An Approximation of the Gilbert-Elliott Channel via a Queue-Based Channel Model[1]

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Abstract — **We investigate the modeling of the well-known burst-noise Gilbert-Elliott channel (GEC) using a recently introduced queue-based channel (QBC) model. The QBC parameters are estimated by minimizing the Kullback-Leibler divergence rate between the probability of error sequences generated by the QBC and the GEC, while maintaining identical bit error rates and correlation coefficients. The accuracy of fitting the GEC via the QBC is evaluated in terms of channel capacity and autocorrelation function. Numerical results show that the QBC provides a very good approximation of the GEC for various channel conditions. It thus offers an interesting alternative to the GEC while remaining mathematically tractable.**

I. Introduction

The GEC is the most commonly used model for binary burst-noise channels. It has an additive hidden Markov (HM) noise source and is described by four parameters (b, g, p_B and p_G) [1]. Due its HM structure, the GEC is often difficult to analyze (e.g., its capacity does not have a closed-form expression and its block transition distribution is not transparently expressed in terms of the channel parameters), particularly when incorporated within an overall coding system.

The QBC, recently introduced in [2], is a binary additive noise channel with memory based on a finite queue. It features a stationary ergodic Mth order Markov noise source and, like the GEC, it is fully characterized by four parameters (ϵ, α, p and M). The channel admits single-letter expressions for its block transition distribution and capacity, which is an attractive feature for mathematical analysis. We herein study the problem of approximating the GEC via the QBC.

II. Fitting the GEC via the QBC

For a given GEC, we construct a QBC whose noise process is statistically "close" in the Kullback-Leibler sense to the noise process generated by the GEC. Specifically, given a GEC with fixed parameters b, g, p_B and p_G resulting in bit error rate $\mathrm{BER}_{\mathrm{GEC}}$ and correlation coefficient $\mathrm{Cor}_{\mathrm{GEC}}$, we estimate the QBC parameters M, p, ε, and α that minimize the Kullback-Leibler divergence rate, $\lim_{n\to\infty} D_n(\mathrm{P}_{\mathrm{GEC}} \parallel \mathrm{P}_{\mathrm{QBC}}^{(M)})$, subject to the constraints $\mathrm{BER}_{\mathrm{QBC}} = \mathrm{BER}_{\mathrm{GEC}}$ and $\mathrm{Cor}_{\mathrm{QBC}} = \mathrm{Cor}_{\mathrm{GEC}}$, where $D_n(\mathrm{P}_{\mathrm{GEC}} \parallel \mathrm{P}_{\mathrm{QBC}}^{(M)})$ is the normalized nth order Kullback-Leibler divergence between the n-fold GEC and QBC noise distributions, $\mathrm{P}_{\mathrm{GEC}}$ and $\mathrm{P}_{\mathrm{QBC}}^{(M)}$, respectively. Since

$$\begin{aligned}&\lim_{n\to\infty} D_n(\mathrm{P}_{\mathrm{GEC}} \parallel \mathrm{P}_{\mathrm{QBC}}^{(M)})\\ &= -\mathcal{H}(\mathrm{P}_{\mathrm{GEC}}) - E_{\mathrm{P}_{\mathrm{GEC}}}[\log \mathrm{P}_{\mathrm{QBC}}^{(M)}(E_{M+1}|E^M)],\end{aligned}$$

where $\mathcal{H}(\cdot)$ denotes the entropy rate and $\mathrm{P}_{\mathrm{QBC}}^{(M)}(e_{M+1}|e^M)$ is the QBC conditional error probability of symbol $M+1$ based on the previous M symbols, then the minimization reduces to maximizing $E_{\mathrm{P}_{\mathrm{GEC}}}[\log \mathrm{P}_{\mathrm{QBC}}^{(M)}(E_{M+1}|E^M)]$ over the QBC parameters. Note that in our approximation, we match the bit error rates and noise correlation coefficients of both channels to guarantee identical noise marginal distributions and identical probabilities of two consecutive errors (ones). Hence, given these constraints, the above optimization problem reduces to an optimization over only two QBC parameters.

III. Modeling Results

We evaluate how well the QBC model fits or approximates the GEC according to two criteria: channel capacity and autocorrelation function (ACF). Typical modeling results are shown in Fig. 1, where the capacity of the GEC and its QBC approximation are shown for different BER values and for $\mathrm{Cor}_{\mathrm{GEC}} = 0.231$. We clearly observe from the figure that the capacity curves of both channels match quite well; the same behavior was observed when comparing the ACF curves. We also obtained nearly identical capacity and ACF curves for several other correlation coefficients. This leads us to conclude that the QBC model provides a very good approximation of the GEC. It hence offers an attractive alternative to the GEC in virtue of its tractability for mathematical analysis as it admits a closed form formula for its capacity and a transparent formula for its n-fold statistics.

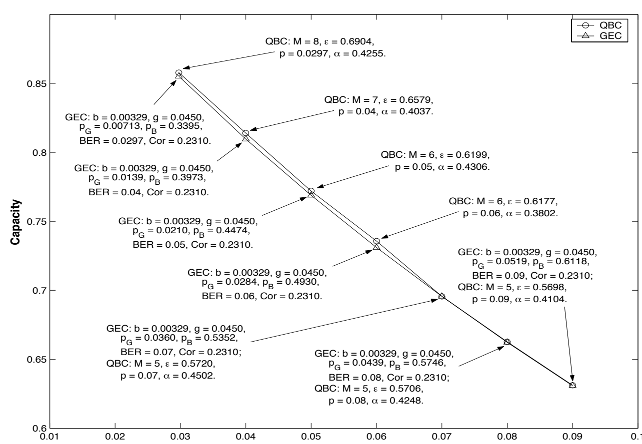

Figure 1: GEC fitting via the QBC: Capacity vs BER.

References

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