Sequence MAP Decoding of Trellis Codes for Gaussian and Rayleigh Channels

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Abstract — **This paper examines the gains that can be obtained when maximum a posteriori (MAP) decoding is used to recover correlated and/or non-equiprobable data that has been channel encoded prior to transmission over very noisy AWGN and Rayleigh fading channels. Most compression algorithms leave some residual redundancy in the output bit stream; this redundancy – in the form of memory and/or a non-uniform distribution – may be exploited at the receiver by adjusting the decoding metric. This approach is demonstrated with a variety of two-state Markov sources; it is shown that sequence MAP decoding provides substantial gain at low channel SNR.**

Trellis coding of the line spectral parameters of FS 1016 CELP modulated with 4D QPSK is also considered. Two MAP decoders are used – one that exploits intra-frame correlation only and another that exploits both inter- and intra-frame correlation. MAP decoding gains (w.r.t. ML decoding) as high as 4 dB are achieved.

I. Introduction

Traditionally, a communication system's source coding (data compression) function and its channel coding (error control) function are designed independently of one another. The justification for this is Shannon's separation principle, which indicates that no performance loss is suffered if the two functions are thus partitioned. However, Shannon's result is an asymptotic one that permits unlimited delay and complexity; given a constraint on complexity/delay, joint source-channel coding may outperform separately designed pairs.

Much of the work in joint source-channel coding has considered the design of source codes that are robust to channel errors. Conversely, the work in [2, 3] has focused on the design of channel decoders that exploit known characteristics of the source code. The work here is in the spirit of [2, 3]; it concerns the performance of trellis codes with sequence maximum a posteriori (MAP) decoding of correlated signals transmitted over very noisy AWGN and Rayleigh channels.

The residual redundancy in the channel encoder input can be exploited at the receiver by adjusting the Viterbi algorithm's metric to use the source's a priori probabilities. This approach, coupled with soft-decision decoding and channel state information (CSI) estimation, can result in a very robust system under very poor channel conditions.

Sequence MAP decoding does not require substantial modification to an existing maximum likelihood (ML) decoder, and as indicated in [2, 3], may be used if necessary when a bad channel environment exists. In [2], Hagenauer showed that coding gains of 2-3 dB could be obtained for PCM transmission with the full rate GSM speech codec; a 16-state rate 1/2 convolutional code with BPSK modulation and a dynamic two state Markov correlation estimator were used. In [3], Alajaji, Phamdo and Fuja used both block and convolutional codes to exploit the residual redundancy in a code excited linear predictive (CELP) speech coder. A 32-state rate 3/4 convolutional code with BPSK modulation and sequence MAP decoding was used, and the CELP line spectral parameters (LSP's) were modeled by first- and second-order Markov chains whose transition probabilities were estimated and then provided to the MAP decoder. Decoding gains of 2-5 dB were obtained.

The work in this paper is divided into two parts. The first part assumes a first order two state Markov source model. This simple model is chosen because its transition probabilities may be easily estimated at the receiver when the channel is "clean". A variety of different systems with different sources, modulation schemes, and trellis codes are simulated. Extensive simulations of these configurations are performed to assess their effect on the sequence MAP decoding gains (henceforth called MAP gains).

In the second part, we consider coding the CELP line spectral parameters using trellis codes with 4D QPSK modulation. Two source models are used. One is based on the intra-frame correlation of the LSP's while the second one models both intra-frame and inter-frame correlations. Both AWGN and Rayleigh channels are considered.

II. Sequence MAP Decoding for Ideal Sources

Assume the input to the channel encoder is a binary sequence $\{u_1, u_2, \ldots\}$, modeled by a stationary first order two-state Markov process. The sequence

$\{u_1, u_2, \ldots\}$ may represent the output of a source encoder, or (if the source is not compressed) the output of the source itself. We denote the transition probabilities $\Pr(u_i = 0|u_{i-1} = 0)$ and $\Pr(u_i = 1|u_{i-1} = 1)$ by $\Pr(0|0)$ and $\Pr(1|1)$, respectively. Let $H_\infty(U)$ be the source entropy rate and let $H(U)$ be the entropy of a memoryless source with the same marginal distribution as the source. Define [4]

$$\begin{aligned} \rho_D &\triangleq 1 - H(U) \\ \rho_M &\triangleq H(U) - H_\infty(U) \\ \rho_T &\triangleq \rho_D + \rho_M = 1 - H_\infty(U). \end{aligned} \tag{1}$$

Then the source redundancy due to the non-uniform distribution is ρ_D, while the redundancy due to memory is ρ_M. The two forms of redundancy are used by the decoder to combat channel errors.

The source bits are arranged in a sequence of binary k-tuples $\{\underline{u}_1, \underline{u}_2, \ldots\}$. At time i, $\underline{u}_i$ is an input to a trellis encoder that produces a binary $(k+1)$-tuple $\underline{c}_i$. The trellis encoder output $\underline{c}_i$ is mapped to the complex vector $\underline{x}_i = \{x_i^1, \ldots, x_i^m\}$, where m is the number of transmitted signals per branch – i.e., the multiplicity of the code. The sequence $\mathbf{x}_N = \{\underline{x}_1, \ldots \underline{x}_N\}$ is transmitted over the channel. This can be described by the discrete time relation

$$y_i^l = a_i^l x_i^l + n_i^l \tag{2}$$

for $1 \le l \le m$. Here, n_i^l is a complex zero mean additive Gaussian noise sample with a single-sided power spectral density of N_o. The distribution of a_i^l depends on the channel:

- For a purely AWGN non-fading channel, $a_i^l = 1$.
- For a fully interleaved Rayleigh fading channel, $\{a_i^l\}$ is a sequence of i.i.d. Rayleigh random variables with $E[(a_i^l)^2] = 1$.

The sequence MAP decision rule is to choose as its estimate of the transmitted sequence the $\hat{\mathbf{x}}_N$ that maximizes $f(\mathbf{y}_N|\hat{\mathbf{x}}_N)\Pr(\hat{\mathbf{x}}_N)$. For the AWGN channel and the afore-mentioned Markov source, this is simplified to choosing $\hat{\mathbf{x}}_N$ that minimizes

$$m_1(\hat{\mathbf{x}}_N, \mathbf{y}_N) = \sum_{i=1}^{N} \left(\sum_{l=1}^{m} |y_i^l - \hat{x}_i^l|^2 - N_o \Pr(\underline{u}_i|\underline{u}_{i-1}) \right).$$

For the Rayleigh fading channel, CSI is incorporated into the metric to choose $\hat{\mathbf{x}}_N$ minimizing

$$m_2(\hat{\mathbf{x}}_N, \mathbf{y}_N) = \sum_{i=1}^{N} \left(\sum_{l=1}^{m} |y_i^l - a_i^l \hat{x}_i^l|^2 - N_o \Pr(\underline{u}_i|\underline{u}_{i-1}) \right).$$

III. Simulation Results for Ideal Sources

Many system configurations were simulated. Five different source distributions were assumed, with varying degrees of memory and/or non-uniformity. BPSK, QPSK and 8-PSK modulation were used and trellis codes with varying rates and complexities were implemented. These different configurations were simulated on two channels – the AWGN model and the Rayleigh fading channel with ideal interleaving. These two channels describe the extremes of channels encountered in practice; hence, the results obtained with other channels (e.g., Rician) will lie in between our results. Table 1 shows the results of the simulation for so-called "Source V" – a source with $\rho_D = 0.496$ and $\rho_M = 0.007$.

For systems with BPSK modulation, rate-1/2 and rate-2/3 trellis codes optimized for the Hamming distance [5] were used; the simulated encoders had 4, 8, and 16 states. For a decoded bit error rate (BER) of 0.02, MAP decoding gains (w.r.t. ML decoding) as high as 1.4 dB were obtained with the rate-1/2 codes over the Gaussian channel; for the Rayleigh channel the gains were as high as 2.0 dB. For the rate-2/3 codes the corresponding gains are 2.5 dB (for the Gaussian channel) and 4.0 (for the Rayleigh channel).

For the QPSK modulated systems, the same rate 1/2 trellis codes were used, with each pair of encoder output bits being mapped to a two-dimensional QPSK signal point via Gray mapping. The resulting gains are very close to those of the BPSK-modulated schemes.

Octal PSK modulation was also simulated to observe the effect of increasing the signal constellation size. Rate-2/3 codes with natural 8-PSK mapping were simulated; these codes are optimal for both the Gaussian and the Rayleigh channels [6]. Higher MAP decoding gains than those of the BPSK-modulated rate 2/3 codes were obtained. For example, at a decoding BER of 0.02 gains as high as 3.3 dB for the Gaussian channel and 4.6 dB for the Rayleigh channel were observed for "Source V".

From the simulations, it is seen that the gains diminish as the BER decreases. At lower error rates ML decoding will have only a slight degradation in performance compared to sequence MAP decoding. This suggests that to reduce the decoding computations MAP decoding should be used only under very poor channel conditions.

Regarding the number of encoder memory elements, it can be seen that most of the decoding gains can be obtained using relatively simple 4-state codes. Although more complex codes improve the performance at low error rates, they do not significantly improve the performance at the relatively high error rates where MAP decoding is most appropriate. This is because codes with more states suffer more seriously from error propagation over low-SNR channels.

Finally, it is observed that increasing the signal constellation size results in greater MAP gain. As noted above, the MAP gains for the rate-2/3 codes are greater when octal PSK is used than when BPSK is used. This is because the 8-PSK signal constellation is more dense and thus more sensitive to noise. The increased MAP decoding gain for dense constellations is promising, given that such constellations will be necessary to satisfy increasing demand for scarce bandwidth.

IV. Channel Coding of CELP LSP's

Code excited linear predictive (CELP) coding is a speech compression algorithm that models segments of speech as the output of a linear filter with a particular input; encoding consists of synthesizing the filter and selecting the appropriate input and then transmitting descriptions of both. The particular implementation we consider is Federal Standard 1016 (FS 1016) 4.8 kbit/s CELP [7]. Among the parameters that are generated by this encoder are 10 quantized line spectral parameters (LSP's), which describe the filter. In FS 1016 CELP, each LSP is scalar-quantized to either three or four bits; specifically, the second through fifth LSP's are quantized to four bits, while the rest are quantized to three bits. The quantized LSP's refer to frequencies that are ordered (LSP-1 $<$ LSP-2 $< \cdots <$ LSP-10). In this work, we consider only the three most significant bits of each LSP, ignoring the least significant bit in the second through fifth parameters.

The modeling of CELP LSP's is described in detail in [3]. The relative frequency of transitions between the values of the three high-order bits of each LSP were compiled to extract Markov transition probabilities. The approach was to use a single "universal" model – constructed from a very large training sequence – to decode *all* the speech samples. Two models for the generation of LSP's distribution were proposed.

- Model 1 – which incorporates only the correlation *within* a CELP frame – indicates that $\rho_T = 9.87$ of the 30 high-order bits in the LSP's are redundant. Approximately $\rho_D = 5.28$ bits of redundancy are due to the non-uniform distribution of the LSP's, while $\rho_M = 4.59$ bits of redundancy are due to the memory within a frame.
- Model 2 – which accounts for both inter-frame and intra-frame correlation – indicates that $\rho_T =$ 12.49 of the 30 high-order bits in the LSP's are redundant. For Model 2, we find that once again $\rho_D = 5.28$ bits while now $\rho_M = 7.21$ bits.

We use three soft-decision decoding algorithms based on the Viterbi algorithm:

- ML – maximum likelihood Viterbi decoding.
- MAP 1 – a MAP decoding algorithm that exploits only the redundancy due to the non-uniform distribution of the LSP's and their correlation within a frame – i.e., the 9.87 bits of redundancy characterized by Model 1.
- MAP 2 – an algorithm that exploits the 12.49 bits of redundancy due to the non-uniform distribution of the LSP's *and* their inter-frame and intra-frame correlation – the redundancy characterized by Model 2.

A decoding buffer length of 10 symbols is used to limit the decoding delay. All algorithms are implemented so as to yield a decoding delay of only one frame.

Since the quantized LSP's are represented by three bits, our channel code must treat three bits as a "unit". The proposed scheme is a 4-state rate 3/4 code with 4D QPSK. Its spectral efficiency is 1.5 bit/s/Hz, and it was designed for Rayleigh fading channels [8]. Moreover, its minimum Euclidean distance is the same as the corresponding code designed for the AWGN channel [9]. This means that the code is optimum for both Gaussian and Rayleigh channels.

In evaluating the performance of the various decoders we used two criteria. The first is the average spectral distortion (SD), one of the most commonly used distortion measure for LSP's; the average spectral distortion induced by CELP's scalar quantizers alone (when the channel is noiseless) is around 1.50 dB. The second measure of the decoders' performance is the symbol error rate (P_s) – i.e., the fraction of LSP's the decoder decodes incorrectly. Table 2 shows the MAP decoding gains for both channels. The gains are shown for P_s values of 1, 5, 10 and 15%. (Error concealment techniques are used for values of P_s as high as 15%.)

It is clear that significant gains are achieved and that the gains for the Rayleigh channel are higher than those for the AWGN channel. For example, at an average spectral distortion of 2.0 dB, MAP decoding gains of 1.5 and 3.1 dB were achieved for the AWGN and Rayleigh channels, respectively. Average spectral distortion curves are shown in Figure 1. It is observed that most of the gains can be achieved using MAP 1; this is not surprising, given that the additional redundancy that MAP 2 exploits is only 2.5 bits.

V. Conclusion

In this paper we considered sequence maximum a posteriori (MAP) decoding of correlated signals transmitted over very noisy AWGN and Rayleigh channels. In the first part of the paper, a first order two-state Markov model was used to model the channel encoder input. A variety of different systems with different sources, modulation schemes and encoders were simulated. Sequence MAP decoding was shown to substantially improve the performance under very noisy channel conditions, relative to ML decoding. Most of the MAP decoding gains are achieved with low complexity encoders. Moreover, trellis coded systems with higher encoder rates have significantly more MAP decoding gains. Also, more decoding gains are obtained for encoders with larger signal constellations.

Trellis encoding of the line spectral parameters of FS 1016 CELP with 4D QPSK modulation were also presented. Two source models were used; one is based on the intra-frame correlation, while the second one models both intra-frame and inter-frame correlations. Coding gains as high as 4 dB are achieved.

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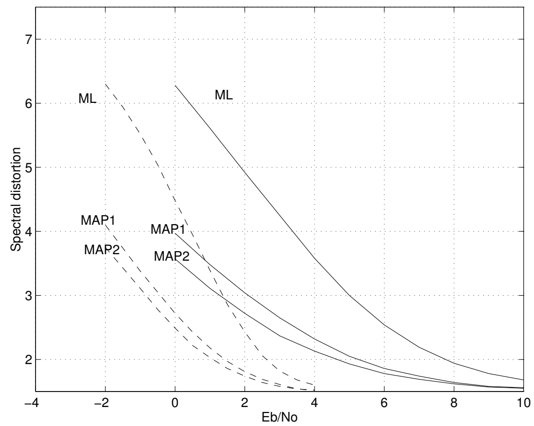

Figure 1: Average spectral distortion vs. Eb/No for the 4-state 4D QPSK scheme - (Solid = Rayleigh; Dashed = AWGN.)

Mod. Type	Rate	ν	Gains (BER) 10%	5%	2%
BPSK	1/2	2	2.6 (3.6)	1.6 (2.3)	1.1 (1.6)
BPSK	1/2	3	2.5 (3.5)	1.6 (2.4)	1.2 (1.7)
BPSK	1/2	4	2.9 (3.9)	1.9 (2.7)	1.4 (2.0)
BPSK	2/3	2	6.7 (8.9)	3.4 (5.0)	2.2 (3.6)
BPSK	2/3	3	6.5 (8.7)	3.4 (5.0)	2.3 (3.6)
BPSK	2/3	4	6.3 (8.5)	3.7 (5.4)	2.5 (4.0)
QPSK	1/2	2	2.9 (3.7)	1.6 (2.2)	1.1 (1.5)
QPSK	1/2	3	2.5 (3.5)	1.6 (2.2)	1.1 (1.6)
QPSK	1/2	4	2.9 (3.9)	1.9 (2.6)	1.4 (2.0)
8PSK	2/3	3	7.5 (9.4)	4.4 (5.4)	3.0 (4.0)
8PSK	2/3	4	7.7 (9.4)	4.7 (5.9)	3.3 (4.6)

Table 1: MAP decoding gains for different trellis codes over AWGN (Rayleigh) channels. (Source V: $\Pr(0|0) = .2, \Pr(1|1) = .9$).

Decoding Gains	Channel Type	$P_s(\%)$ 1%	5%	10%	15%
MAP 1 vs. ML	AWGN	0.8	1.1	1.3	1.6
	Rayleigh	2.0	2.1	2.2	2.3
MAP 2 vs. MAP 1	AWGN	0.2	0.3	0.3	0.3
	Rayleigh	0.3	0.6	0.6	0.6
MAP 2 vs. ML	AWGN	1.0	1.4	1.6	1.9
	Rayleigh	2.3	2.6	2.8	2.9

Table 2: Sequence MAP decoding gains for the CELP encoded speech with 4-state 4D QPSK TCM schemes over AWGN and Rayleigh channels