Channel Optimized Vector Quantization Based on Maximum a Posteriori Hard Decision Demodulation*

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***Abstract*— We design a *Maximum a Posteriori* hard-decision demodulated channel optimized vector quantizer (COVQ) that exploits the non-uniformity of the quantized source. We consider memoryless Gaussian and Gauss-Markov sources transmitted over a binary phase-shift keying modulated additive white Gaussian noise channel. Our scheme which has less decoding complexity than soft decoded COVQ systems, is shown to provide a notable signal-to-distortion ratio gain (up to 0.4 dB for memoryless Gaussian sources and up to 0.8 dB for Gauss-Markov sources) over the conventional COVQ designed for hard-decision demodulated channels.**

I. Introduction

Communication systems designed on the basis of Shannon's separation theorem are called *tandem* source-channel coding (TSCC) systems. This classical approach to the problem for sending information reliably over a noisy channel is under the implicit assumption of asymptotically large codeword lengths resulting in large system delay. Furthermore, in many wireless communication situations involving non-stationary sources/channels, the separation theorem may not hold. Thus, studying joint source-channel coding (JSCC) for either cases has attracted much recent interest. The advantages of JSCC over TSCC were studied quantitatively in [7] and in terms of the JSCC error exponent in [12]. In the latter it was shown that under some conditions, the JSCC error exponent can be as large as twice that of TSCC.

Channel optimized vector quantization (COVQ) is a JSCC technique in which the analog source is quantized by taking into consideration the characteristics of both the source and the channel. COVQ has been thoroughly studied under different approaches (e.g., see [1]-[5], [9], [10] and [13]).

COVQ designs usually employ a discrete memoryless channel (DMC) corresponding to a memoryless analog-valued channel used in conjunction with hard-decision demodulation. However in these designs, little attention has been paid to optimize the discrete channel by properly choosing the modulation constellation or exploiting the non-uniformity of the source encoder indices arriving at the channel input. Some notable exceptions include [11] where non-iterative (one step) hard decision maximum a posteriori (MAP) decoding is considered and [6] where joint optimization of the codebooks and constellation is studied.

In this work, we examine how to improve the design of a COVQ scheme for a binary phase-shift keying (BPSK) modulated additive white Gaussian noise (AWGN) channel used with hard-decision demodulation, while keeping system complexity moderately low. Such a scheme may be appealing for wireless applications where resources such as processing power and storage capability are limited. Since we restrict the system to employ hard-decision demodulation (e.g. due to complexity constraints), we cannot exploit the channel's soft (or soft-decision) information in our design as was done in [1], [2] and [8]-[10]. Instead, we focus on iteratively optimizing the discrete channel (having identical input and output alphabets) representing the concatenation of the modulator, AWGN channel and hard-decision demodulator together with its correspondingly designed COVQ encoder/decoder pair. This is achieved by using a symbol MAP hard-decision detector instead of the standard maximum likelihood (ML) detector, motivated by the fact that the COVQ encoder indices arriving at the modulator are non-uniformly distributed (hence the MAP decoder will be optimal in terms of minimizing the discrete channel's symbol error rate). We thus propose a three-phase COVQ design algorithm which is based on first designing a conventional COVQ (for the ML decoded channel), then computing the input distribution to use in MAP decoding, and redesigning

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the COVQ for the new channel defined in terms of an updated transition matrix. Numerical results indicate that the proposed algorithm achieves notable coding gains over the conventional COVQ scheme designed for the discrete (ML hard-decision demodulated) channel. This performance gain does come however with a slight increase in computational complexity at the decoder as MAP decoding is more complex than ML decoding.

II. COVQ SYSTEM

The purpose of the system is to transmit the random vector $\boldsymbol{X}_n \in \mathbb{R}^k$ of dimension k over the noisy channel and form an estimate $\hat{\boldsymbol{X}}_n$ of $\boldsymbol{X}_n$ based on the channel output $\boldsymbol{R}_n$, such that the distortion $\mathbb{E}\|\boldsymbol{X}_n - \hat{\boldsymbol{X}}_n\|^2$ is minimized. Here, n represents the time index of the vector which consists of k single source outputs. The general block diagram of the system is depicted in Fig. 1. The source $\{\boldsymbol{X}_n\} \in \mathbb{R}^k$ is assumed to be a stationary ergodic process, with zero mean and unit variance. The COVQ encoder encodes $\{\boldsymbol{X}_n\}$ at a rate of r bits per sample (bps). Therefore, the COVQ encoder is a mapping $\mathscr{E} : \mathbb{R}^k \rightarrow \mathcal{I}_n \triangleq \{0, 1, \cdots, N_e - 1\} = \{0, 1\}^{kr}$, such that $\mathscr{E}(\boldsymbol{X}_n) = I_n$ is sent over the AWGN channel with noise power $N_0/2$ after modulation. We use BPSK modulation, although other memoryless modulation techniques can also be considered. The encoding is done using the decision regions $\{\mathcal{S}_i\}_{i=0}^{N_e-1}$ ($N_e = 2^{kr}$) via the encoding rule: $\boldsymbol{X}_n \in \mathcal{S}_i \Rightarrow I_n = \mathscr{E}(\boldsymbol{X}_n) = i$. The *a priori* probability of the indices to be chosen are denoted by P_i, where $P_i = \Pr[\boldsymbol{X}_n \in \mathcal{S}_i]$. The concatenation of the modulator, the actual channel and the detector form a DMC. We refer to this discrete channel as the "equivalent DMC,". The transition probabilities of the equivalent DMC can be determined in terms of the actual channel parameters. The input alphabet of the DMC is $\mathcal{I}_n = \{0, 1, \cdots, N_e - 1\}$. Each input index is transmitted over the channel and is received through a transition matrix $P_{Y|X}$. The output alphabet set is $\mathcal{J}_n = \{0, 1, \cdots, N_d - 1\}$. Since the system uses hard-decision demodulation $\mathcal{J}_n = \mathcal{I}_n$ and $N_d = N_e$. Hard-decision decoding has less decoding complexity than systems employing soft decoding or soft-decision (e.g., [1], [2] and [8]-[10]). The decoder is the combination of two functions $\mathscr{D}_1$ and $\mathscr{D}_2$. Thus the decoder can be written as $\mathscr{D} = \mathscr{D}_2 \circ \mathscr{D}_1$, where:

$$\mathscr{D}_1 : \mathbb{R}^{kr} \rightarrow \mathcal{J}_n = \mathcal{I}_n = \{0, 1, \cdots, N_e - 1\},$$

$$\mathscr{D}_2 : \mathcal{J}_n \rightarrow \mathbb{R}^k$$

and $\circ$ denotes composition.

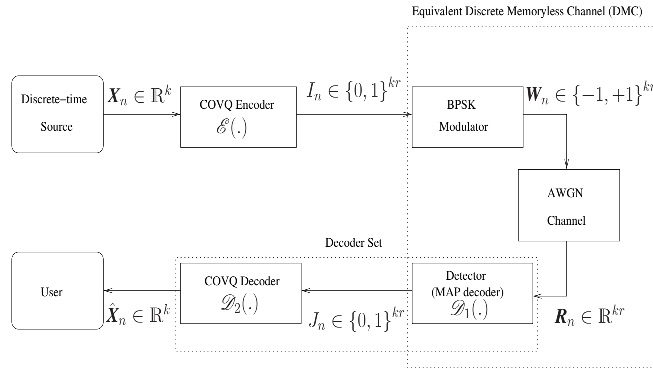

Fig. 1. Block diagram of the iterative MAP decoded COVQ system.

The received vector $\boldsymbol{R}_n$ consists of kr consecutive received values: $R_n^1, R_n^2, \cdots, R_n^{kr}$ that can each be written as

$$R_n^t = W_n^t + \nu_t, \quad t = 1, 2, \cdots, kr, \tag{1}$$

where $\boldsymbol{W}_n \in \{-1, +1\}^{kr}$ and $\nu_t \sim \mathcal{N}(0, \frac{N_0}{2})$. The COVQ decoder $\mathscr{D}_2 : \mathcal{J}_n = \{0, 1, \cdots, N_e - 1\} \rightarrow \mathbb{R}^k$ maps each received index into a codevector estimate $\hat{\boldsymbol{X}}_n$.

III. THREE PHASE ITERATIVE MAP DECODED (IMD) ALGORITHM

The main contribution of this paper is a simple algorithm that jointly optimizes $\mathscr{D}_1$ and the pair $\{\mathscr{D}_2, \mathscr{E}\}$. The IMD algorithm consists of three phases. The first phase is the ordinary COVQ design algorithm. The problem of designing COVQ for a DMC is well known ([3], [4], and [8]). The COVQ encoder $\mathscr{E} : \mathbb{R}^k \rightarrow \mathcal{I}_n$ is characterized in terms of a partition [4] $\mathcal{P} = \{S_i \subset \mathbb{R}^k : i \in \mathcal{I}_n\}$. The DMC takes the input index I_n and produces the output J_n according to the channel transition matrix $P_{J_n|I_n}(j|i)$.

The decoder mapping $\mathscr{D}_2 : \mathcal{J}_n \rightarrow \mathbb{R}^k$ is represented by the codebook $\mathcal{C} = \{\boldsymbol{c}_j \in \mathbb{R}^k : j \in \mathcal{J}_n\}$. The COVQ encoder and decoder are iteratively optimized based on two conditions [4] that are necessary for the squared-error distortion to be minimized, making sure that the procedure ends up with a locally optimal solution.

Thus, in the first phase $\mathscr{D}_1$ is fixed and $\mathscr{E}$ and $\mathscr{D}_2$ are alternatingly optimized in an iterative fashion. For the above system, the average distortion per sample is given by [4]

$$D_n = \frac{1}{k}\sum_i \int_{S_i} f(\mathbf{x}) \sum_j P_{J_n|I_n}(j|i) \|\mathbf{x} - \boldsymbol{c}_j\|^2 d\mathbf{x} \tag{2}$$

where $f(\mathbf{x})$ is the k-dimensional source density. From (2), it can be shown [4] that for a fixed $\mathcal{C}$, the optimal partition

$\mathcal{P}^* = \{S_i^*\}$ is given by

$$S_i^* = \left\{ \mathbf{x} : \sum_j P_{J_n|I_n}(j|i) \|\mathbf{x} - \boldsymbol{c}_j\|^2 \right. \left. \leq \sum_j P_{J_n|I_n}\left(j|\hat{i}\right) \|\mathbf{x} - \boldsymbol{c}_j\|^2 \quad \forall \hat{i} \in \mathcal{I}_n \right\}$$

for every $i \in \mathcal{I}_n = \{0,1\}^{kr}$.

The second necessary condition, on the other hand determines the optimal codebook $\mathcal{C}^* = \{\boldsymbol{c}_j^*\}$ for a given partition [4] with

$$\boldsymbol{c}_j^* = \frac{\sum_j P_{J_n|I_n}(j|i) \int_{S_i} \mathbf{x} f(\mathbf{x})\, d\mathbf{x}}{\sum_j P_{J_n|I_n}(j|i) \int_{S_i} f(\mathbf{x})\, d\mathbf{x}}. \tag{3}$$

In practice, only samples from the source are available and we use the training data replacing the integrals with summations and the density function with empirical weights. Note that in the first step of the iteration, we assume a uniform input index distribution resulting in ML decoding. In this case the DMC is a binary symmetric channel (BSC) used kr times independently and the symbol transition probabilities are kr long products of the bit transition probabilities.

Once Phase 1 is complete, the encoder index distribution is fed to the MAP decoder to start the second phase of the algorithm. Thus we use the computed input distribution to replace the ML detector by a symbol based MAP decoder and redesign the COVQ. So, given $\mathscr{D}_2$ and $\mathscr{E}$ from the first phase, we find $\mathscr{D}_1$ such that

$$\begin{aligned} J_n &= \arg\max_{\mathcal{I}_n} \; P(I_n|\boldsymbol{R}_n) = \arg\max_{\mathcal{I}_n} \; P(\boldsymbol{R}_n|I_n) P_{i_n} \\ &= \arg\min_{\mathcal{I}_n} \left[\frac{1}{N_0} \|\boldsymbol{R}_n - \boldsymbol{W}_n\|^2 - \ln P_{i_n} \right], \end{aligned} \tag{4}$$

where P_{i_n} is given from the first phase.

After updating the detector, which results in a new DMC, and finding an updated codebook according to (3) (based on the new DMC probability distribution), we design the new COVQ (the pair $\{\mathscr{D}_2, \mathscr{E}\}$), using the updated codebook found by (3) as the initial codebook. The above process makes the second phase of the algorithm. We calculate the distortion D_n at the end of the second phase. In the third phase, we repeat Phase 2 and terminate when the signal-to-noise ratio (SDR) is maximized.

The three-phase COVQ algorithm can be summarized as follows:

1) Design a (conventional) COVQ encoder/decoder pair for the DMC under ML (hard-decision) decoding.
2) Compute the source encoder index distribution, use MAP (hard-decision) decoding, update the DMC's transition distribution and redesign the COVQ encoder/decoder pair for the updated channel. This begins with first updating the codebook (using the last encoding partition) and then the encoding regions.
3) Repeat Phase 2 until the distortion is minimized (by monitoring the system's distortion and stopping the iterative process when the distortion is increased).[1]

Channel SNR	Conventional COVQ	IMD COVQ	SDD COVQ ($q=2$) [8]
8.0	8.64	8.69	8.76
6.0	6.89	6.99	7.21
4.0	5.17	5.48	5.74
3.0	4.38	4.77	5.08
2.0	3.77	4.03	4.36
1.0	3.17	3.41	3.71
0.0	2.66	2.84	3.14
-1.0	2.21	2.35	2.69
-2.0	1.82	1.94	2.26
-3.0	1.50	1.58	1.88
-4.0	1.22	1.28	1.53
-6.0	0.82	0.85	1.02

TABLE I

SDR IN dB FOR ML DECODED CONVENTIONAL, ITERATIVE MAP DECODED (IMD) AND SOFT DECISION DECODED (SDD) COVQS FOR THE MEMORYLESS GAUSSIAN SOURCE. THE VECTOR QUANTIZER RATE IS $r = 2$ BPS AND THE QUANTIZATION DIMENSION IS $k = 2$.

IV. Numerical Results

In the first phase (ML decoded COVQ), we employ the transition matrix calculated from (1) and derived from kr uses of a BSC with crossover probability $Q(\sqrt{\text{SNR}})$, where SNR$= \frac{2}{N_0}$ is the channel signal-to-noise ratio. For designing the COVQ, 100,000 source training vectors are generated. After designing the COVQ (Phase 1), we generate 400,000 noise vectors, use MAP decoding and from the resulting empirical distribution compute the new $2^{kr} \times 2^{kr}$ transition matrix. Then the COVQ is redesigned as described in Phases 2 and 3 above.

We compare the performance of the ordinary COVQ with the proposed IMD-COVQ and also with the soft-decision demodulated (SDD) COVQ proposed in [8]. Table 1 presents SDR results for the memoryless Gaussian source

[1]It is worth pointing out that the system's distortion is not always monotonically decreasing with the number of iterations. This is due to the fact that minimizing the channel's symbol error rate under MAP decoding is not necessarily equivalent to minimizing the end to end distortion.

Channel SNR	Conventional COVQ	IMD COVQ	SDD COVQ ($q = 2$) [8]
8	10.99	11.09	11.20
6	8.72	9.33	9.72
4	6.71	7.51	7.70
3	6.03	6.70	6.86
2	5.15	5.81	5.86
1	4.40	5.00	5.06
0	3.62	4.24	4.42
-1	2.99	3.42	3.83
-2	2.47	2.78	3.29
-3	2.12	2.49	2.80
-4	1.94	2.15	2.42
-6	1.18	1.40	1.65

TABLE II
SDR IN DB FOR ML DECODED CONVENTIONAL, IMD AND SDD COVQS FOR THE GAUSS-MARKOV SOURCE WITH CORRELATION COEFFICIENT $\rho = 0.9$. THE VECTOR QUANTIZER RATE IS $r = 2$ BPS AND THE QUANTIZATION DIMENSION IS $k = 2$.

while Table 2 is devoted to the Gauss-Markov source with correlation coefficient 0.9. Interestingly we observed that generally the non-uniform input distribution, after applying the IMD algorithm tends to be even more non-uniform which is desirable [4]. Since there are already many empty decision regions for the conventional COVQ at low SNRs (SNR < -2 dB), the IMD algorithm does not provide much gain in that region. However, for SNR ranging from -2 dB to 4 dB, it considerably outperforms the conventional COVQ system. For high SNRs, MAP decoding does not yield much gain compared with ML decoding as in this case both decoding methods are nearly equivalent.

As expected, the proposed system provides larger SDR gains over the conventional COVQ system when the source has memory as opposed to being memoryless (compare Table II with Table I). Observe that in Table II, the gain is 0.8 dB for SNR = 4 dB. Also, the IMD COVQ performs almost as well as the SDD COVQ for SNRs from 0 to 3 dB. Note that the main advantage of the IMD COVQ system over the SDD COVQs of [1], [2] and [8] is its reduced storage complexity due to the significantly smaller amount of memory needed in the COVQ decoder. The SDD COVQ (with soft-decision resolution q) has a codebook size of 2^{qkr} k-dimensional codevectors while the IMD COVQ has a codebook size of 2^{kr} codevectors (just as the conventional COVQ). Note, however, that since it uses MAP decoding the IMD COVQ system has higher computational decoding complexity (but still of order 2^{kr}) than SDD COVQ and conventional COVQ.

V. Conclusion

A design algorithm for COVQ based on MAP hard-decision decoding is proposed. The algorithm is applied to a BPSK modulated AWGN channel. The COVQ is firstly designed for the ML (hard-decision) decoded channel. Using the computed index distribution in conjunction with MAP decoding, the DMC is updated and the COVQ is redesigned for the updated channel. The process continues until the SDR is maximized. It is demonstrated that an SDR gain of up to 0.8 dB can be achieved over conventional COVQ while its decoding complexity stays moderately low.

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