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Hybrid Digital-Analog Joint Source-Channel Coding for Broadcasting Correlated Gaussian Sources*

Hamid Behroozi, Fady Alajaji and Tamás Linder
Department of Mathematics and Statistics, Queen's University, Kingston, Ontario, Canada, K7L 3N6
Email: {behroozi, fady, linder}@mast.queensu.ca

***Abstract*— We consider the transmission of a bivariate Gaussian source $S = (S_1, S_2)$ across a power-limited two-user Gaussian broadcast channel. User i $(i = 1, 2)$ observes the transmitted signal corrupted by Gaussian noise with power σ_i^2 and wants to estimate S_i. We study hybrid digital-analog (HDA) joint source-channel coding schemes and analyze these schemes to obtain achievable (squared-error) distortion regions. Two cases are considered: 1) source and channel bandwidths are equal, 2) broadcasting with bandwidth compression. We adapt HDA schemes of Wilson *et al*. [1] and Prabhakaran *et al*. [2] to provide various achievable distortion regions for both cases. Using numerical examples, we demonstrate that for bandwidth compression, a three-layered coding scheme consisting of analog, superposition, and Costa coding performs well compared to the other provided HDA schemes. In the case of matched bandwidth, a three-layered coding scheme with an analog layer and two layers, each consisting of a Wyner-Ziv coder followed by a Costa coder, performs best.**

I. Introduction

This paper considers broadcasting correlated Gaussian sources and aims to characterize mean squared-error (MSE) distortion pairs that are simultaneously achievable at two receivers using hybrid digital-analog (HDA) coding schemes. It is known that the separate design of source and channel coding due to Shannon does not in general lead to the optimal performance theoretically attainable (OPTA) in networks. On the other hand, for the point-to-point transmission of a single Gaussian source through an additive white Gaussian noise (AWGN) channel it is well known that if the channel and source bandwidths are equal, simple uncoded transmission achieves OPTA. Uncoded (or analog) transmission in this case (and in the rest of this paper) means scaling the encoder input subject to the channel power constraint and transmitting it without explicit channel coding. In order to exploit the advantages of both analog transmission and digital techniques, various HDA schemes have been introduced in the literature, see e.g., [1], [3]–[9]. Broadcasting a single memoryless Gaussian source under bandwidth mismatch using HDA schemes is considered in [5], [8]. Bross *et al.* [10] show that there exists a continuum of HDA schemes with optimal performance for the transmission of a Gaussian source over an average-power-limited Gaussian channel with matched bandwidth. Tian and Shamai [11] generalize this result to the mismatched bandwidth case. Broadcasting a Gaussian source with memory is analyzed in [9].

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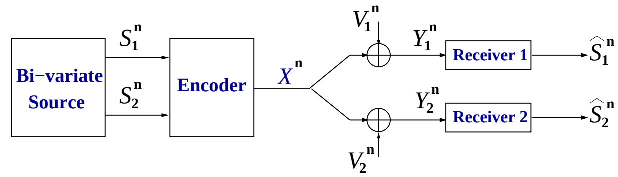


Fig. 1. Broadcasting a bivariate Gaussian source over a two-user power-limited Gaussian broadcast channel.

Our system model is illustrated in Fig. 1. We aim to determine achievable distortion regions using HDA schemes for two cases; 1) the source bandwidth equals the channel bandwidth, 2) broadcasting with bandwidth compression. To our knowledge, apart from [12] in which Bross *et al.* analyzed uncoded transmission for broadcasting correlated Gaussian sources, no explicit distortion-regions have been established in the literature for broadcasting correlated Gaussian sources. We are also not aware of any prior work on HDA schemes for broadcasting correlated Gaussians either when the source and channel bandwidths are equal or when there is a bandwidth mismatch. Note that the source-channel separation theorem does not hold in broadcasting correlated sources.

II. Problem Statement

We consider broadcasting a bivariate Gaussian source across a two-user power-limited Gaussian broadcast channel. User i $(i = 1, 2)$ receives the transmitted signal corrupted by Gaussian noise with power σ_i^2 and wants to estimate the ith component of the source. We assume $\sigma_1^2 > \sigma_2^2$ and call user 1 the weak user and user 2 the strong user. Let S_1 and S_2 be correlated Gaussian random variables and let $\{(S_1(t), S_2(t))\}_{t=1}^{\infty}$ be a stationary Gaussian memoryless vector source with marginal distribution that of (S_1, S_2). We assume that $S_1(t)$ and $S_2(t)$ have zero mean and variance $\sigma_{S_1}^2$ and $\sigma_{S_2}^2$, respectively, and correlation coefficient $\rho \in (-1, 1)$.

We represent the first n source samples by the data sequences $S_1^n = \{S_1(1), S_1(2), \cdots, S_1(n)\}$ and $S_2^n = \{S_2(1), S_2(2), \cdots, S_2(n)\}$, respectively. The system is shown in Fig. 1. The source sequences S_1^n and S_2^n are jointly encoded to $X^n = \varphi(S_1^n, S_2^n)$, where the encoder function is of the form $\varphi : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$. The transmitted sequence X^n is average-power limited to $P > 0$, i.e.,

$$\frac{1}{n} \sum_{t=1}^{n} E\left[|X(t)|^2\right] \leq P. \tag{1}$$

User i observes the transmitted signal $X(t)$ corrupted by Gaussian noise $V_i(t)$ with power σ_i^2 so that each observation time $t = 1, 2, 3, \ldots$ receiver i observes

$$Y_i(t) = X(t) + V_i(t), \qquad i = 1, 2 \tag{2}$$

where the $V_i(t) \sim \mathcal{N}\left(0, \sigma_i^2\right)$ are independently distributed over i and t, and are independent of the $X(t)$. Based on its channel output Y_i^n, user i provides an estimate $\widehat{S}_i^{\,n} = \psi_i\left(Y_i^n\right)$, where $\psi_i : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a decoding function. The quality of the estimate is measured by the average MSE distortion $\Delta_i = \frac{1}{n} E[\sum_{t=1}^{n} |S_i(t) - \widehat{S}_i(t)|^2]$. Let $\mathcal{F}^{(n)}(P)$ denote all encoder and decoder functions $(\varphi, \psi_1, \psi_2)$ defined as above. For a particular coding scheme $(\varphi, \psi_1, \psi_2)$, the performance is determined by the channel power constraint P and the incurred distortions Δ_1 and Δ_2 at the receivers . For any given power constraint $P > 0$, the distortion region $\mathcal{D}$ is defined as the convex closure of the set of all distortion pairs (D_1, D_2) for which (P, D_1, D_2) is achievable, where a power-distortion pair (P, D_1, D_2) is achievable if for any $\delta > 0$, there exists $n_0(\delta)$ such that for any $n \geq n_0(\delta)$ there exists $(\varphi, \psi_1, \psi_2) \in \mathcal{F}^{(n)}(P)$ with distortions $\Delta_i \leq D_i + \delta$ $(i = 1, 2)$.

III. Distortion Regions with Matched Bandwidth

A. Uncoded Transmission

In [12] for the above problem an achievable distortion region is obtained based on analyzing the uncoded transmission in broadcasting a bivariate Gaussian source. In this approach, a linear combination of both components of a bivariate Gaussian source is transmitted across a power-limited Gaussian broadcast channel. The transmitted signal can be expressed as

$$X_a(t) = \tilde{\alpha} \sum_{i=1}^{2} a_i S_i(t), \tag{3}$$

where $\tilde{\alpha} = \sqrt{\frac{P}{\mathrm{Var}(\sum_{i=1}^{2} a_i S_i(t))}}$, $a_i \geq 0$ and $\mathrm{Var}(\sum_{i=1}^{2} a_i S_i(t)) = a_1^2 \sigma_{S_1}^2 + a_2^2 \sigma_{S_2}^2 + 2 a_1 a_2 \rho \sigma_{S_1} \sigma_{S_2}$. The scale factor $\tilde{\alpha}$ is chosen such that the channel power constraint is satisfied with equality. The received signal at receiver i is then given by

$$Y_i(t) = X_a(t) + V_i(t) = \tilde{\alpha} \sum_{i=1}^{2} a_i S_i(t) + V_i(t). \tag{4}$$

By evaluating the resulting MSE distortion, the set of simultaneously achievable distortion pairs at two users are as follows:

$$D_i = \sigma_{S_i}^2 - \frac{\tilde{\alpha}^2 (a_i \sigma_{S_i}^2 + a_j \rho \sigma_{S_i} \sigma_{S_j})^2}{P + \sigma_i^2}, \;\; i, j = 1, 2, \;\; j \neq i \tag{5}$$

It is shown in [12] that the uncoded scheme is optimal below a certain SNR-threshold.

B. Joint Source-Channel Coding Schemes

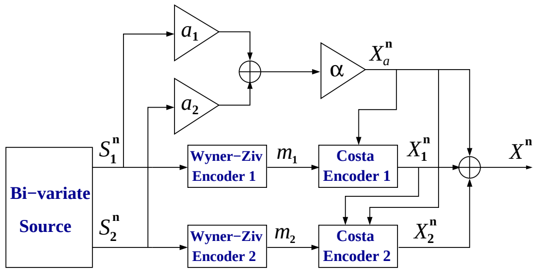
In our schemes, we will closely follow the notation and code constructions in [1]. Here we only give a high-level description and analyses of the schemes without detailed proofs. In particular, in many steps of the analysis we treat finite-blocklength coding schemes as idealized systems with asymptotically large blocklengths.

1) Layering with Analog and Costa Coding: This coding scheme has three layers and is similar to the scheme in [1] for broadcasting a single memoryless Gaussian source. The only difference between the two schemes is that we use a Wyner-Ziv encoder followed by a Costa encoder in the second layer, while the second layer of the scheme in [1] employs an HDA Costa coder (which will be explained in Section IV-A). Block diagrams of the encoder and the decoder are shown in Fig. 2. The first layer is the analog transmission layer. Here $X_a(t) = \alpha \sum_{i=1}^{2} a_i S_i(t)$, where $\alpha = \sqrt{\frac{P_a}{\mathrm{Var}(\sum_{i=1}^{2} a_i S_i(t))}}$. This layer is meant for both strong and weak users. Now fix P_1 and P_2 to satisfy $P = P_a + P_1 + P_2$.

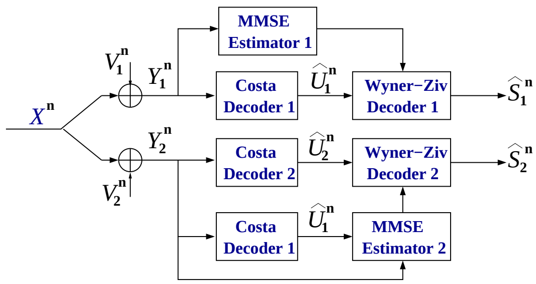
In the second layer, the first component of the source is first Wyner-Ziv coded at rate $R_1^{'} = \frac{1}{2}\log(1 + \frac{P_1}{P_2 + \sigma_1^2})$ using an estimate of S_1^n at the receiver as side information. The Wyner-Ziv index, $m_1 \in \{1, 2, \cdots, 2^{nR_1^{'}}\}$, is then encoded using Costa's "dirty paper" coding treating the analog transmission layer, X_a^n, as an interference. Let U_1 be an auxiliary random variable given by $U_1 = X_1 + \alpha_1 X_a$, where $X_1 \sim \mathcal{N}(0, P_1)$ is independent of $X_a \sim \mathcal{N}(0, P_a)$ and the scaling factor α_1 is set to be $\frac{P_1}{P_1 + P_2 + \sigma_1^2}$. We generate a length n i.i.d. Gaussian codebook $\mathcal{U}_1$ with $2^{nI(U_1;Y_1)}$ codewords, where each component of the codeword is Gaussian with zero mean and variance $P_1 + \alpha_1^2 P_a$, and each codeword is then randomly placed into one of $2^{nR_1^{'}}$ bins. Let $i(U_1^n)$ be the index of the bin containing U_1^n. For a given m_1, we look for an U_1^n such that $i(U_1^n) = m_1$ and U_1^n and X_a^n are jointly typical. Then, we transmit $X_1^n = U_1^n - \alpha_1 X_a^n$, where U_1^n is meant to be decoded by the weak user.

In the third layer, which is meant for the strong user, the second component of the source, S_2^n, is also Wyner Ziv coded at rate $R_2^{'} = \frac{1}{2}\log(1 + \frac{P_2}{\sigma_2^2})$ using the estimate of S_2^n at the receiver as side information. The Wyner-Ziv index, $m_2 \in \{1, 2, \cdots, 2^{nR_2^{'}}\}$, is then encoded using digital Costa coding that treats both X_a^n and X_1^n as interference and uses power P_2. Let U_2 be an auxiliary random variable given by $U_2 = X_2 + \alpha_2(X_a + X_1)$, where $X_2 \sim \mathcal{N}(0, P_2)$, X_1 and X_a are independent from each other and $\alpha_2 = \frac{P_2}{P_2 + \sigma_2^2}$. Here we also create a length n i.i.d. Gaussian codebook $\mathcal{U}_2$ with $2^{nI(U_2;Y_2)}$ codewords, where each component of the codeword is Gaussian with zero mean and variance $P_2 + \alpha_2^2(P_a + P_1)$ and (randomly) evenly distribute them over $2^{nR_2^{'}}$ bins. Let $i(U_2^n)$ be the index of the bin containing U_2^n. For a given m_2, we look for an U_2^n such that $i(U_2^n) = m_2$ and (U_2^n, X_a^n, X_1^n) are jointly typical. Then, we transmit $X_2^n = U_2^n - \alpha_2(X_a^n + X_1^n)$. As shown in Fig. 2.(a), we merge all three layers and transmit $X^n = X_a^n + X_1^n + X_2^n$.

An achievable distortion-region can be obtained by varying P_a, P_1 and P_2 subject to $P = P_a + P_1 + P_2$. For a given P_a, P_1 and P_2, the achievable distortion pairs can be computed as follows. At the receiver (Fig. 2.(b)), an estimate of the first component of the source, S_1^n, is first obtained from the analog layer. This estimate acts as side information that can be used in refining the estimate of S_1^n for the weak user using the $R_1^{'}$ decoded Wyner-Ziv bits (obtained by the Costa decoder of the second layer). Since $R_1^{'}$ equals the capacity of the channel with known interference at the encoder only, $I(U_1; Y_1) - I(U_1; X_a) = \frac{1}{2}\log(1 + \frac{P_1}{P_2 + \sigma_1^2})$, the distortion in estimating S_1^n at the weak user is given by the Wyner-Ziv distortion-rate function, $D_1^* 2^{-2R_1^{'}}$, where $D_1^* = E[(S_1 - E[S_1|Y_1])^2]$ is the (idealized) MMSE from



(a) Encoder



(b) Decoder

Fig. 2. Broadcasting a bivariate source (S_1^n, S_2^n) by adopting the layering scheme with analog and Costa coding layers in [1].

the received Y_1^n. So the overall distortion seen at the weak user can be expressed as $D_1 = D_1^* \left(1 + \frac{P_1}{P_2+\sigma_1^2}\right)^{-1}$, where

$$D_1^* \;=\; \sigma_{S_1}^2 - \frac{\alpha^2(a_1\sigma_{S_1}^2 + a_2\rho\sigma_{S_1}\sigma_{S_2})^2}{P_a + P_1 + P_2 + \sigma_1^2}. \tag{6}$$

Then, an estimate of S_2^n can be determined from the first and the second layers. This estimate acts as side information for estimating S_2^n (for the strong user) from the $R_2^{'}$ decoded Wyner-Ziv bits. Here, again, $R_2^{'}$ equals the capacity of the channel with known interference, X_a^n and X_1^n, at the encoder only, i.e., $R_2^{'} = I(U_2;Y_2) - I(U_2;X_a,X_1) = \frac{1}{2}\log(1+\frac{P_2}{\sigma_2^2})$. Thus, the distortion in estimating S_2^n at the strong user is given by the Wyner-Ziv distortion-rate function, $D_2^* 2^{-2R_2^{'}}$, where D_2^* is the MMSE from the received Y_2^n and the decoded U_1^n. So the overall distortion for the strong user is given by $D_2 = D_2^* \left(1 + \frac{P_2}{\sigma_2^2}\right)^{-1}$, where $D_2^* = \sigma_{S_2}^2 - \Gamma_2^T \Upsilon_2^{-1} \Gamma_2$,

$$\Gamma_2 = \begin{bmatrix} \alpha(a_2\sigma_{S_2}^2 + a_1\rho\sigma_{S_1}\sigma_{S_2}) \\ \alpha_1\alpha(a_2\sigma_{S_2}^2 + a_1\rho\sigma_{S_1}\sigma_{S_2}) \end{bmatrix},$$

and

$$\Upsilon_2 = \begin{bmatrix} P_a + P_1 + P_2 + \sigma_2^2 & P_1 + \alpha_1 P_a \\ P_1 + \alpha_1 P_a & P_1 + \alpha_1^2 P_a \end{bmatrix}. \tag{7}$$

2) Layering with Analog, Superposition and Costa Coding: This scheme also has three coding layers: analog, superposition, and Costa coding. In the second layer, we have two merged streams, similar to the case of broadcasting a single memoryless source over a broadcast channel [4], [13]. The first component of the source is broadcasted to two users. The first source encoder is an optimal Wyner-Ziv encoder with rate $R_1^{''} = \frac{1}{2}\log(1 + \frac{(1-\lambda)P_1}{\lambda P_1 + P_a + P_2 + \sigma_1^2})$, and the second source encoder is an optimal Wyner-Ziv encoder for the residual error of the first encoder with rate $R_2^{''} - R_1^{''} = \frac{1}{2}\log(1 + \frac{\lambda P_1}{P_a + P_2 + \sigma_2^2})$. Then, we encode the Wyner-Ziv bits with capacity-achieving channel codes and transmit with

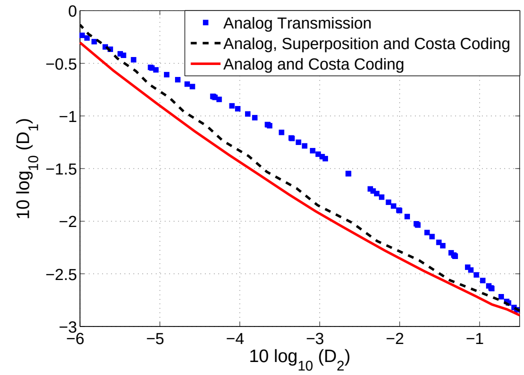


Fig. 3. Distortion regions in broadcasting a bivariate source with the correlation coefficient $\rho = 0.2$.

powers $(1-\lambda)P_1$ and λP_1, respectively. Since we require a rate of one channel use per source symbol, and the Gaussian source is successively refinable, by combining the Wyner-Ziv rate-distortion function with the pair of achievable rates for a broadcast channel $(R_1^{''}, R_2^{''})$, the corresponding achievable distortion pairs are [4]: $D_1^* 2^{-2R_1^{''}}$ and $D_1^* 2^{-2R_2^{''}}$, where D_1^* is given in (6). The coding scheme in the third layer is similar to that in the previous scheme.

The final distortion in estimating S_1^n at the weak user is

$$D_1 = D_1^* 2^{-2R_1^{''}} = \frac{D_1^*}{1 + \frac{(1-\lambda)P_1}{\lambda P_1 + P_a + P_2 + \sigma_1^2}}. \tag{8}$$

At the strong user, first an estimate of the first component of the source can be obtained within distortion

$$D_{12}^* = D_1^* 2^{-2R_2^{''}} = \frac{D_1^* 2^{-2R_1^{''}}}{1 + \frac{\lambda P_1}{P_a + P_2 + \sigma_2^2}} = \frac{D_1}{1 + \frac{\lambda P_1}{P_a + P_2 + \sigma_2^2}}.$$

Then we obtain an estimate of S_2^n from the above estimate of S_1^n with the following distortion:

$$D_2^* = \sigma_{S_2}^2 \left(1 - \rho^2 \left(1 - \frac{D_{12}^*}{\sigma_{S_1}^2}\right)\right). \tag{9}$$

This estimate of S_2^n acts as side information in refining the estimate of S_2^n (for the strong user) using the decoded Wyner-Ziv bits. The overall distortion for the strong user in estimating S_2^n is thus given by $D_2 = D_2^* \left(1 + \frac{P_2}{\sigma_2^2}\right)^{-1}$.

3) Numerical Example: We transmit n samples of a bivariate Gaussian source with the covariance matrix $\Lambda = \begin{bmatrix} 1 & 0.2 \\ 0.2 & 1 \end{bmatrix}$ in n uses of a power-limited broadcast channel to two users with observation noise variances $\sigma_1^2 = -5\,\text{dB}$ and $\sigma_2^2 = 0\,\text{dB}$, respectively. The two-user broadcast channel has the power constraint $P = 0\,\text{dB}$. The boundaries of the distortion regions for the schemes presented in this section are shown in Fig. 3. We observe that the layering with analog transmission and Costa coding outperforms all other JSCC schemes, including analog transmission.

IV. Distortion Regions with Bandwidth Compression

We next consider the problem of broadcasting a bivariate Gaussian source with 2:1 bandwidth compression. We want to transmit $k = 2n$ samples of a bivariate Gaussian source (S_1^k, S_2^k) in n uses of a power-limited broadcast channel to

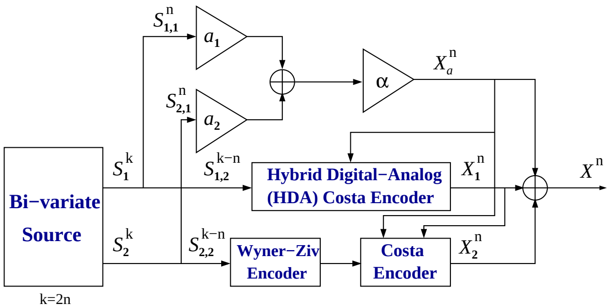


Fig. 4. Broadcasting a bivariate source (S_1^k, S_2^k) with bandwidth compression using a three-layered coding provided in [1].

two users. The two-user broadcast channel has the power constraint P. We split both components of the bivariate Gaussian source into two equal length parts, i.e., we split $2n$ samples of each source vector S_i^{2n} into two vectors of length n: $S_{i,1}^n$ and $S_{i,2}^n$.

A. Layering with Analog, HDA Costa and Costa Coding

This scheme is introduced in [1] for broadcasting a memoryless Gaussian source with bandwidth compression; see Fig. 4. In the first (analog) transmission layer, a linear combination of the first n samples of the bivariate Gaussian source components are scaled such that the power of the transmitted signal in this layer X_a^n becomes P_a. Here $X_a(t) = \alpha \sum_{i=1}^{2} a_i S_{i,1}(t)$ where $\alpha = \sqrt{\frac{P_a}{a_1^2\sigma_{S_1}^2 + a_2^2\sigma_{S_2}^2 + 2a_1a_2\rho\sigma_{S_1}\sigma_{S_2}}}$.

In the second and the third layers, we work on the remaining n samples of the source components, i.e., $S_{1,2}^n$ and $S_{2,2}^n$, respectively. In the second layer, we apply the HDA Costa coding, presented in [1], to $S_{1,2}^n$ in order to produce X_1^n with power P_1. Here, the source is not explicitly quantized and it appears in an analog form in the transmitted signal [1]. Let U_1 be an auxiliary random variable given by $U_1 = X_1 + \alpha_1 X_a + K_1 S_{1,2}$, where $X_1 \sim \mathcal{N}(0, P_1)$, $X_a \sim \mathcal{N}(0, P_a)$, and $S_{1,2}$ are independent of each other, $\alpha_1 = \frac{P_1}{P_1+P_2+\sigma_1^2}$, and $K_1^2 = \frac{P_1^2}{(P_1+P_2+\sigma_1^2)\sigma_{S_1}^2}$. As in [1], we generate a random i.i.d. codebook $\mathcal{U}_1$ with 2^{nR_1} codewords, where each component of each codeword is Gaussian with zero mean and variance $P_1 + \alpha_1^2 P_a + K_1^2\sigma_{S_1}^2$ and $R_1 = \frac{1}{2}\log(\frac{P_1+\alpha_1^2P_a+K_1^2\sigma_{S_1}^2}{P_1})$. For given $S_{1,2}^n$ and X_a^n, we find a U_1^n such that $(U_1^n, S_{1,2}^n, X_a^n)$ is jointly typical and transmit $X_1^n = U_1^n - \alpha_1 X_a^n - K_1 S_{1,2}^n$.

In the third layer, n samples of the second component of the source, $S_{2,2}^n$ are Wyner Ziv coded at rate $R_2' = \frac{1}{2}\log(1 + \frac{P_2}{\sigma_2^2})$ using the estimate of $S_{1,2}^n$ at the receiver as side information. The Wyner-Ziv index is then encoded using Costa coding that treats both X_a^n and X_1^n as interference and uses power $P_2 = P - P_a - P_1$. The code construction as well as the encoding and decoding procedures are analogous to the ones described in Section III-B.1. Therefore, we transmit $X_2^n = U_2^n - \alpha_2(X_a^n + X_1^n)$. We merge all three layers and transmit $X^n = X_a^n + X_1^n + X_2^n$.

At the decoder, we look for an U_1^n that is jointly typical with Y_1^n. The weak user estimates $S_1^k = (S_{1,1}^n S_{1,2}^n)$ by MMSE estimation from the received signal Y_1^n and the decoded U_1^n. Thus, the overall distortion seen at the weak user is [1]:

$$D_1 = \frac{n}{k}D_{11} + (1 - \frac{n}{k})D_{12} = \frac{1}{2}D_{11} + \frac{1}{2}D_{12}, \tag{10}$$

where D_{1j} $(j = 1, 2)$, the MMSE distortion in estimating $S_{1,j}^n$ from Y_1^n and U_1^n, is given by

$$D_{1j} = \sigma_{S_1}^2 - \Gamma_{1j}^T \Upsilon_{1HDA}^{-1} \Gamma_{1j},$$

where

$$\Gamma_{11} = \begin{bmatrix} \alpha(a_1\sigma_{S_1}^2 + a_2\rho\sigma_{S_1}\sigma_{S_2}) \\ \alpha_1\alpha(a_1\sigma_{S_1}^2 + a_2\rho\sigma_{S_1}\sigma_{S_2}) \end{bmatrix}, \Gamma_{12} = \begin{bmatrix} 0 \\ K_1\sigma_{S_1}^2 \end{bmatrix}, \tag{11}$$

and

$$\Upsilon_{1HDA} = \begin{bmatrix} P_a + P_1 + P_2 + \sigma_1^2 & P_1 + \alpha_1 P_a \\ P_1 + \alpha_1 P_a & P_1 + \alpha_1^2 P_a + K_1^2\sigma_{S_1}^2 \end{bmatrix}.$$

Then, an estimate of S_2^k is obtained from the first and the second layers. This estimate acts as side information for estimating S_2 (for the strong user) using the decoded Wyner-Ziv bits. The strong user estimates the second component of the source $S_2^k = (S_{2,1}^n S_{2,2}^n)$ from Y_2^n, the decoded U_1^n and U_2^n. Hence the overall distortion for the strong user is given by $D_2 = \frac{1}{2}D_{21} + \frac{1}{2}D_{22}$, where D_{2j} $(j = 1, 2)$, the distortion in estimating $S_{2,j}^n$, is determined via the Wyner-Ziv distortion-rate function:

$$D_{2j} = \left(\sigma_{S_2}^2 - \Gamma_{2j}^T \Upsilon_{2HDA}^{-1} \Gamma_{2j}\right) \left(1 + \frac{P_2}{\sigma_2^2}\right)^{1-j},$$

where

$$\Gamma_{21} = \begin{bmatrix} \alpha(a_2\sigma_{S_2}^2 + a_1\rho\sigma_{S_1}\sigma_{S_2}) \\ \alpha_1\alpha(a_2\sigma_{S_2}^2 + a_1\rho\sigma_{S_1}\sigma_{S_2}) \end{bmatrix}, \Gamma_{22} = \begin{bmatrix} 0 \\ K_1\rho\sigma_{S_1}\sigma_{S_2} \end{bmatrix}, \tag{12}$$

and

$$\Upsilon_{2HDA} = \begin{bmatrix} P_a + P_1 + P_2 + \sigma_2^2 & P_1 + \alpha_1 P_a \\ P_1 + \alpha_1 P_a & P_1 + \alpha_1^2 P_a + K_1^2\sigma_{S_1}^2 \end{bmatrix}.$$

B. Layering with Analog and Costa Coding

Here, we also use three coding layers and they are the same as the ones in Section IV-A, except for the second layer. In the second layer, the n samples of the second half of the first component of the source, $S_{1,2}^n$, are quantized at rate $R_1' = \frac{1}{2}\log(1 + \frac{P_1}{P_2+\sigma_1^2})$. The quantization index is then encoded using Costa coding that treats X_a^n as interference and uses power P_1. Therefore, we transmit $X_1^n = U_1^n - \alpha_1 X_a^n$, where $\alpha_1 = \frac{P_1}{P_1+P_2+\sigma_1^2}$. We merge all three layers and transmit $X^n = X_a^n + X_1^n + X_2^n$.

At the receiver, the weak user estimates $S_1^{2n} = (S_{1,1}^n S_{1,2}^n)$ by MMSE estimation from the received signal Y_1^n and the decoded U_1^n. Thus the overall distortion seen at the weak user is given by

$$D_1 = \frac{1}{2}\left(\sigma_{S_1}^2 - \Gamma_{11}^T \Upsilon_1^{-1} \Gamma_{11}\right) + \frac{1}{2}\frac{\sigma_{S_1}^2}{1 + \frac{P_1}{P_2+\sigma_1^2}}, \tag{13}$$

where Γ_{11} is given in (11) and

$$\Upsilon_1 = \begin{bmatrix} P_a + P_1 + P_2 + \sigma_1^2 & P_1 + \alpha_1 P_a \\ P_1 + \alpha_1 P_a & P_1 + \alpha_1^2 P_a \end{bmatrix}.$$

The strong user estimates the second component of the source $S_2^{2n} = (S_{2,1}^n S_{2,2}^n)$ within the overall distortion

$$D_2 = \frac{1}{2}\left(\sigma_{S_2}^2 - \Gamma_{21}^T \Upsilon_2^{-1} \Gamma_{21}\right) + \frac{1}{2}\sigma_{S_2}^2 \left(1 - \rho^2\left(1 - \frac{D_{12}^*}{\sigma_{S_1}^2}\right)\right)\left(1 + \frac{P_2}{\sigma_2^2}\right)^{-1} \quad (14)$$

where Γ_{21} is given in (12), Υ_2 is provided in (7) and

$$D_{12}^* = \frac{\sigma_{S_1}^2}{1 + \frac{P_1}{P_a + P_2 + \sigma_2^2}}.$$

C. Layering with Analog, Superposition and Costa Coding

Analogously to the previous coding schemes, this scheme is three-layered with its layers identical to the ones presented in Section IV-A, except for the second layer. In the second layer, as in Section III-B.2, we use two merged streams. The second part of the first component of the source, $S_{1,2}^n$, is broadcasted to two users. The first source encoder is an optimal source encoder with rate $R_1^{''} = \frac{1}{2}\log(1 + \frac{(1-\lambda)P_1}{\lambda P_1 + P_a + P_2 + \sigma_1^2})$, and the second source encoder is an optimal encoder for the residual error of the first encoder with rate $R_2^{''} - R_1^{''} = \frac{1}{2}\log(1 + \frac{\lambda P_1}{P_a + P_2 + \sigma_2^2})$. Then, we encode the quantization bits with capacity-achieving channel codes and transmit the resulting streams under powers $(1-\lambda)P_1$ and λP_1, respectively.

The weak user forms an MMSE estimate of S_1^{2n} with the following distortion:

$$D_1 = \frac{1}{2}\left(\sigma_{S_1}^2 - \frac{\alpha^2(a_1\sigma_{S_1}^2 + a_2\rho\sigma_{S_1}\sigma_{S_2})^2}{\lambda P_1 + P_a + P_2 + \sigma_1^2}\right) + \frac{1}{2}\frac{\sigma_{S_1}^2}{1 + \frac{(1-\lambda)P_1}{\lambda P_1 + P_a + P_2 + \sigma_1^2}}. \quad (15)$$

At the strong user, first an estimate of $S_{1,2}^n$ can be obtained within distortion

$$D_{12}^* = \frac{1}{1 + \frac{\lambda P_1}{P_a + P_2 + \sigma_2^2}} \times \frac{\sigma_{S_1}^2}{1 + \frac{(1-\lambda)P_1}{\lambda P_1 + P_a + P_2 + \sigma_1^2}}.$$

This estimate acts as side information for obtaining the estimate of $S_{2,2}^n$ using the decoded Wyner-Ziv bits. The resulting distortion for the strong user is thus given by

$$D_2 = \frac{1}{2}\left(\sigma_{S_2}^2 - \Gamma_{21}^T \Upsilon_2^{-1} \Gamma_{21}\right) + \frac{1}{2}\sigma_{S_2}^2 \left(1 - \rho^2\left(1 - \frac{D_{12}^*}{\sigma_{S_1}^2}\right)\right)\left(1 + \frac{P_2}{\sigma_2^2}\right)^{-1} \quad (16)$$

Finally, note that if we set $\rho = 1$ and $\sigma_{S_1}^2 = \sigma_{S_2}^2$, then the results of [1], [9], which currently appear to be the best known results for broadcasting a Gaussian source with bandwidth compression, are obtained.

D. Numerical Results

We transmit $k = 2n$ samples of a bivariate Gaussian source (S_1^k, S_2^k) with the covariance matrix $\Lambda = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$ in n uses of a power-limited broadcast channel to two users with observation noise variances $\sigma_1^2 = -5\,\text{dB}$ and $\sigma_2^2 = 0\,\text{dB}$, respectively. The distortion regions for the schemes presented in this section are shown in Fig. 5 for two different correlation coefficients, $\rho = 0.2$ and $\rho = 0.8$.

We observe that the layering with analog, superposition

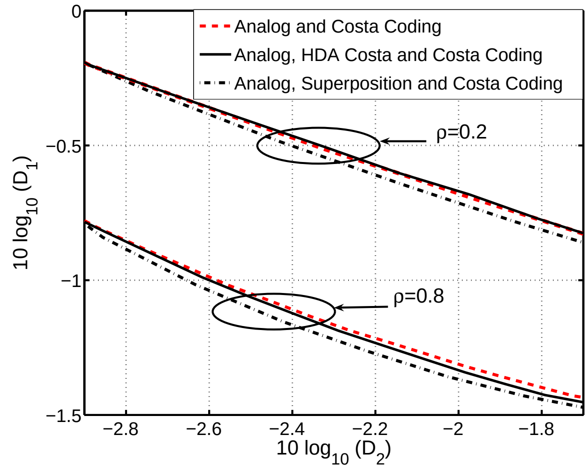


Fig. 5. Distortion regions of different HDA coding schemes. System parameters are $P = 0\,\text{dB}$, $\sigma_1^2 = -5\,\text{dB}$ and $\sigma_2^2 = 0\,\text{dB}$.

and Costa coding of Section IV-C outperforms all other schemes in both cases. When the source components are highly correlated, layering with analog, HDA Costa, and Costa coding scheme performs better than the layering with analog and Costa coding scheme; however, the two two schemes perform similarly for small values of the correlation coefficient.

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