Hybrid Digital–Analog Coding for Bandwidth Compression/Expansion Using VQ and Turbo Codes

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I. Introduction

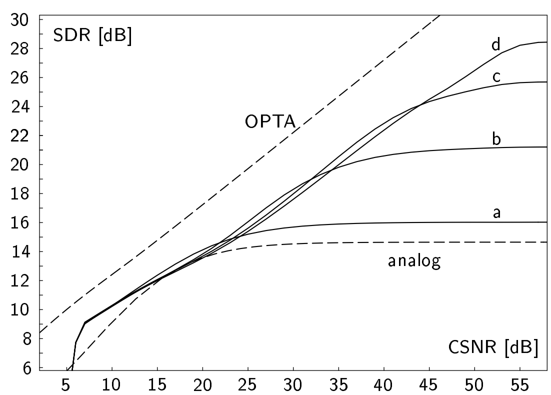
In [1] we proposed a *hybrid-digital-analog* (HDA) system based on vector quantization (VQ). One drawback of the system proposed in [1] is that it only works for transmission rates larger than one (bandwidth expansion). Motivated by this fact, we herein present a general system that can be used at any transmission rate (bandwidth compression/expansion).

II. System Description and Performance

Consider a system conveying a vector $\mathbf{X} \in \mathbb{R}^d$ over a Gaussian channel. The vector $\mathbf{X}$ is first fed to a *low-delay source or source–channel* encoder mapping ε_1, and $I = \varepsilon_1(\mathbf{X}) \in \{0, \ldots, N-1\}$, where $N = 2^L$, is then fed (in its L-bit binary form) to a *high-delay channel encoder* ε_2, of rate $r_c < 1$. (Typically, several consecutive outputs from ε_1 are blocked and encoded by ε_2.) The output K of ε_2 is then assigned a channel symbol $\mathbf{s}_K$. The index I also chooses a vector $\mathbf{z}_I$ from the *encoder codebook* $\{\mathbf{z}_i\}$ forming the "error vector" $\mathbf{E} = \mathbf{X} - \mathbf{Z}_I$, and $\mathbf{E}$ is then used as input to a mapping α, with output $\mathbf{Z} = \alpha(\mathbf{E})$. Ideally, α is an *analog* mapping. The vectors $\mathbf{s}_K$ and $a \cdot \mathbf{Z}$, where a is a scaling constant, are added and then fed to a channel with AWGN $\mathbf{W}$ of variance σ^2 per component. The resulting channel output is denoted by $\mathbf{R}$.

At the receiver, a *decoder* maps $\mathbf{R}$ into an estimate $\hat{\mathbf{X}}$. Ideally $\hat{\mathbf{X}} = E[\mathbf{X}|\mathbf{R}]$. In general, however, the complexity of implementing this decoder prohibits its use. Therefore, we propose a suboptimal decoder, where $\mathbf{R}$ is fed to a decoder δ_2 for ε_2, and the output J is encoded by ε_2 and assigned a channel symbol. The result is then subtracted from $\mathbf{R}$ and scaled by a constant b, forming an estimate $\hat{\mathbf{Z}}$ of $\mathbf{Z}$. This estimate is fed to a mapping β with output $\hat{\mathbf{E}}$, where $\hat{\mathbf{E}}$ is an estimate of $\mathbf{E}$. The index J is also fed to a decoder δ_1 for ε_1. A source vector estimate is then formed as $\hat{\mathbf{X}} = \delta_1(J) + \hat{\mathbf{E}}$.

Below we illustrate the performance of the system implemented at a rate of 0.5 channel use/source sample. The vector $\mathbf{X}$ is of dimension $d = 32$, and drawn from a Gauss-Markov source with correlation 0.9 between samples. The encoder ε_1 is a VQ encoder of size $L = 8$ and trained for a noiseless channel, $\{\mathbf{z}_i\}$ is identical to the codebook defining ε_1, and ε_2 is a $(1024, 2048)$, rate $r_c = 1/2$, Turbo encoder, with generators $(37, 21)$ (punctured to rate $1/2$) and with a random interleaver. The 8-bit blocks from ε_1 are blocked into one 1024-bit block, which is fed to ε_2, and the output bits are then mapped into BPSK symbols. The scaling constant a is chosen so that a fraction $0 \leq \Delta \leq 1$ of the total input power is assigned to the analog part. The mapping δ_2 is a Turbo decoder for the encoder ε_2, and δ_1 is defined by a table look-up in $\{\mathbf{z}_i\}$. The constant b is chosen to minimize $E\|\mathbf{Z} - \hat{\mathbf{Z}}\|^2$, assuming that δ_2 is powerful enough to correct *all* errors in the digital part. Regarding the analog part, the system employs a "discrete approximation" of the mappings (α, β), described as follows. The components of $\mathbf{Z}$ belong to a discrete set of 256 equally spaced signal points (equidistant PAM). The decoder β implements ML detection of the transmitted symbols, based on the input $\hat{\mathbf{Z}}$, and then performs a table look-up in a codebook. The pair (α, β) is trained to minimize $E\|\mathbf{E} - \hat{\mathbf{E}}\|^2$, for a fixed σ^2, a given Δ, a fixed a and under a constraint on the total transmit power. The encoder α has dimension two in and one out. That is, $\mathbf{E}$ is divided into 16 two-dimensional subvectors, and one PAM symbol is generated for each of these. Similarly, β produces two-dimensional vectors, grouped into one $\hat{\mathbf{E}}$.



The figure shows signal-to-distortion ratio versus channel SNR. *Solid lines*: (α, β) trained at (a) CSNR = 20 dB and $\Delta = 0.2$; (b) CSNR = 30 dB and $\Delta = 0.2$; (c) CSNR = 40 dB and $\Delta = 0.3$; (d) CSNR = 50 dB and $\Delta = 0.3$. In (a)–(d) the performance is evaluated over different CSNRs and with the power allocation set at $\Delta = 0.3$ in all cases. *Dashed lines* from above: OPTA (via the distortion-rate function at channel capacity) and analog (a linear analog system based on an eigenvalue decomposition of the source vector $\mathbf{X}$ and transmitting only the 16 "strongest directions").

We see that the HDA systems offer a robust and graceful performance over the entire range of the CSNRs. In particular this holds for system (d) which is able to perform well over an approximately 50 dB wide CSNR range. Note also that the performance can be made to saturate at an arbitrarily high SDR by increasing the resolution of the maps (α, β).

References

[1] M. Skoglund, N. Phamdo, and F. Alajaji, "VQ-based hybrid digital–analog joint source–channel coding," in *Proc. IEEE International Symposium on Information Theory*, Sorrento, Italy, June 2000, p. 403.

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