VII}) &= Q\left(\frac{-\bar{d}_2}{2\sigma}\right) - (p_{01}+p_{10})Q\left(\frac{\bar{d}_1 - \bar{d}_2}{2\sigma}\right) \\ &= 1 - Q\left(\frac{\bar{d}_2}{2\sigma}\right) - (p_{01}+p_{10})Q\left(\frac{\bar{d}_1 - \bar{d}_2}{2\sigma}\right),\end{aligned}$$

where $\bar{d}_j \in (0, d_{j,\max}]$. This expression is of the same form as $\tilde{P}_\mathrm{c}$(Case I) with the correspondences $d_1 \leftrightarrow \bar{d}_1$ and $d_2 \leftrightarrow \bar{d}_2$. Moreover, the domain of the parameters are identical, i.e., $d_j, \bar{d}_j \in (0, d_{j,\max}]$, $j = 1, 2$. Therefore, for any σ^2, we always have $\max_{d_1,d_2} \tilde{P}_\mathrm{c}(\text{Case I}) = \max_{\bar{d}_1,\bar{d}_2} \tilde{P}_\mathrm{c}(\text{Case VII})$. ■

The argument of Lemma 3 can be extended to Cases II and VIII, Cases III and V, and Cases IV and VI. Since the corresponding proofs are almost identical, we omit the details. Based on these results, we can exclude Cases IV, V, VII, and VIII from consideration.

Lemma 4. *The maximum of* $\tilde{P}_c$*(Case II) cannot be less than the maximum of* $\tilde{P}_c$*(Case I).*

Proof: Define $G(y_1, y_2) = 1 - Q\left(\frac{y_1}{2\sigma}\right) - (p_{10}+p_{01})Q\left(\frac{y_2}{2\sigma}\right)$ for $y_1, y_2 > 0$. For the maximum of correct decoding we have

$$\tilde{P}_\mathrm{c}^*(\text{Case I}) \triangleq \max_{y_1 \in (0, d_{2,\max}]} \max_{y_2 \in (0, d_{1,\max} - y_1]} G(y_1, y_2)$$

and

$$\tilde{P}_\mathrm{c}^*(\text{Case II}) \triangleq \max_{y_1 \in (0, d_{2,\max}]} \max_{y_2 \in (0, d_{2,\max} - y_1]} G(y_1, y_2).$$

Since $d_{1,\max} \geq d_{2,\max}$, we obtain $\tilde{P}_\mathrm{c}^*(\text{Case I}) \geq \tilde{P}_\mathrm{c}^*(\text{Case II})$. ■

A similar argument can be made to show that $\tilde{P}_\mathrm{c}^*$(Case VI) $\leq \tilde{P}_\mathrm{c}^*$(Case III). Consequently, it suffices to optimize constellations for Cases I and III, and the design with the larger $\tilde{P}_\mathrm{c}$ among the two cases is the best design. The next two lemmas help us derive the optimized constellations for both cases.

Lemma 5. *For fixed* d_2*,* $\tilde{P}_c$*(Case I) is increasing in* d_1*.*

Proof: Taking the partial derivative of $\tilde{P}_\mathrm{c}$(Case I) with respect to d_1 yields

$$\frac{\partial \tilde{P}_\mathrm{c}(\text{Case I})}{\partial d_1} = (p_{01}+p_{10}) \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(d_1-d_2)^2}{8\sigma^2}\right) \geq 0.$$

■

Lemma 6. $\tilde{P}_c$*(Case I) is concave in* d_2 *for* $d_1 = d_{1,\max}$*.*

Proof: By taking partial derivatives of $\tilde{P}_\mathrm{c}$(Case I) with respect to d_2, we have

$$\frac{\partial \tilde{P}_\mathrm{c}(\text{Case I})}{\partial d_2} = \frac{1}{2\sqrt{2\pi\sigma^2}} \left(\exp\left(\frac{-d_2^2}{8\sigma^2}\right) - (p_{01}+p_{10}) \exp\left(\frac{-(d_{1,\max}-d_2)^2}{8\sigma^2}\right)\right)$$

and

$$\frac{\partial^2 \tilde{P}_\mathrm{c}(\text{Case I})}{\partial d_2^2} = \frac{1}{16\sigma^2\sqrt{2\pi}} \left(\exp\left(\frac{-d_2^2}{8\sigma^2}\right)(-2d_2) - (p_{01}+p_{10}) \exp\left(\frac{-(d_{1,\max}-d_2)^2}{8\sigma^2}\right) 2(d_{1,\max}-d_2)\right). \quad (5)$$

Since $d_{2,\max} \leq d_{1,\max}$, the second derivative given in (5) is non-positive for all $d_2 \in (0, d_{2,\max}]$. Hence, $\tilde{P}_\mathrm{c}$(Case I) is a concave function of d_2 on the interval $(0, d_{2,\max}]$. ■

Similarly, one can easily show that for any $d_1 \in (0, d_{2,\max}]$, $\tilde{P}_\mathrm{c}$(Case III) is increasing in d_2, where d_1 is upper bounded by $d_{2,\max}$ due to the condition $|d_1| < |d_2|$. Also, for $d_2 = d_{2,\max}$, $\tilde{P}_\mathrm{c}$(Case III) is a concave function in d_1 for $d_1 \in (0, d_{2,\max}]$, Since the proofs of these statements are almost identical to the proof for Case I, the details are omitted. Based on these results, we can now prove Theorem 2.

Proof of Theorem 2: First, we show that if $p_{00}+p_{11} \geq p_{10}+p_{01}$, then $\tilde{P}_\mathrm{c}^*(\text{Case I}) \geq \tilde{P}_\mathrm{c}^*(\text{Case III})$, which implies that we only need to consider Case I. By letting $\bar{d}_2 = -d_2$, we can rewrite $\tilde{P}_\mathrm{c}$(Case III) as

$$\begin{aligned}\tilde{P}_\mathrm{c}(\text{Case III}) &= Q\left(\frac{-\bar{d}_2}{2\sigma}\right) - (p_{00}+p_{11})Q\left(\frac{d_1 - \bar{d}_2}{2\sigma}\right) \\ &= 1 - Q\left(\frac{\bar{d}_2}{2\sigma}\right) - (p_{00}+p_{11})Q\left(\frac{d_1 - \bar{d}_2}{2\sigma}\right), \quad (6)\end{aligned}$$

TABLE I: The probability of correct decoding in the high SNR regime, where the arrows in the second column indicate the relationships $d_1 \gtrless 0$, $d_2 \gtrless 0$, and $|d_1| \gtrless |d_2|$, respectively, and $Q(\cdot)$ denotes the Gaussian Q-function.

Case		$\tilde{P}_{\rm c}$
I	(>,>,>)	$1 - Q\left(\frac{d_2}{2\sigma}\right) - (p_{10}+p_{01})Q\left(\frac{d_1-d_2}{2\sigma}\right)$
II	(>,>,<)	$1 - Q\left(\frac{d_1}{2\sigma}\right) - (p_{10}+p_{01})Q\left(\frac{d_2-d_1}{2\sigma}\right)$
III	(>,<,>)	$Q\left(\frac{d_2}{2\sigma}\right) - (p_{00}+p_{11})Q\left(\frac{d_1+d_2}{2\sigma}\right)$
IV	(>,<,<)	$(p_{10}+p_{01}) - Q\left(\frac{d_1}{2\sigma}\right) + (p_{00}+p_{11})Q\left(\frac{d_1+d_2}{2\sigma}\right)$
V	(<,>,>)	$(p_{10}+p_{01}) - Q\left(\frac{d_2}{2\sigma}\right) + (p_{00}+p_{11})Q\left(\frac{d_1+d_2}{2\sigma}\right)$
VI	(<,>,<)	$Q\left(\frac{d_1}{2\sigma}\right) - (p_{00}+p_{11})Q\left(\frac{d_1+d_2}{2\sigma}\right)$
VII	(<,<,>)	$Q\left(\frac{d_2}{2\sigma}\right) - (p_{10}+p_{01})Q\left(\frac{d_2-d_1}{2\sigma}\right)$
VIII	(<,<,<)	$Q\left(\frac{d_1}{2\sigma}\right) - (p_{10}+p_{01})Q\left(\frac{d_1-d_2}{2\sigma}\right)$

where $d_1 \in (0, d_{1,\max}]$ and $\bar{d}_2 \in (0, d_{2,\max}]$. Based on this expression and the correspondence $\bar{d}_2 \leftrightarrow d_2$, the feasible set of d_1 and $\bar{d}_2$ for $\tilde{P}_{\rm c}$(Case III) is observed to be identical to that for $\tilde{P}_{\rm c}$(Case I). Moreover, the new expression for $\tilde{P}_{\rm c}$(Case III) only differs from $\tilde{P}_{\rm c}$(Case I) in the coefficient of the third term. Therefore, when $p_{00}+p_{11} \geq p_{10}+p_{01}$, we have $\max_{d_1,d_2} \tilde{P}_{\rm c}(\text{Case I}) \geq \max_{d_1,\bar{d}_2} \tilde{P}_{\rm c}(\text{Case III})$ and the optimal constellations for Case I should be selected. In contrast, when $p_{00}+p_{11} < p_{10}+p_{01}$, the optimal constellations for Case III are chosen. Next, we explicitly derive the optimal constellations for Cases I and III to complete the proof.

To find the optimal $\mathcal{S}_1$ that maximizes $\tilde{P}_{\rm c}$(Case I), d_1 should be set to its maximum possible value $d_1 = d_{1,\max}$ according to Lemma 5. By Lemma 1, this choice immediately gives the optimal constellation $\mathcal{S}_1$ in (2). Moreover, based on the concavity property in Lemma 6, the maximum of $\tilde{P}_{\rm c}$(Case I) in the variable d_2 is known to occur at either where the partial derivative is zero or at the boundary of its support interval. Solving $\partial\tilde{P}_{\rm c}(\text{Case I})/\partial d_2 = 0$ for d_2, we obtain $d_2^* = (-4\sigma^2/d_{1,\max})\ln(p_{10}+p_{01}) + d_{1,\max}/2$. By substituting $S_{21} - S_{20} = d_2^*$ into $p_2 S_{20}^2 + (1-p_2)S_{21}^2 = E_2$, the optimal $\mathcal{S}_2$ is obtained. When $(d_2^*)^2 p_2(p_2-1) + E_2 > 0$, there are two optimal constellations as shown in (4). When $(d_2^*)^2 p_2(p_2-1) + E_2 \leq 0$, the optimal constellation for sender 2 is given in (3), which follows from the result that the maximum of $\tilde{P}_{\rm c}$(Case I) occurs at $d_2 = d_{2,\max}$ and from Lemma 1.

With the help of the expression in (6) and the above derivation, the optimal constellations for Case III can be easily derived. For sender 1, the optimized constellation $\mathcal{S}_1$ is the same as the one given in (2) because the choice $d_1 = d_{1,\max}$ also maximizes $\tilde{P}_{\rm c}$(Case III). For sender 2, solving $\partial\tilde{P}_{\rm c}(\text{Case III})/\partial\bar{d}_2 = 0$ gives $\bar{d}_2 = (-4\sigma^2/d_{1,\max})\ln(p_{00}+p_{11}) + d_{1,\max}/2$. By substituting $S_{20} - S_{21} = \bar{d}_2$ into $p_2|S_{20}|^2 + (1-p_2)|S_{21}|^2 = E_2$, there are two optimized constellations $\mathcal{S}_2$ in the case of $\bar{d}_2^2 p_2(p_2-1) + E_2 > 0$ as shown in (4). When $\bar{d}_2^2 p_2(p_2-1) + E_2 \leq 0$, the optimized constellation $\mathcal{S}_2$ is given in (3). ■

To optimize the joint symbol error rate, the proposed design not only attempts to enlarge the minimum Euclidean distance between combined signals but also manages to lower the number of neighboring signals for either subsets $\{A_{00}, A_{11}\}$ or $\{A_{10}, A_{01}\}$ of $\mathcal{A}$. The latter effort is the most important difference in constellation design between single and two-sender systems. Since the two users cannot cooperate (being separately located), we design $\mathcal{S}_1$ and $\mathcal{S}_2$ rather than $\mathcal{A}$ to minimize the system's joint error rate. Hence, reducing the number of neighboring signals for all high probability symbols A_{uv}'s in $\mathcal{A}$ as in conventional 4-PAM design for single sender systems does not apply to our two-sender systems.

As a final remark, we note that our optimized BPAM design for uniformly distributed sources can be used in conjunction with channel coding schemes such as [18]-[20] to enhance their decoding performance. This topic is, however, beyond the scope of this paper, and hence is not covered here.

IV. Simulation Results

In this section, we evaluate the effectiveness of our constellation designs via simulations. We set $E_1 = E_2 = 1$, and the SNR is defined as $(E_1+E_2)/N_0 = 2/\sigma^2$. For performance comparison, the conventional antipodal BPAM is considered, i.e., $S_{10} = S_{20} = 1$ and $S_{11} = S_{21} = -1$. The optimal constellation designed for a single sender AWGN channel with a non-uniform binary source is also included [13], i.e., separately substitute $W = U$ and $W = V$ in Lemma 1. Such constellations are called individually optimized BPAM constellations. We also implement the scaled antipodal BPAM scheme proposed in [8]. The scheme's power allocation is performed subject to $E_1 + E_2 = 2$ for a fair comparison with our system and the scaled antipodal constellation is of the form $\mathcal{S}_j = \{-\sqrt{C_j}, \sqrt{C_j}\}$ for $j = 1, 2$, in which C_j's are optimized to minimize the system's error rate in the high SNR regime. The decoding performance of numerically optimized BPAM constellations, i.e., constellations which minimize $P_{\rm err}^{\rm MAP}$ for each SNR value and which are obtained by exhaustive search, is further provided for reference. Our proposed design given in Theorem 2 is called a jointly optimized BPAM constellation. Let $\underline{p}_{UV} \triangleq [p_{00}, p_{01}, p_{10}, p_{11}]$.

For the source $\underline{p}_{UV,\text{Case1}} = [0.091, 0.009, 0.009, 0.891]$ with $p_1 = p_2 = 0.1$, we observe in Fig. 2 that the conventional antipodal and individually optimized BPAM schemes exhibit poor decoding performance. This is due to the fact that the identical BPAM constellations at both senders introduce an ambiguity for the transmitted signals, i.e., $A_{01} = A_{10} = 0$ (recall that $A_{uv} = S_{1u} + S_{2v}$). Although the scaled BPAM scheme brings some improvement, the jointly optimized constellation derived from our analysis has a considerably better performance which is also quite close to that of the numerically optimized constellation. The same phenomenon holds for $\underline{p}_{UV,\text{Case2}} = [0.18, 0.02, 0.32, 0.48]$ with $p_1 = 0.2$ and $p_2 = 0.5$ except that the individually optimized BPAM scheme performs better than the earlier case; indeed since $p_1 \neq p_2$, the senders' constellations do not coincide. Overall, our jointly optimized scheme has a superior performance for all SNRs.

Lastly, we present an example in which the two senders transmit their signals with different average energy $E_1 = 2E_2$. Here, the joint source with $\underline{p}_{UV,\text{Case1}}$ is considered, and the simulation results are depicted in Fig. 3. Clearly, due to the

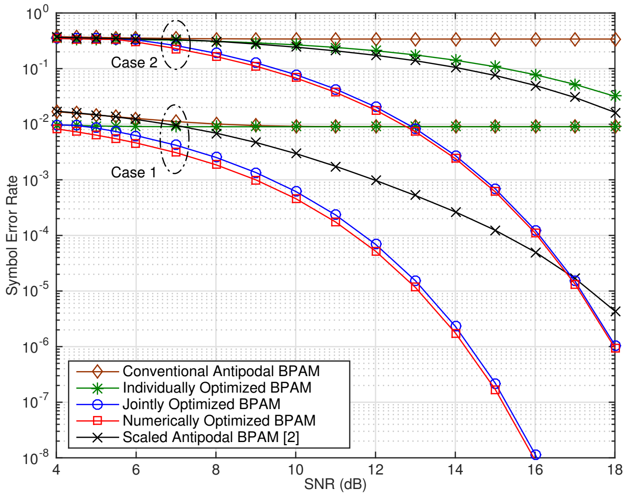

Fig. 2: The decoding performance of various BPAM scheme for correlated sources $\underline{p}_{UV,\text{Case1}}$ and $\underline{p}_{UV,\text{Case2}}$.

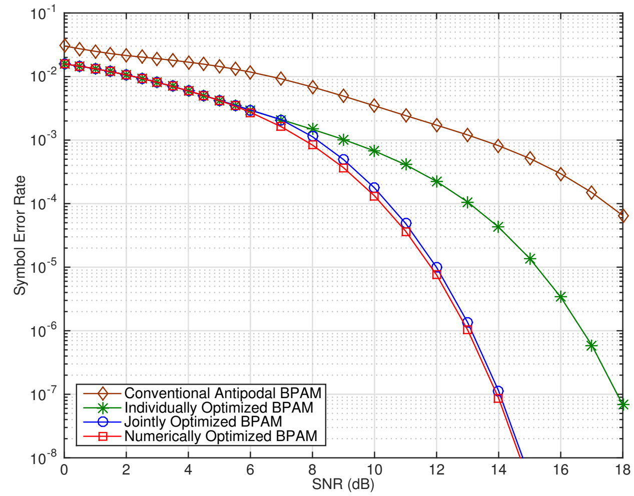

Fig. 3: The decoding performance of various BPAM scheme for $\underline{p}_{UV,\text{Case1}}$ with unequal energy allocation: $E_1 = 2E_2$.

unequal energy allocation, the signal sets of both conventional antipodal BPAM and the individually optimized BPAM schemes for the two senders are distinct, thereby yielding a better error rate performance than that of equal energy allocation (see Fig. 2). At an error rate of 10^{-5}, the jointly optimized constellation achieves about 3dB gain over the individually optimized design. Also, there is about 1dB SNR gain from the unequal energy allocation for our design (compare Figs. 2 and 3). This example demonstrates that combining the idea of unequal energy allocation [8] with our design for the two-sender GMAC can further improve the decoding performance.

V. Conclusions

In this work, we investigated the design of optimized binary signaling schemes for sending correlated binary sources over a non-orthogonal GMAC. For a wide range of SNRs and various correlated source distributions, the error rate performance of the analytically derived signaling schemes was found to be close to the optimal performance under joint MAP decoding. The SNR gain achieved by our scheme is more than 4dB over the scaled scheme of [8] in the high SNR regime. More discussions on the BPAM constellation design for GMACs by using partially correlated BPAM waveforms can be found in [21]. Future research directions include the transmission of correlated non-binary sources and GMAC systems with more senders.

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