Problem Set #19
Due: 1 March 2019 at noon

1. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Define the linear transformation $T : \mathcal{M}_2 \to \mathcal{M}_2$ by $T(X) = AX -XA$. Find the matrix of $T$ in the basis $B = \{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \}$. 

2. Let $\mathcal{M}_n$ be the vector space of complex $(n \times n)$-matrices and consider the linear operator $T : V \to V$ defined by $T(A) = A^T$.
   (a) Show that $\pm 1$ are the only eigenvalues of $T$.
   (b) Describe the eigenvectors corresponding to each eigenvalue of $T$.
   (c) Find a basis $B$ for $\mathcal{M}_2$ such that $[T]_B$ is a diagonal matrix.

3. Let $\mathcal{T}$ be the $\mathbb{C}$-vector space with basis $B = \{ 1, \cos x, \sin x \}$. Define $J : \mathcal{T} \to \mathcal{T}$ by $(J f)(x) = \int_0^\pi f(x-t) \, dt$. Show that $J$ is diagonalizable and find a basis of $\mathcal{T}$ consisting of eigenvectors of $J$. 

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