1. Consider \( Q = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -1 \\ -1 & 0 & 1 \end{bmatrix} \).

(a) Verify that \( Q^3 - 5Q^2 + 9Q - 4I = 0 \).

(b) If \( P = \frac{1}{4}(Q^2 - 5Q + 9I) \), then prove \( P = Q^{-1} \).

(c) Explain how \( P \) in part (b) can be obtained from the equation in part (a).

2. Assume that \( A, I + B, \) and \( C \) are invertible \( n \times n \)-matrices. Solve the matrix equation

\[
(A(I + X)A^{-1})^{-1} = C(I + B)C^{-1}
\]

for the matrix \( X \), i.e., express \( X \) in terms of the matrices \( A, B, \) and \( I \).

3. Let \( A \) be an invertible \( m \times m \)-matrix, \( D \) - an invertible \( n \times n \)-matrix, and \( C \) - an arbitrary \( n \times m \)-matrix. Prove that the \((m + n) \times (m + n)\)-matrix

\[
\begin{bmatrix} A & 0 \\ C & D \end{bmatrix}
\]

is invertible and find its inverse. In the formula above 0 stands for the zero matrix of size \( m \times n \).

**Hint.** Read the subsection *Partitioned matrices* on p. 145 and especially Example 3.12 on p. 148. Also compare with Problem 64 on p. 179.