

-1/2

```
load_attach_path("/home/catherine/Dropbox/SageThierry/")
```

```
attach "inverse_alphabet.py"  
attach "free_group.py"  
attach "free_group_automorphism.py"  
attach "free_group_word.py"  
attach "inverse_graph.py"  
attach "graph_map.py"  
attach "marked_graph.py"  
attach "topological_representative.py"  
attach "test.py"
```

```
N=3  
M=20  
L=50  
F=FreeGroup(N)  
good=0  
reducible=0  
perm=0  
pnielsen=0  
for i in xrange(L):  
    print i,":",  
    phi=F.random_automorphism(M)  
    done=False  
    while not done:  
        f=phi.train_track(stable=True,relative=False)  
        if len(f._strata)>1:  
            done=True  
            print "not irreducible"  
            reducible+=1  
        else:  
            if f.expansion_factor()==1:  
                done=True  
                print "permutation automorphism"  
                perm+=1  
            else:  
                pnps=f.periodic_nielsen_paths()  
                if len(pnps)==0:  
                    done=True  
                    good+=1  
                    print "good automorphism"  
                else:  
                    done=True  
                    print "pINPs exist:",len(pnps)  
                    pnielsen+=1
```


Edge map: a->BABABDCBDEBABABDCBDEBABDC,
 b->BABDCBABABDCBDEBABABDCBDEcddbabedbcdbababedbcdbababedABABDCBD
 dbabedbcdbababedbcdbababedABABDCBDEBABABDCBDEBABDCBDEBABABDCBDEBAB
 DEBABDCBDE,
 c->edABABDCBDEBABABDCBDEBABDCBABABDCBDEBABABDCBDEcddbabedbcdbaba|
 dbcdababedABABDCBDEcddbabedbcdbababedbcdbababedABABDCBDEBABABDC|
 BDCB, d->ABABDCBDEBABABDCBDE, e->ed
 77.14530740078947?
 Multifold: [['a', 7], ('c', 'B')]
 Safe Fold: ('c', 'B') avoiding ['a', 7]
 Fold: ('c', 'B') common prefix ed

Graph with inverses: a: 0->1, b: 1->3, c: 3->2, d: 2->1,
 e: 0->2, f: 0->3
 Marking: a->eCFaDE, b->edbF, c->fcE
 Edge map: a->fBAfBAfBDCBDEfBAfBAfBDCBDEfBAfBDCF,
 b->fBAfBDCBAfBAfBDCBDEfBAfBAfBDCBDEfcdBfabFedbcdbFabFabFedbcdbF
 FabFedAfBAfBDCBDEfcdBfabFedbcdbFabFabFedbcdbFabFabFedAfBAfBDCBD
 BAfBDCBDEfBAfBDCBAfBAfBDCBDEfBAfBAfBDCBDEfBAfBDCB,
 c->AfBAfBDCBDEfBAfBAfBDCBDEfBAfBDCBAfBAfBDCBDEfBAfBAfBDCBDEfcdB
 bFedbcdbFabFabFedbcdbFabFabFedAfBAfBDCBDEfcdBfabFedbcdbFabFabFe
 FabFabFedAfBAfBDCBDEfBAfBAfBDCBDEfBAfBDCB,
 d->AfBAfBDCBDEfBAfBAfBDCBDE, e->ed, f->ed
 77.0975368439764?
 Multifold: [['a', 2], ('b', 'A')]
 Safe Fold: ('b', 'A') avoiding ['a', 2]
 Fold: ('b', 'A') common prefix f

Graph with inverses: a: 0->4, b: 4->3, c: 3->2, d: 2->1,
 e: 0->2, f: 0->3, g: 1->4
 Marking: a->eCFaGDE, b->edgbF, c->fcE
 Edge map: a->fBAfBAfBGDCBGDEfBAfBAfBGDCBGDEfBAfBGDC,
 b->BAfBGDCBAfBAfBGDCBGDEfBAfBAfBGDCBGDEfcdgbFabFedgbcdbFabFabF
 gbcdbFabFabFedgAfBAfBGDCBGDEfcdgbFabFedgbcdbFabFabFedgbcdbFab
 edgAfBAfBGDCBGDEfBAfBAfBGDCBGDEfBAfBGDCBAfBAfBGDCBGDEfBAfBAfBGDC
 fBAfBGDCBG,
 c->gAfBAfBGDCBGDEfBAfBAfBGDCBGDEfBAfBGDCBAfBAfBGDCBGDEfBAfBAfBG
 BGDEfcdgbFabFedgbcdbFabFabFedgbcdbFabFabFedgAfBAfBGDCBGDEfcdg|
 edgbcdbFabFabFedgbcdbFabFabFedgAfBAfBGDCBGDEfBAfBAfBGDCBGDEfB
 CBG, d->gAfBAfBGDCBGDEfBAfBAfBGDCBGDE, e->ed, f->ed,
 g->f
 Fusion lines: [['G', 'D']]
 Keep edges: ['D']

Graph with inverses: a: 0->4, b: 4->3, c: 3->2, d: 2->4,
 e: 0->2, f: 0->3
 Marking: a->eCFaDE, b->edbF, c->fcE
 Edge map: a->fBAfBAfBDCBDEfBAfBAfBDCBDEfBAfBDC,
 b->BAfBDCBAfBAfBDCBDEfBAfBAfBDCBDEfcdBfabFedbcdbFabFabFedbcdbFa|

babcbdbcbdbcbdbcbdbcbabcbabcbdbdABDCBABDCBDCBDcdbabcbdbcbdbcbdbcbcb
dbabcbdbcbdbcbdbcbdbcbabcbabcbdbdABDCBABDCBDCBDBDCBDBDCBABDCBABDCB
BDCBDBDCBABDCBDCBDBDCB,
d->ABDCBABDCBDCBDBDCBDBDCBABDCBABDCBDCBDBDCBDB
72.99758349393871?
Multifold: [['c', 37], ('c', 'd')]
Safe Fold: ('c', 'd') avoiding ['c', 37]
Fold: ('c', 'd') common prefix
ABDCBABDCBDCBDBDCBDBDCBABDCBABDCBDCB

Graph with inverses: a: 0->4, b: 4->0, c: 1->0, d: 1->4,
e: 0->1

Marking: a->CEBDEaDE, b->edbecedb, c->edbec

Edge map:

a->DEBDECEBABDECEBABDECEBDECEBDEBDECEBDEBDECEBABDECEBABDECEBDEC
DEBDECEBDEBDECEBABDECEBDECEBDE, b->BDECEBABDECEBDECEBDEBDECEB,
c->Dcedbabecedbedbecedbedbecedbecedbabecedbabecedbedbecedbedbec
becedbabecedbabecedbedABDECEBABDECEBDECEBDcedbabecedbedbecedbed
becedbabecedbabecedbedbecedbedbecedbecedbabecedbabecedbedABDECE
CEBDECEBDEBDECEBDEBDECEBABDECEBABDECEBDECEBDEBDECEBDEBDECEBABDE
CEBDEBDECEB, d->DEBDECEBDE,
e->ABDECEBABDECEBDECEBDEBDECEBDEBDECEBABDECEBABDECEBDECEB

72.5581316284908?

Multifold: [['c', 1], ('c', 'd')]

Safe Fold: ('c', 'd') avoiding ['c', 1]

Subdivide edges: ['D']

Graph with inverses: a: 0->4, b: 4->0, c: 1->0, d: 2->4,
e: 0->1, f: 2->1

Marking: a->CEBDfEaDfE, b->eFdbceceFdb, c->eFdbec

Edge map:

a->DfEBDfECEBABDfECEBABDfECEBDfECEBDfEBDfECEBDfEBDfECEBABDfECEB.
DfECEBDfECEBDfEBDfECEBDfEBDfECEBABDfECEBDfECEBDfE,
b->BDfECEBABDfECEBDfECEBDfEBDfECEB,
c->DfceFdbabeceFdbeFdbceceFdbeFdbceceFdbabeceFdbabeceFdbeFdb
ceFdbeFdbceceFdbabeceFdbabeceFdbabeceFdbefdABDfECEBABDfECEBDfECEBD
babeceFdbeFdbceceFdbeFdbceceFdbabeceFdbabeceFdbefdBABDfECEBABDfECEBDfECEBDfE
FdbceceFdbabeceFdbabeceFdbefdBABDfECEBABDfECEBDfECEBDfEBDfECEBDfE
EBABDfECEBABDfECEBDfECEBDfEBDfECEBDfEBDfECEBABDfECEBDfECEBDfEBD
, d->E,
e->ABDfECEBABDfECEBDfECEBDfEBDfECEBDfEBDfECEBABDfECEBABDfECEBDf
EB, f->FdbceceFdbefD

Fold: ('c', 'F') common prefix D

Graph with inverses: a: 0->4, b: 4->0, c: 3->0, d: 2->4,
e: 0->1, f: 2->3, g: 1->3

Marking: a->CGEBDFGEaDfGE, b->egFdbegcegFdb, c->egFdbegc

Edge map:

a->DfGEBDFGECGEBABDfGECGEBABDfGECGEBDFGECGEBDFGEBDFGECGEBDFGEBD

ECGEBABDfGECGEBABDfGECGEBDfGECGEBDfGEBDfGECGEBDfGEBDfGECGEBABDf
 BDfGECGEBDfGE, b->BDfGECGEBABDfGECGEBDfGECGEBDfGEBDfGECGEB,
 c->fcegFdbabegcegFdbegFdbegcegFdbegFdbegcegFdbabegcegFdb
 begcegFdbegFdbegcegFdbegFdbegcegFdbegcegFdbabegcegFdbabegcegFdb
 BDfGECGEBABDfGECGEBDfGECGEBDfcegFdbabegcegFdbegFdbegcegFdbegFdb
 FdbegcegFdbabegcegFdbabegcegFdbegFdbegcegFdbegFdbegcegFdbegcegF
 gcegFdbabegcegFdbegFdABDfGECGEBABDfGECGEBDfGECGEBDfGEBDfGECGEBD
 fGECGEBABDfGECGEBABDfGECGEBDfGECGEBDfGEBDfGECGEBDfGEBDfGECGEBAB
 GEBDfGECGEBDfGEBDfGECGEB, d->E,
 e->ABDfGECGEBABDfGECGEBDfGECGEBDfGEBDfGECGEBDfGEBDfGECGEBABDfGE
 EBABDfGECGEBDfGECGEB, f->gFdbegcegFdbegF, g->D
 Fusion lines: [['G', 'E'], ['D', 'f']]
 Keep edges: ['E', 'D']

Graph with inverses: a: 0->4, b: 4->0, c: 3->0, d: 3->4,
 e: 0->3

Marking: a->CEBDEaDE, b->edbecedb, c->edbec

Edge map:

a->DEBDECEBABDECEBABDECEBDECEBDEBDECEBDEBDECEBABDECEBABDECEBDEC
 DEBDECEBDEBDECEBABDECEBDECEBDE, b->BDECEBABDECEBDECEBDEBDECEB,
 c->cedbabecedbedbecedbedbecedbedbecedbabecedbabecedbedbecedbedbece
 cedbabecedbabecedbedABDECEBABDECEBDECEBDcedbabecedbedbecedbedbe
 cedbabecedbabecedbedbecedbedbecedbedbecedbabecedbabecedbedABDECEB
 EBDECEBDEBDECEBDEBDECEBABDECEBABDECEBDECEBDEBDECEBDEBDECEBABDEC
 EBDEBDECEB, d->EBDECEBDE,
 e->ABDECEBABDECEBDECEBDEBDECEBDEBDECEBABDECEBABDECEBDECEBD
 72.5572724907789?

Multifold: [['c', 83], ('A', 'D')]

Safe Fold: ('A', 'D') avoiding ['c', 83]

Fold: ('A', 'D') common prefix edbecedbe

Graph with inverses: a: 0->3, b: 4->0, c: 3->0, d: 3->4,
 e: 0->3

Marking: a->CEBDEaE, b->edbecedb, c->edbec

Edge map:

a->DEBDECEBDABDECEBDABDECEBDECEBDEBDECEBDEBDECEBDABDECEBDABDECE
 ECEBDEBDECEBDEBDECEBDABDEC, b->BDECEBDABDECEBDECEBDEBDECEB,
 c->cedbadbecedbedbecedbedbecedbedbecedbadbecedbadbecedbedbecedbedbe
 edbecedbadbecedbadbecedbeABDECEBDABDECEBDECEBDcedbadbecedbedbec
 becedbedcedbadbecedbadbecedbedbecedbedbecedbedbecedbadbecedbadbeced
 ECEBDABDECEBDECEBDEBDECEBDABDECEBDABDECEBDECEBDEBDECEBD
 EBDABDECEBDECEBDEBDECEB, d->EBDECEBDE,
 e->DABDECEBDABDECEBDECEBDEBDECEBDEBDECEBDABDECEBDABDECEBDECEBD

Fusion lines: [['B', 'D']]

Keep edges: ['B']

Graph with inverses: a: 0->4, b: 4->0, c: 4->0, e: 0->4

Marking: a->CEBEaE, b->ebeceb, c->ebec

Edge map:

a->EBECEBABECEBABECEBEBECEBEBECEBEBECEBABECEBABECEBEBECEBEBECEBEBEC
 ABEC, b->EBECEBEBECEBABECEBEBECEBEBECEB,
 c->cebabecebebecebebecebecebecebecebecebecebecebecebecebecebecebeceba|
 cebeABECEBABECEBEBECEBcebebecebebecebecebecebecebecebecebecebecebecebe|
 becebecebecebecebeABECEBABECEBEBECEBEBECEBEBECEBABECEBABECEBEBECEBE|
 EBECEBABECEBEBECEBEBECEB,
 e->ABECEBABECEBEBECEBEBECEBEBECEBABECEBABECEBEBECEB
 72.49487434273410?
 Train-track !
 INPs: []
 Graph with inverses: a: 0->4, b: 4->0, c: 4->0, e: 0->4
 Marking: a->CEBEaE, b->ebeceb, c->ebec
 Edge map:
 a->EBECEBABECEBABECEBEBECEBEBECEBEBECEBABECEBABECEBEBECEBEBECEBEBEC
 ABEC, b->EBECEBEBECEBABECEBEBECEBEBECEB,
 c->cebabecebebecebebecebecebecebecebecebecebecebecebecebecebecebeceba|
 cebeABECEBABECEBEBECEBcebebecebebecebecebecebecebecebecebecebecebecebe|
 becebecebecebecebeABECEBABECEBEBECEBEBECEBEBECEBABECEBABECEBEBECEBE|
 EBECEBABECEBEBECEBEBECEB,
 e->ABECEBABECEBEBECEBEBECEBEBECEBABECEBABECEBEBECEB
 Irreducible representative
[full output.txt](#)

```
print f.periodic_nielsen_paths()
```

[]

```
G=f.domain()
G.plot()
```

