1. Let $U$ be a finite dimensional vector space, and let $V$ and $W$ be subspaces of $U$. Recall that $V + W$ and $V \cap W$ are subspaces of $U$, see Problem 3 from Assignment 14. Prove that
\[ \dim(V + W) = \dim V + \dim W - \dim(V \cap W). \]

*Hint.* Start with a basis of $V \cap W$, complete it to bases of $V$ and $W$ respectively and prove that all vectors obtained in this way form a basis of $V + W$.

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be counterclockwise rotation through angle $\pi/3$ about the origin and let $S : \mathbb{R}^2 \to \mathbb{R}^2$ be reflection in the line $x = y$. Write down the standard matrices $[T], [S]$, and $[S \cdot T]$ of the linear transformations $T$, $S$, and $S \cdot T$. Verify that $[S \cdot T] = [S][T]$.

3. Let $P : \mathbb{R}^3 \to \mathbb{R}^3$ be the projection onto the plane $x + y + z = 0$. Write down the matrix $[P]$ of $P$ and verify that $[P]^2 = [P]$. Explain why $[P]^2 = [P]$. 