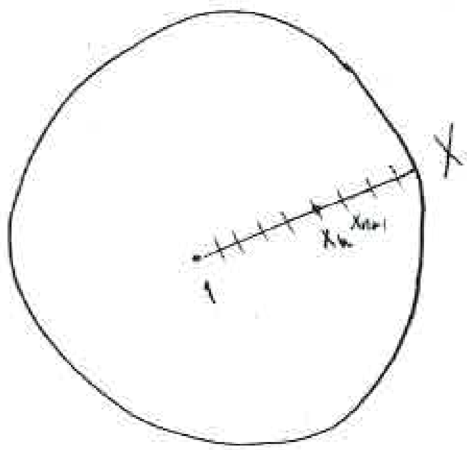
$Out(F_N) \curvearrowright Curr(F_N)$

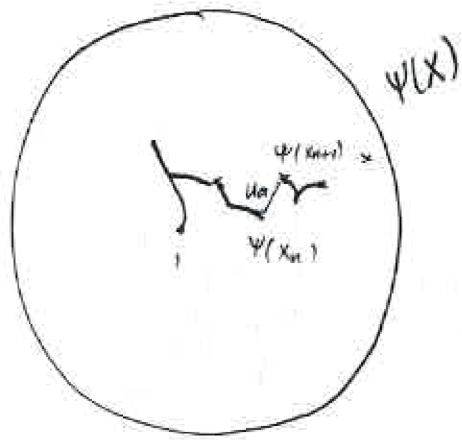
Fact: $\forall g \in F_N \quad g \neq 1$

$\forall \varphi \in Out(F_N)$ then $\varphi \eta_g = \eta_{\varphi(g)}$

How does $Aut(F_N)$ act on ∂F_N?

$\psi \in Aut(F_N)$, $F_N = F(A)$



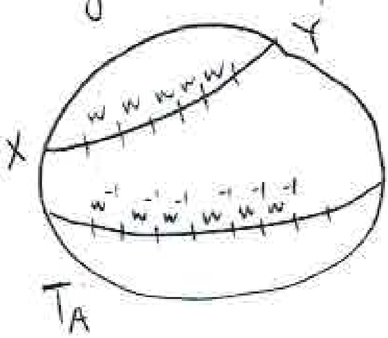


$\psi(a) = u_a \in F(A)$

- Look at the image, and then tighten the path.

Let $g \in F_N$, $g \neq 1$, not a proper power

w - cycl. reduced form of g in $F(A)$



$(X, Y) \in \partial^2 F_N$

$\Lambda_g = \{ (X,Y) \in \partial^2 F_N \mid$ the geod. from X to Y in T_A is labelled by $\ldots wwww \ldots$ or $\ldots w^{-1}w^{-1}w^{-1}w^{-1} \ldots \}$

$S \subseteq \partial^2 F_N$

$\eta_g(S) = \# \{ (X,Y) \in S \mid (X,Y) \in \Lambda_g \}$

<u>Facts:</u> Let $\Psi \in Aut(F_N)$ then:

1) $\forall g \in F_N$, $\forall X \in \partial F_N$

$$\Psi(gX) = \Psi(g)\Psi(X)$$

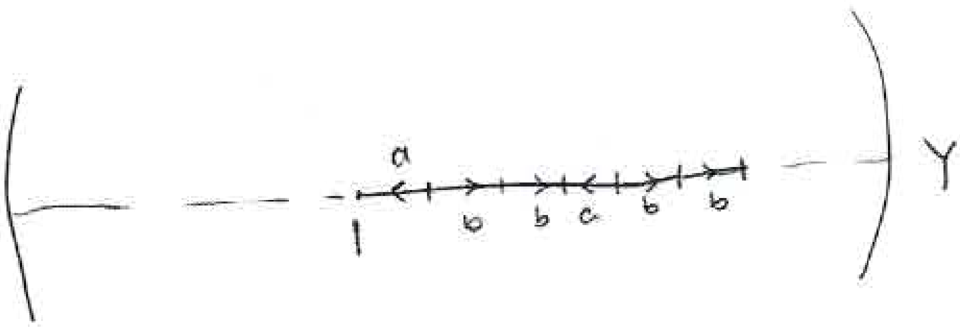
Assume $g \neq 1$, not a proper power

2) $\Psi(\Lambda_g) = \Lambda_{\Psi(g)}$

<u>EX:</u>

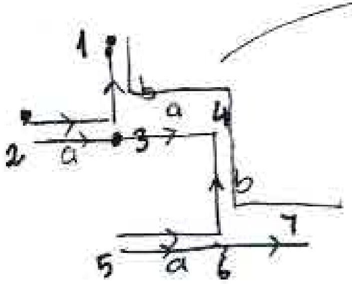
$F_2 = F(a,b)$, $\varphi(a) = ab$ $\varphi(b) = a$

$\varphi(a^{-1}) = b^{-1}a^{-1}$, $\varphi(b^{-1}) = a^{-1}$



$g = a^{-1}b^2$

$\varphi(g) = b^{-1}\cancel{a^{-1}}\cancel{a}a = b^{-1}a$



after cancelling

semiinfite power of $b^{-1}a$

$\mu \in Curr(F_N)$, $\varphi \in Aut(F_N)$

$S \subseteq \partial^2 F_N$, $(\varphi\mu)(S) := \mu(\varphi^{-1} S)$

$\Psi(uX) = \Psi(u)\Psi(X)$

Let $g \neq 1$, not a proper power.

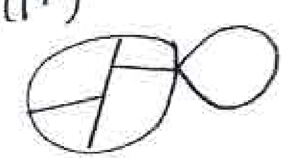
$\mu = \eta_g$, $(\varphi\eta_g)(S) = \eta_g(\varphi^{-1} S)$

$$= \#\{(X,Y) \in \varphi^{-1}(S) \mid (X,Y) \in \Lambda_g\}$$

$$= \#\{(X',Y') \in S \mid (X',Y') \in \Lambda_{\varphi(g)}\}$$

$$= \eta_{\varphi(g)}(S).$$

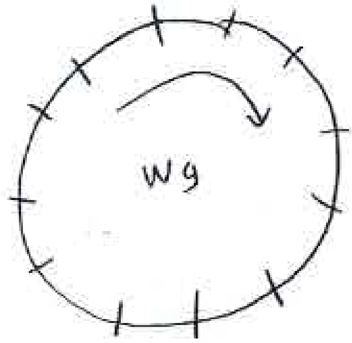
$\alpha : F_N \xrightarrow{\sim} \pi_1(\Gamma)$



$g \in F_N$, $g \neq 1$.

$g \rightsquigarrow w_g \rightarrow$ immersed circuit in Γ representing $[g]$

$w_g \rightsquigarrow W_g$
cyclic word / path



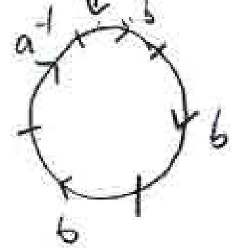
Ex: $F_2 = F(a,b)$

Γ :

$g = ab^2a^{-1}ba^{-1}$

$w_g = b^2a^{-1}b$

W_g

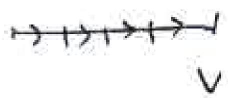


$v_1 = b^2$

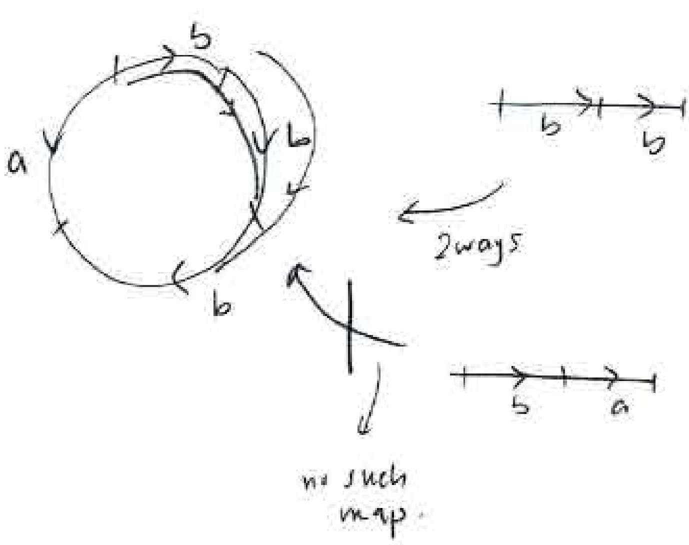
$v_2 = ba$

Let $v \in \Omega(\Gamma)$

J_v



<u>An occurance of v in W_g</u> is a map $f : J_v \to W_g$ sending vertices to vertices, edges to edges an respecting labels.



We can wind around the circle as many times we want.

Notation:

$\langle v, w_g \rangle_\alpha$:= # of occurances of v in w_g.

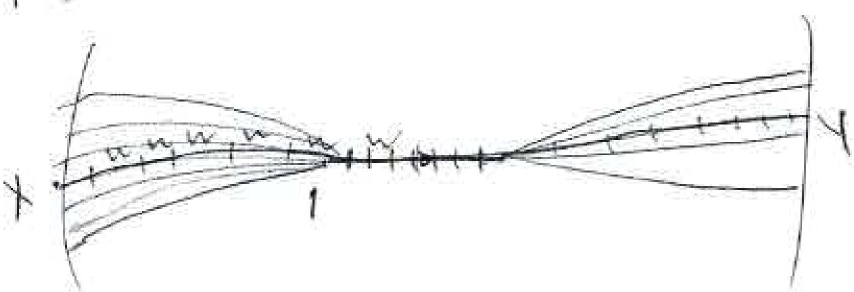
Prop: $\forall g \in F_N$, $g \neq 1$, $\forall v \in \Omega(\Gamma)$

$$\langle v, \eta_g \rangle_\alpha = \langle v, w_g \rangle_\alpha$$

weight $\eta_g(\mathrm{Cyl}(\gamma))$ where γ is any lift of v to $\tilde{\Gamma}$.

as above.

· A free basis



$T/_{+}$

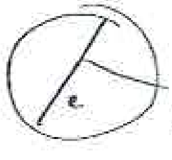
Theorem: $\{ c\eta_g \mid c \geq 0, g \in F_N, g \neq 1 \}$ is dense in $\mathrm{Curr}(F_N)$.

rational currents

INTERSECTION FORM

Construct $\langle \ , \ \rangle : cv_N \times \mathrm{Curr}(F_N) \to \mathbb{R}_{\geq 0}$

Let $T \in cv_N$, Γ / F_N - finite connected metric graph Γ with $\alpha : F_N \xrightarrow{\sim} \pi_1(\Gamma)$



$L(e) > 0$

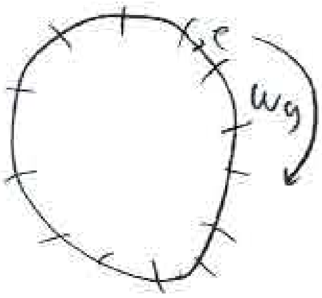
For $\mu \in Curr(F_N)$, define

$$\langle T, \mu \rangle = \sum_{e \in E^+\Gamma} \langle e, \mu \rangle_\alpha \cdot L(e)$$

Prop: $\forall T \in cv_N$, $\forall g \in F_N$ $g \neq 1$

$$\langle T, \eta_g \rangle = \|g\|_T$$

$g \rightsquigarrow w_g$, $\|g\|_T = L(w_g)$



$$\langle T, \eta_g \rangle = \sum_{e \in E^+\Gamma} \langle e, \eta_g \rangle L(e)$$

$$= \sum_{e \in E^+\Gamma} \langle e, w_g \rangle_\alpha \cdot L(e) \quad = L(w_g) = \|g\|_T.$$

$$= \|g\|_T$$

End of Lecture-3.