1. For any complex number $z$, define $\cos z$ and $\sin z$ by the formulas

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

and

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}.$$

Prove that

(a) $\cos^2 z + \sin^2 z = 1$;

(b) if $z = iy$ is purely imaginary, then $\cos z$ is a real number greater or equal to 1.

2. Solve the following equations

(a) $2x^2 + x + 4 = 0$ in $\mathbb{Z}_5$;

(b) $3x = 4$ in $\mathbb{Z}_7$;

(c) $6x = 2$ in $\mathbb{Z}_{10}$.

3. Solve the following linear system in $\mathbb{Z}_5$:

$$\begin{cases}
2x_1 + 4x_2 + x_3 + 3x_4 + 3x_5 = 4 \\
x_1 + 2x_2 + x_4 = 0 \\
x_1 + 2x_2 + x_3 + x_4 + x_5 = 1 \\
2x_1 + 4x_2 + x_3 + 2x_5 = 0.
\end{cases}$$