Image Coding for Binary Bursty Noise Channels*

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Abstract- **We present a new progressive method for image transmission over binary channels with additive bursty noise. The method is based on transform coding, subband modeling, and channel-optimized scalar quantization (COSQ). It requires a negligible amount of side information (less than 0.00009 bpp) and offers superior performance over a similar system designed for the fully interleaved channel, due to the exploitation of channel memory. Increased correlation among channel noise samples leads to a better performance in our system. Comparisons are made with a competing system which employs separate source and channel coding over the fully interleaved channel and uses adaptive bit allocation between the two codes. Our proposed method outperforms this substantially more complex system for the whole range of considered bit rates and a wide range of channel conditions.**

Index Terms - Subband coding, joint source-channel coding, channel optimized scalar quantization, channels with memory, convolutional codes.

1 Introduction

Traditionally, Shannon's separation theorem has been used to justify independent design of source and channel codes [1]. The performance of the resulting systems – often called tandem systems – may be far from optimal when resources are restricted. As a result, it is beneficial to perform source compression and error protection jointly. Several methods have been proposed for joint source-channel coding, which may be categorized as unequal error protection [2]-[6], channel-optimized scalar and vector quantization (COVQ) [7]-[11], index assignment optimization [12, 13], and exploitation of the residual redundancy of the source coder via maximum *a posteriori* (MAP) decoding [14, 15].

In this paper we present a COSQ-based image coder for transmission of images over noisy channels with memory. Memory is an important property of many real life channels and is usually combated using channel interleaving. However, interleaving causes delay and increases complexity. Furthermore, the resulting associated memoryless channel has a lower capacity than the original channel with memory for the case of information stable channels [16]. Therefore, a COVQ designed for the channel with memory is expected to achieve a better performance than one designed for the equivalent memoryless channel.

Recently, we presented a discrete wavelet transform (DWT) based image coder which was designed for image transmission over binary bursty noise channels [17]. In that method, the image was divided into a number of blocks and was transformed into the frequency domain. Then, a number of sub-sources were formed, each of which contained one coefficient at a specific location of each block. In this paper, we present another low-complexity image compression method for channels with memory which outperforms the previous method. The algorithm employs quadrature mirror filter (QMF) transformation and COSQ. One salient feature of this method is that its performance improves as channel noise becomes more correlated, making it attractive for wireless applications. Also, the bit allocation problem has negligible computational complexity. Moreover, it provides reasonable image quality at bit rates as low as 0.125 bpp and bit error rates as high as 0.1. Our system performs better than a similar system designed for a memoryless channel and used in series with an ideal interleaver, which increases delay. Also, it outperforms UEP schemes which use scalar quantization, convolutional coding and ideal interleaving.

2 COSQ-Based Image Coding for Bursty Noise Channels

A. Structure

Figure 1 shows the block diagram of the employed image coding system. First, the average value of the pixel intensities is removed. Next, the image is decomposed into different non-overlapping frequency subbands using 32-D QMF banks [18]. This is done four times, every time

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on the lowest frequency subband of the previously decomposed data, giving 13 subbands. Similar to the schemes in [2]-[4], [10, 19], we aim to exploit the intra-block dependencies by considering groups of coefficients which are expected to have high correlation and call them "sub-sources". Like [10], we choose our sub-sources to be the subbands themselves. This results in a very small amount of side information, as addressed in section 2-D. Depending on the available bit rate, some of the sub-sources are then normalized (to have a unit variance) and quantized, using a COSQ for channels with memory. The resulting bit-stream is sent directly over the channel. The receiver is simply the inverse of the transmitter.

For COSQ design, we need to know the distribution of the samples to be quantized. It is well-known that the distribution of the coefficients of every subband approximately follows the generalized Gaussian distribution [10], with a probability density function given by

$$f(x) = \frac{\alpha\eta(\alpha,\sigma)}{2\Gamma(1/\alpha)} \exp\{-[\eta(\alpha,\sigma)|x|]^{\alpha}\}$$

where $\eta(\alpha,\sigma) = \frac{1}{\sigma}\left(\frac{\Gamma(3/\alpha)}{\Gamma(1/\alpha)}\right)^{\frac{1}{2}}$ is the rate of decay, σ^2 is the variance, and $\Gamma(\cdot)$ is the Gamma function. For α=1 and 2, the above yields the Laplacian and Gaussian distributions, respectively. For simplicity, we assume here that the sub-sources in all subbands have the Laplacian distribution and we quantize them using a COSQ trained for such a source with a unit variance.

B. Model for Channel with Memory

Based on [16], we use the Polya-contagion channel model which assumes that any noise sample depends only on the sum of the M previous samples. The resulting noise process is a stationary ergodic Markov source of order M. If X_i, Y_i, and Z_i represent the input, output, and noise in that order and $\oplus$ is addition modulo 2, the channel input-output relationship is described by $Y_i = X_i \oplus Z_i$. Assuming that the input and noise are independent, for $i \geq M$ and any $e_{i-M}^{i-1} \in \{0,1\}^M$, we have, (see [16]):

$$\begin{aligned}
&\Pr\{ Z_i = 1 |\, Z_{i-M}^{i-1} = e_{i-M}^{i-1}\} \\
&= \Pr\left\{ Z_i = 1 \,\middle|\, \sum_{j=i-M}^{i-1} Z_j = \sum_{j=i-M}^{i-1} e_j \right\} \\
&= \frac{\epsilon + \delta \sum_{j=i-M}^{i-1} e_j}{1 + M\delta}
\end{aligned}$$

where ϵ is the bit error rate (BER) and $\delta \geq 0$ controls the correlation coefficient of the noise given by $\frac{\delta}{\delta+1}$. The channel capacity (whose closed-form expression is derived in [16]) increases with δ, showing that COVQ may achieve less distortion for channels with memory. This is supported by the simulation results in [20] for generalized Gaussian sources. If δ is set to zero, the noise process becomes memoryless and the channel reduces to a binary symmetric channel (BSC). Remark also that this model is less complex than the Gilbert-Elliott channel model and is completely specified with only three parameters.

C. COSQ Design

Let $d = d_H(\mathbf{x},\mathbf{y})$ be the Hamming distance between the binary channel input block $\mathbf{x}=(x_1,\ldots,x_n)$ and the output block $\mathbf{y}=(y_1,\ldots,y_n)$. We have [16]:

- For $n \leq M$, $\mathrm{P}(\mathbf{y}|\mathbf{x}) = L(n,d,\epsilon,\delta)$, where

$$L(n,d,\epsilon,\delta) = \frac{\prod_{i=0}^{d-1}(\epsilon + i\delta)\prod_{i=0}^{n-d-1}(1-\epsilon+i\delta)}{\prod_{i=0}^{d-1}(1+i\delta)}.$$

- For $n > M$,

$$\mathrm{P}(\mathbf{y}|\mathbf{x}) = L(M, s_{M+1}, \epsilon, \delta) \times \prod_{i=M+1}^{n} \left[\frac{\epsilon + s_i\delta}{1+M\delta}\right]^{e_i} \left[1 - \frac{\epsilon + s_i\delta}{1+M\delta}\right]^{1-e_i}$$

where $e_i = x_i \oplus y_i$ and $s_i = e_{i-M} + \cdots + e_{i-1}$.

The significance of the above formulas is that unlike many other channel models in the literature, they provide easy and computationally inexpensive tools to implement the modified generalized Lloyd algorithm (GLA) for noisy channels which we used together with simulated annealing to generate the required codebooks.

D. Bit Allocation

It is well-known that in subband coding, the end-to-end distortion is more sensitive to errors in the low resolution subbands. Therefore, when allocating bits to the sub-sources, the subbands at which they are located should be taken into account.

Usually, the distortion of sub-source i is weighted by the L_2 norm of the wavelet basis functions of the subband to which it belongs, denoted by w_i. Denoting the sensitivity of the overall distortion to errors in the i^{th} subband by w_i and using the mean-square error distortion measure, we write the end-to-end distortion as

$$D = \sum_{i=1}^{13} w_i d_i, \quad d_i = \frac{1}{N_i^2}\sum_{m=1}^{N_i}\sum_{n=1}^{N_i}(c_{m,n} - \hat{c}_{m,n})^2 \quad (1)$$

for the i^{th} subband having size $N_i \times N_i$. We employ dynamic programming for bit allocation. In particular, we extend the work in [9] for the Markov channel and for the case where the overall distortion has different sensitivities to different sub-sources. The bit allocation problem is to minimize (1) subject to $\sum_{i=1}^{13} r_i \leq B$ and $0 \leq r_i \leq r_{\max}$, where r_i is the number of bits allocated to the i^{th} sub-source and B is the total number of bits available. $r_{\max}$ is the maximum number of bits which may be allocated to a sub-source.

Note that the overall bit rate is $\sum_{i=1}^{13} \frac{r_i}{N_i^2}$. In this paper, we choose $r_{\max} = 9$ bits to have relatively small codebooks and fast encoding. Modeling the sub-sources as independent Laplacian sources, we can write each d_i in (1) as $\sigma_i^2 d_L(r_i)$ where $d_L(r_i)$ is the distortion of a unit-variance Laplacian source quantized for a set of channel conditions (*i.e.*, ϵ, δ, M) and σ_i^2 is the variance of the i^{th} sub-source. The problem now is to allocate the available bits to 13 Laplacian sources, each with variance $w_i\sigma_i^2$, given the channel conditions. We use the algorithm in [9] to solve this problem which is guaranteed by [9] to achieve optimal bit allocation.

Note that $d_L(r_i)$ is calculated off-line. Also, although σ_i^2 is image-dependent, it is not computed inside the algorithm. Indeed, the computational complexity of this algorithm is favorable compared to [3, 4].

3 Simulation Results

We implemented the proposed image coder for the compression and transmission of gray-scale images over the contagion channel with $M = 1$ and tested it for the image Lena (tests performed on other images such as Goldhill, Baboon, and Peppers gave results consistent with the Lena experiments).

The simulation results of all tested systems are shown in Figure 2. Figure (2.a) compares our method, denoted by "IntraBlock+QMF", with three other coders for noiseless channels. They are indicated by "Chen and Fisher", which is the method in [10], "IntraBlock+DWT", whose structure is identical to "IntraBlock+QMF" but uses DWT, and "InterBlock+DWT", which is our previous method described in [17]. For DWT, we used the 9/7 Daubechies filters which are used in the JPEG2000 image compression standard [22]. This figure shows that our system outperforms – in terms of peak signal-to-noise ratio (PSNR) – the best reported COSQ-based method [10] which has *exactly* the same structure and filters as ours. This is due to our improved bit allocation and selection of the w_i weights in (1). Note that the method in [10] is not suitable for channels with memory and requires channel interleaving. It also demonstrates that using QMF banks yields better results than DWT filter banks (at the expense of higher arithmetic complexity). Finally, we see that our recent method is an improvement over our previous method; namely, exploiting the intra-block redundancy in image coding is more efficient.

The rest of the simulations, shown in Figures (2.b)-(2.d), compare our method in noisy channel conditions with the best among three different tandem systems, denoted by "CC best", which comprises scalar quantization and convolutional coding with three different rates followed by ideal channel interleaving. The Polya-contagion (correlated noise) channel in [16] is characterized with three parameters, the noise memory, M, the correlation coefficient $\frac{\delta}{\delta+1}$, and the bit error rate (BER) ϵ. We refer to our system as COSQ, followed by the δ of the channel it is designed for (*e.g.*, COSQ-5 and COSQ-10). COSQ-IL denotes the same system which uses an ideal channel interleaver ($\delta = 0$), and hence it is designed for the BSC with the same BER as the channel with memory. The plots show that the performance curve of the image coders designed for the correlated channel are higher than those of the interleaved channel. This shows that substantial gain may be obtained from exploiting the channel memory instead of using interleaving, which increases delay and memory requirements.

In Table I, we compare the amount of the side information of our current method with three other coders and show that this amount is identical to that of the method in [10]. Note that side information is the part of data in which no single bit error can be tolerated. Therefore, this amount must be kept as small as possible to avoid data misinterpretation. We see that the coder in [6] requires *error-free* transmission of above 33,500 bits of side information for a 512×512 image which may not be affordable for low bit-rate communications.

Table II compares our coders in the presence of BER mismatch with our previous method [17] and that of Sherwood and Zeger [4], which is the best UEP scheme reported and is designed for the ideally interleaved channel. It is observed that our scheme is quite robust to BER mismatch and it slightly outperforms the system in [17]. It is also seen that although the Sherwood-Zeger scheme performs better than ours at no mismatch, it performs nearly 10 dB worse than our method in presence of mismatch. This is expected, because it employs variable length coding for source compression. If such coders are designed for a smaller BER than the actual value, the channel coder would not be strong enough and the decoder cannot correct all channel errors. This causes loss of synchronization which typically results in loss of a large part of information.

Throughout this work, we considered binary channels with memory which model physical channels used in conjunction with hard-decision demodulation. Future work might address the design of efficient COSQ-based image coding schemes for soft-decision demodulated channels with memory. It is expected that additional coding gains can be obtained via the use of the channel soft-decision information; this was indeed observed in [23, 24] for the case of ideal Gaussian sources.

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Proposed method	8.3×10^{-5}
DCC'02 [17]	2.5×10^{-4}
Chen and Fisher [10]	8.3×10^{-5}
Cai and Chen [6]	1.25×10^{-2}

Table I- Overhead data in bpp.

Actual BER	0	0.01	0.03	0.04	0.05
Proposed method (COSQ-10)	31.15	30.43	29.18	28.79	28.25
DCC'02 (COSQ-10) [17]	30.77	30.08	28.81	28.16	27.81
SPIHT+CRC+RCPC [4]	34.97	34.97	28.33	21.90	18.33

Table II- PSNR at 0.5 bpp, design BER = 0.01.

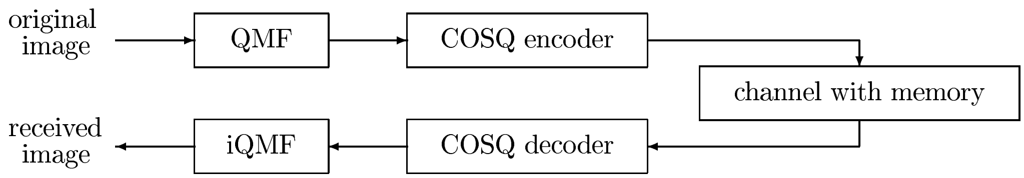

Figure 1: The structure of the proposed communication system.

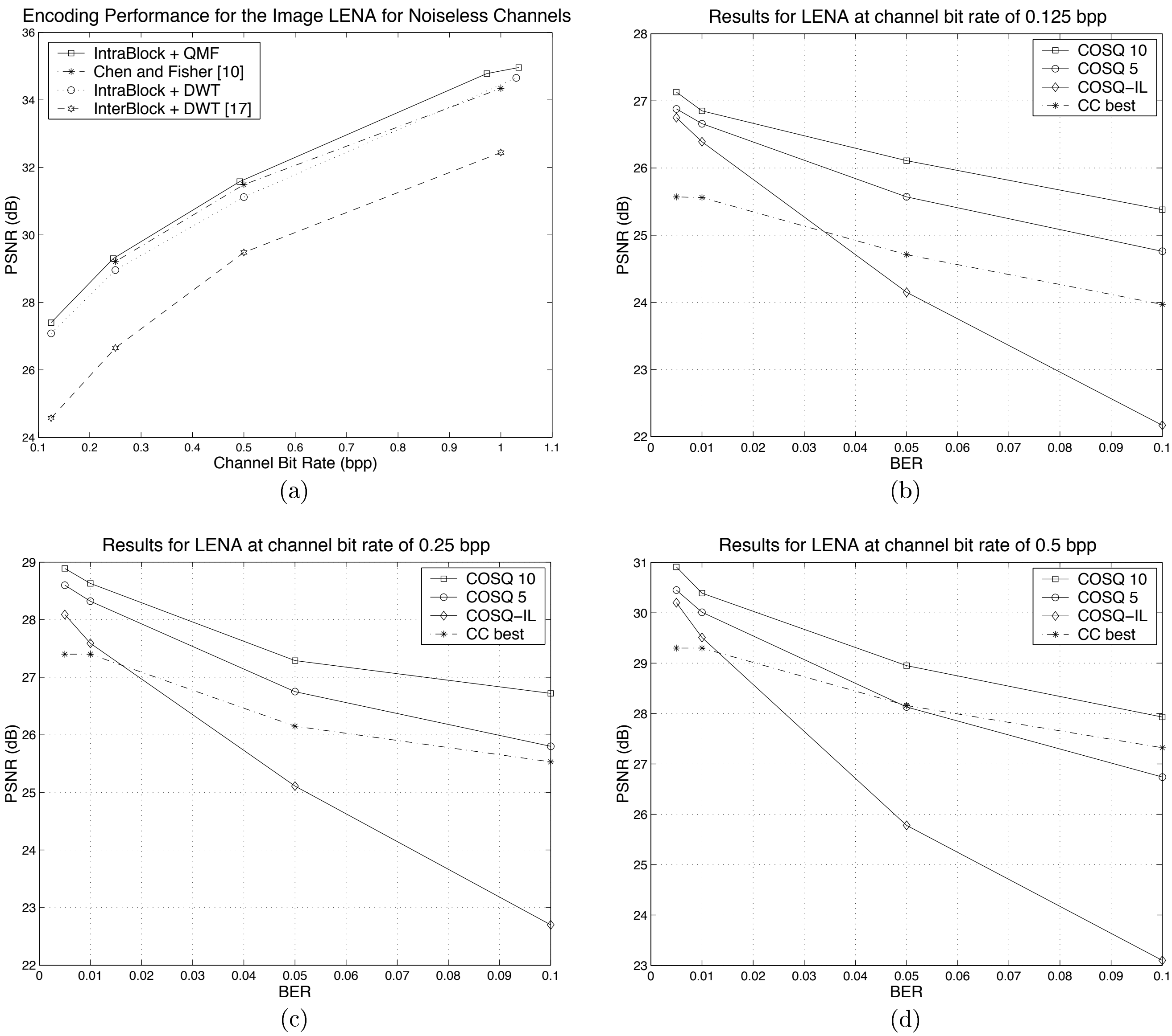

Figure 2: Performance of the proposed image coder at various conditions.