MAP Decoding of Quantized Sources over Soft-Decision Fading Channels with Memory

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***Abstract*—We study a joint source-channel decoding scheme that exploits the channel's statistical memory and soft-decision information in fading channels. The channel considered is a recently introduced binary input 2^q-ary output channel with Markovian ergodic noise based on a finite queue (called NBNDC-QB). This model has been shown to effectively represent soft-decision demodulated correlated Rayleigh fading channels. The coding scheme consists of a scalar quantizer, a proper index assignment, and a sequence maximum a posteriori (MAP) decoder designed to harness the redundancy left in the quantizer's indices, the channel's soft-decision output, and correlation in the channel noise process. We first consider the simple case where the quantized indices form a binary symmetric Markov source and establish a necessary and sufficient condition under which the sequence MAP decoder is reduced to a simple instantaneous symbol-by-symbol decoder. We next assess the signal-to-distortion ratio (SDR) performance of our general system. Our numerical results confirm that this system can successfully take advantage of the channel memory and outperforms systems that use channel interleaving by as much as 2.6 dB in SDR. In addition, SDR gains of up to 2.8 dB are achieved using as few as 2 bits for soft-decision quantization over hard quantized output schemes. Finally, the NBNDC-QB channel model is validated in terms of SDR performance by fitting the NBNDC-QB model to a discrete correlated Rayleigh fading channel, designing a system for this matched NBNDC-QB model, and comparing this system's performance over both the NBNDC-QB and the Rayleigh fading channels.**

I. INTRODUCTION

It is well known that the separate treatment of source and channel coding, as in Shannon's source-channel coding theorem [1], is not optimal in the presence of complexity and delay constraints. For lossy coding, a variety of different joint-source channel coding schemes have been proposed to address this problem (such as [2]–[6] and many others). It is also known that if a channel is well-behaved (ergodic) and has memory, then its capacity is strictly greater than the capacity of its memoryless counterpart (a channel with identical one-dimensional transition distribution) realized via ideal (infinite-depth) block interleaving [7], [8]. Consequently, a communication system can be designed to take advantage of the channel's memory so that it can outperform systems that discard such memory via interleaving. Furthermore, effective use of the channel's soft-decision information can improve capacity and system performance over hard-decision decoded schemes (e.g., see [9]–[12]).

In this paper, we study the sequence maximum-a-posteriori (MAP) decoding problem of quantized sources over a non-binary noise discrete channel (NBNDC) and the correlated Rayleigh fading channel used with soft-decoding demodulation. This extends the work of [4] where only binary output channels with Markov noise were considered. Our system uses a scalar quantizer (SQ) designed for a noiseless channel and applied to an analog-valued source; the SQ output is passed through an index assignment mapping (without the use of algebraic channel coding) and then sent over the channel. The channel output is soft-demodulated with resolution of q bits and delivered to a sequence MAP detector to combat channel errors. As in [4], we refer to such a coding scheme as SQ-MAP. We use scalar quantization, rather than vector quantization (VQ), since although VQ achieves better signal-to-distortion (SDR) performance than SQ when the channel is noiseless, it retains less redundancy in the index codewords at the quantizer output that can be exploited (together with the channel memory) by the MAP decoder. Consequently, in this channel uncoded system the overall performance of VQ is not necessarily better than that of SQ. It is important to mention that the SQ-MAP scheme is designed to minimize the sequence error probability, while we evaluate the performance of the system via the signal-to-distortion ratio (SDR) with the mean square error (MSE) distortion measure. Hence, the SQ-MAP is not necessarily optimal in terms of achieving minimum mean square error (MMSE). However, this system has tractably low complexity as well as good performance according to simulations results, which makes it an efficient joint source-channel coding scheme. We also prove a necessary and sufficient condition under which the sequence MAP detector in the SQ-MAP system with rate one over the NBNDC-QB reduces to an instantaneous symbol-by-symbol mapping.

The NBNDC channel model we consider for representing the fading channel was recently introduced in [13]. This model is more general than the binary Markov channel used in [4] and subsumes it as a special case. The channel has a binary input and 2^q-ary output. The noise process is a generalization of the finite queue based (QB) noise model introduced in [8]. We show that the NBNDC with QB noise, which we denote by NBNDC-QB, is able to model the Rayleigh discrete fading

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channel (DFC) in terms of SDR performance. Note that no closed form expression for the block transition probabilities of the correlated Rayleigh DFC is known for block lengths greater than three [13], which makes the implementation of MAP decoding on this channel very hard. On the other hand, the NBNDC-QB admits a tractable closed form block transition distribution in terms of a few parameters.

The rest of the paper is organized as follows. In Section II, the channel models are explained in detail. The coding scheme is described in Section III. Details of the system implementation as well as a theoretical result about the MAP detector are provided. Section IV is devoted to numerical results and conclusions are given in Section V.

II. NBNDC-QB AND DFC CHANNEL MODELS

In this section we review two channel models: the NBNDC with queue-based noise (NBNDC-QB) and the Rayleigh DFC. Furthermore, we observe that the Rayleigh DFC is a special case of the NBNDC.

A. NBNDC with queue-based noise

The NBNDC-QB is a binary-input and 2^q-ary-output channel model [13]. The channel noise is modeled via a 2^q-ary stationary and ergodic M^{th}-order Markov process described by 2^q+2 independent parameters. We note that the number of model parameters does not depend on the channel memory M, keeping the complexity of the model independent of the memory order. On the other hand, the number of model parameters is exponentially proportional to q, although typical values for q are as low as 2 or 3. Specifically, the input data bits X_j are affected by the noise Z_j via the relation

$$Y_j = (2^q-1)X_j + (-1)^{X_j} Z_j, \tag{1}$$

where $Y_j, Z_j \in \{0,1,\dots,2^q-1\}$ for $j=1,2,\dots$, with $\{X_j\}$ denoting the channel input binary process and $\{Y_j\}$ denoting the channel output 2^q-ary process. Also, the 2^q-ary noise process $\{Z_j\}$ is assumed to be independent of $\{X_j\}$, so that

$$\Pr\{Y^m = y^m \mid X^m = x^m\} = \Pr\{Z^m = z^m\}, \tag{2}$$

where $x^m = (x_1,x_2,\dots,x_m)$, $y^m = (y_1,y_2,\dots,y_m)$, and $z^m = (z_1,z_2,\dots,z_m)$ is given by

$$z_k = \frac{y_k-(2^q-1)x_k}{(-1)^{x_k}}, \quad k=1,2,\dots,m. \tag{3}$$

The noise process is a non-binary generalization of the queue-based (QB) noise [8]. In this model, each noise symbol is considered as a numbered ball, either selected from an urn (with probability $1-\varepsilon$) or from a finite queue of length M (with probability ε). The urn contains a large set of numbered balls, such that a ball with number i (representing the noise symbol i) is selected from the urn with probability ρ_i, $i \in \{0,1,\dots,2^q-1\}$. The finite queue is updated every time a noise symbol is generated. See [13] for a detailed description of the procedure. The resulting QB noise process is a stationary ergodic Mth order Markov source and has only 2^q+2 independent parameters: the size of the queue M, the probability distribution of the balls in the urn, and correlation parameters $0 \leqslant \varepsilon < 1$ and $\alpha \geqslant 0$.

The state process of the queue based noise $\{\mathbf{S}_n\}_{-\infty}^{\infty}$, which is defined by $\mathbf{S}_n \triangleq (Z_n, Z_{n-1},\dots,Z_{n-M+1})$, is a homogeneous first-order Markov process. Define the noise state transition probability by

$$Q(\mathbf{s}_n|\mathbf{s}_{n-1}) \triangleq \Pr\{\mathbf{S}_n = \mathbf{s}_n | \mathbf{S}_{n-1} = \mathbf{s}_{n-1}\},$$

According to [13],

$$\begin{aligned} Q(\mathbf{s}_n|\mathbf{s}_{n-1}) &= \left(\sum_{\ell=1}^{M-1} \delta_{z_n,z_{n-\ell}} + \alpha\delta_{z_n,z_{n-M}}\right)\frac{\varepsilon}{M-1+\alpha} \\ &+ (1-\varepsilon)\rho_{z_n}, \end{aligned} \tag{4}$$

where, $\delta_{i,j}=1$ if $i=j$ and $\delta_{i,j}=0$ if $i \neq j$.

The m-fold channel transition probability $\Pr\{Z^m = z^m\} \triangleq P^{(m)}_{\text{NBNDC}}(z^m)$ is given by (17) of [13] and the channel noise correlation is given by

$$\text{Cor} = \frac{\mathbf{E}[Z_k Z_{k+1}] - \mathbf{E}[Z_k]^2}{Var(Z_k)} = \frac{\frac{\varepsilon}{M-1+\alpha}}{1-(M-2+\alpha)\frac{\varepsilon}{M-1+\alpha}}.$$

B. Rayleigh DFC.

The Rayleigh DFC consists of a binary phase-shift keying (BPSK) modulator, a time-correlated flat Rayleigh fading channel with additive white Gaussian noise (AWGN), and a q-bit soft-quantized demodulator. Let the input and output alphabets be $\mathcal{X} = \{0,1\}$ and $\mathcal{Y} = \{0,1,\dots,2^q-1\}$, respectively. Denoting the DFC binary input process by $\{X_k\}$, the received channel symbols are given by

$$R_k = \sqrt{E_s}A_k S_k + N_k, \qquad k=1,2,\dots$$

where E_s is the energy of signal sent over the channel, $S_k = 2X_k - 1 \in \{-1,1\}$ is the BPSK modulated signal and N_k is an additive white noise, represented by a sequence of independent and identically distributed (i.i.d.) Gaussian random variables of variance $N_0/2$. Here $\{A_k\}$ is the channel's fading process with $A_k = |G_k|$, where $\{G_k\}$ is a time-correlated complex wide-sense stationary Rayleigh process with autocorrelation function given by $R[k] = J_0(2\pi f_D T|k|)$ from Clarke's model [14], where $f_D T$ is the normalized maximum doppler frequency and $J_0(\cdot)$ is the zeroth-order Bessel function of first kind. Therefore, each A_k is Rayleigh distributed, with unit second moment. The fading process $\{A_k\}$ is assumed to be independent of the noise and input processes. The channel signal-to-noise ratio (SNR) is given by SNR$= E_s/N_0$.

As the last part of the DFC model, a soft-decision demodulator consisting of a uniform quantizer with resolution q bits, takes the output R_k to produce the discrete channel output:

$$Y_k = j, \quad \text{if}\, R_k \in [T'_{j-1}, T'_j),$$

where T'_j are uniformly spaced thresholds with step-size Δ, such that

$$T'_j = \begin{cases} -\infty, & \text{if} \quad j=-1 \\ (j+1-2^{q-1})\Delta, & \text{if} \quad j=0,1,\dots,2^q-2 \\ \infty, & \text{if} \quad j=2^q-1. \end{cases}$$

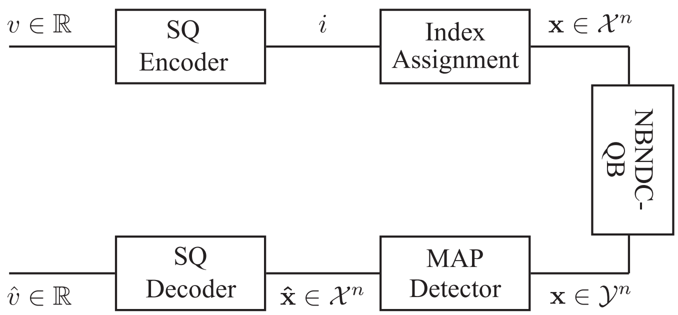

Fig. 1. Block diagram of a joint source-channel coding system using scalar quantization and MAP decoder (SQ-MAP).

Let $\delta \triangleq \Delta/\sqrt{E_s}$ and $T_j \triangleq T'_j/\sqrt{E_s}$. The channel m-fold conditional probability for the DFC,

$$P_{\text{DFC}}^{(m)}(y^m \mid x^m) \triangleq \Pr\{Y^m = y^m \mid X^m = x^m\}, \tag{5}$$

can be calculated via (2) in [13]. For $m = 1$, a closed form expression for $P_{\text{DFC}}^{(1)}(j),\ j \in \mathcal{Y}$ is given by

$$P_{\text{DFC}}^{(1)}(j) = n(-T_{j-1}) - n(-T_j), \tag{6}$$

where

$$n(T_j) = 1 - Q(T_j\sqrt{2\text{SNR}}) - \frac{\left[1 - Q\left(\frac{T_j\sqrt{2}}{\sqrt{\frac{1}{\text{SNR}}+1}}\right)\right] e^{-\frac{T_j^2}{\left(\frac{1}{\text{SNR}}+1\right)}}}{\sqrt{\frac{1}{\text{SNR}}+1}},$$

where $Q(\cdot)$ is the Gaussian Q-function. In general, for $m \leqslant 3$, $P_{\text{DFC}}^{(m)}(y^m \mid x^m)$ can be calculated in closed form. For $m > 3$, since the joint probability density functions of arbitrarily correlated Rayleigh and Rician random variables are not known in closed form, it can only be determined via numerical methods. It can be shown that the DFC is actually an NBNDC as given by (1) with a stationary ergodic noise process [13].

III. Joint source-channel MAP decoding of the NBNDC-QB

Consider the communication system depicted in Fig. 1. The analog source $\mathcal{V} = \{V_i\}_{i=1}^{\infty}$ is assumed to be a real-valued stationary and ergodic process. The scalar quantizer (SQ) encoder is a mapping γ from the real domain of source symbols to the index set $\{0, 1, \ldots, 2^n - 1\}$, such that

$$\gamma(v) = i \quad \text{if} \quad v \in S_i,$$

where $\{S_i : i \in \{0, 1, \ldots, 2^n - 1\}\}$ is a partition of $\mathbb{R}$. The partitions are chosen according to Lloyd-Max formulation in [15], with the initial codebook selection obtained via the splitting algorithm [16].

The index assignment module is a one-to-one mapping, which maps each index i to a binary vector $\mathbf{x} \in \{0, 1\}^n$

$$b : \{0, 1, \ldots, 2^n - 1\} \rightarrow \{0, 1\}^n, \quad b(i) = \mathbf{x}$$

where $\mathbf{x}$ is represented in binary form. Since the mapping is one-to-one, for a given index mapping b, we can present the quantization regions by $S_{\mathbf{x}}$ instead of S_i, where $b(i) = \mathbf{x}$.

To assign a binary n-tuple codeword to each index, different index assignment methods such as the natural binary code (NBC), the folded binary code (FBC) [4], simulated annealing, and some heuristic assignment methods were tested. The FBC was selected because of its simplicity and good performance. The n-tuple codeword $\mathbf{x}$ is then sent bit-by-bit over the NBNDC-QB channel where it is affected by the error n-tuple z^n. The channel output $\mathbf{y} \in \mathcal{Y}^n$ is fed to a MAP decoder where the data redundancy is used for error correction. Finally, the SQ decoder β maps the decoder output $\hat{\mathbf{x}}$ into output levels of the quantizer codebook

$$\beta(\hat{\mathbf{x}}) = c_{\hat{\mathbf{x}}},\ c_{\hat{\mathbf{x}}} \in \mathbb{R},\ \hat{\mathbf{x}} \in \{0, 1\}^n.$$

The MAP decoder is designed to minimize the sequence error probability by exploiting the residual redundancy of the source and channel model statistics to combat channel errors. The redundancy ρ_T, in general, is due to a combination of non-uniformity of the distribution (ρ_D) and memory (ρ_M), such that $\rho_T = \rho_D + \rho_M$. Similar to [4], we first assume an i.i.d. source and then we modify the metric for sources with memory. If $\mathcal{V}$ is i.i.d. then the SQ encoder output process, $\{X_i\}$, is also i.i.d. Hence $\rho_M = 0$ and the only remaining redundancy is due to source non-uniformity.

The MAP detector can be viewed as a system observing a sequence of 2^q-ary n-tuples $\mathbf{y}^N = (\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_N) \in \mathcal{Y}^{nN}$, where N denotes the number of source symbols to be transmitted over the channel and n is the codeword length. $\mathbf{y}^N$ is a noisy observation of the source sequence $\mathbf{x}^N = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N) \in \mathcal{X}^{nN}$. The channel contaminates the source bits via 2^q-ary error symbols $z^{nN} = (z_1, z_2, \ldots, z_{nN}) \in \mathcal{Y}^{nN}$. Note that since the transmission over the channel is done bit-by-bit (and not n-tuple by n-tuple), we represent the noise sequence using a bit-by-bit notation so that the noise symbols $(z_{ni+1}, z_{ni+2}, \ldots, z_{(n+1)i})$, $i \in \{0, 1, \ldots, N-1\}$ correspond to the input n-tuple $\mathbf{x}_{i+1}$ and output n-tuple $\mathbf{y}_{i+1}$. The MAP decoder estimates $\mathbf{x}^N$ by $\hat{\mathbf{x}}^N$ according to

$$\hat{\mathbf{x}}^N = \arg\max_{\mathbf{x}^N} \Pr\{\mathbf{X}^N = \mathbf{x}^N \mid \mathbf{Y}^N = \mathbf{y}^N\}.$$

It can be shown that the above equation is equivalent to

$$\begin{aligned}
\hat{\mathbf{x}}^N &= \arg\max_{\mathbf{x}^N \in \{0,1\}^{nN}} \Pr\{\mathbf{Y}^N = \mathbf{y}^N \mid \mathbf{X}^N = \mathbf{x}^N\} \times \\
&\quad \Pr\{\mathbf{X}^N = \mathbf{x}^N\} \\
&= \arg\max_{\mathbf{x}^N \in \{0,1\}^{nN}} \Pr\{Z^{nN} = z^{nN}\} \Pr\{\mathbf{X}^N = \mathbf{x}^N\} \\
&= \arg\max_{\mathbf{x}^N \in \{0,1\}^{nN}} \qquad\qquad\qquad\qquad (7) \\
&\quad \left[Q(z_1^n) P(\mathbf{x}_1) \prod_{i=1}^{N-1} \Big(Q(z_{ni+1}^{ni+n} | z_1^{ni}) P(\mathbf{x}_{i+1}) \Big) \right],
\end{aligned}$$

where each $z_i,\ i = 1, 2, \ldots, nN$ is given by (3), $Q(z_{i+1}^{i+j} | z_{i-k}^{i}) \triangleq \Pr\{Z_{i+1} = z_{i+1}, Z_{i+2} = z_{i+2}, \ldots, Z_{i+j} = z_{i+j} | Z_i = z_i, \ldots, Z_{i-k} = z_{i-k}\},\ i, j, k \in \{1, 2, \ldots, nN - 1\},\ i + j \leq nN,\ i - k \geq 1$, and $P(\mathbf{x}_i) \triangleq \Pr\{\mathbf{X}_i = \mathbf{x}_i\}$ is the probability mass function (pmf) of the n-tuple codewords.

Note that since the NBNDC-QB is Markovian with memory order M, for $nN \geqslant M$ (which is always the considered case since N is assumed to be large) it can be shown that (7) is equivalent to

$$\begin{aligned}\hat{\mathbf{x}}^N &= \arg\max_{\mathbf{x}^N}\{\log[P^{(n)}_{\text{NBNDC-QB}}(z_1^n)p(\mathbf{x}_1)] + \\ &\sum_{i=1}^{N-1}\log[Q(z_{in+1}^{(i+1)n} \mid z_{in-(M-1)}^{in})p(\mathbf{x}_{i+1})]\},\end{aligned} \tag{8}$$

where

$$Q(z_{j+1}^{j+n} \mid z_{j-(M-1)}^{j}) = \tag{9}$$
$$\prod_{i=j+1}^{j+n}\left[\left(\sum_{\ell=i-(M-1)}^{i-1}\delta_{z_i,z_\ell}+\alpha\delta_{z_i,z_{i-M}}\right)\times\frac{\varepsilon}{M-1+\alpha}+(1-\varepsilon)\rho_{z_i}\right],$$

which is obtained from (4), $z_i \triangleq 0$ if $i < 1$, $z_i^j = (z_i, z_{i+1}, \dots, z_j), j \geq i$, $P^{(n)}_{\text{NBNDC-QB}}(z_1^n) = \Pr\{Z_1^n = z_1^n\}$ is given via (17) of [13], and each z is related to its corresponding symbols x, y via (3).

In view of (8) and (9), the MAP detection can be implemented using a modified version of the Viterbi algorithm. We consider the state space to be the set of all possible n-tuple codewords. Therefore, the trellis has 2^n states, each having 2^n incoming and outgoing branches and the path metric at step i is

$$\log[Q(z_{in+1}^{(i+1)n} \mid z_{in-(M-1)}^{in})P(\mathbf{x}_i)].$$

When the source has memory, we assume that it forms a discrete Markov chain of order 1 with state transition probability matrix $P(\mathbf{x}_i \mid \mathbf{x}_{i-1})$, and the path metric will be updated to

$$\log[Q(z_{in+1}^{(i+1)n} \mid z_{in-(M-1)}^{in})P(\mathbf{x}_i \mid \mathbf{x}_{i-1})].$$

The pmf $P(\mathbf{x}_i)$ and state transition matrix $[P(\mathbf{x}_i \mid \mathbf{x}_{i-1})]$ of the source codewords are calculated from a training set of symbols (the same training set used for designing the SQ).

A special case of the MAP decoder

It is useful to know when it is possible to replace the MAP detector with an instantaneous (symbol-by-symbol) decoding rule, without sacrificing the system's optimality in terms of the probability of sequence error.

The answer to this question is partly given in [17], for $n = M = q = 1$. To be more specific, for $q = 1$ the NBNDC model is identical to the queue based channel (QBC) model which is introduced in [8]. It is shown there that for $\alpha = 1$ (which is the case here since $M = 1$), the channel reduces to the binary Markov channel introduced in [18]. Theorem 1 of [17] states necessary and sufficient conditions for the MAP decoder to be *useless* over a binary Markov channel and for binary Markov sources. In this case, a MAP decoder is defined to be useless when it decodes what is sees (i.e., $\hat{X}^N = Y^N$). As a result, [17] shows that under certain conditions, it is optimal to skip the MAP decoder and believe in what is seen at the receiver. Note that skipping the decoder can only be applied for $q = 1$ where the output sequence is also binary. On the other hand, for $q \geq 2$, the received sequence is not binary; hence, we use a mapping $\theta(y_i) = \tilde{y}_i,\ y_i \in \mathcal{Y},\ \tilde{y}_i \in \mathcal{X}$, in order to convert the 2^q-ary received sequence Y^N, into a binary sequence $\tilde{Y}^N$, using the rule

$$\tilde{y}_i = \begin{cases} 0, & \text{if } y_i < 2^{q-1}; \\ 1, & \text{if } y_i \geq 2^{q-1}. \end{cases}$$

We note that θ is the optimum instantaneous symbol-by-symbol detector for a symmetric Markov source. Since θ is also (trivially) defined for $q = 1$ ($\tilde{y} = y$), Theorem 1 of [17] yields necessary and sufficient conditions for the mapping θ to be an optimal sequence detection rule for $q = 1$. We herein establish the following theorem which gives a necessary and sufficient condition for the mapping θ to be an optimal sequence detection rule for $q > 1$, where the Markov source is binary symmetric and $n = 1$.

Theorem 1: For a symmetric binary Markov source with $p_{00} = p_{11} \in [\frac{1}{2}, 1]$, where $p_{ij} = \Pr\{X_n = j | X_{n-1} = j\},\ i, j \in \{0, 1\}$, and the NBNDC-QB with correlation parameter $\varepsilon \geq 0$, memory order $M = 1$, $q > 1$, and satisfying $\rho_0 \geq \rho_1 \geq \rho_2 \geq \dots \geq \rho_{2^q-1}$, assume that sequence length $N \geq 3$, $X_1 = \tilde{Y}_1$, and $X_N = \tilde{Y}_N$. Then $\hat{X}^N = \tilde{Y}^N$ is an optimal sequence MAP detection rule if and only if

$$\frac{\rho_{2^{q-1}-1}}{\rho_{2^{q-1}}} \times \left[\frac{1-p_{00}}{p_{00}}\right]^2 \geq 1, \tag{10}$$

where $\tilde{Y}^N = \theta(Y^N)$ is obtained via applying the mapping θ component-wise to Y^N.

Theorem 1 is illustrated in Table I for a binary symmetric Markov source with $p_{00} = 0.6$ and 0.7, where C is the left-hand term of (10). In the table, the NBNDC-QB's one-dimensional noise distribution is calculated by matching it to that of the underlying DFC; i.e., by setting $\rho_j = P^{(1)}_{\text{DFC}}(j)$ as given in (6) in terms of SNR, q and δ, where the values of δ are chosen so that the capacity of the DFC is maximized. (This is also done in the numerical results section below.) From the table we clearly observe that when $C < 1$ the MAP decoder is performing better than the mapping θ, while for the cases with $C \geq 1$ the MAP decoder and the instantaneous mapping θ are performing identically.

IV. Numerical Results

We now present numerical results on the performance of the described communication system over both the NBNDC-QB model and the underlying Rayleigh DFC.

Several source distributions are tested, including memoryless (i.i.d.) Gaussian and Laplacian sources and correlated Gauss-Markov sources. All of the sources have zero mean and unit variance. The correlated source is modeled via a Markov process of first-order : $V_i = \phi V_{i-1} + U_i$ where $\phi \in (-1, 1)$ is the correlation parameter and $\{U_i\}$ is a Gaussian i.i.d. process.

For each simulation, the SQ training and statistics collection is done over a set of 10^6 source symbols. For testing, independently generated $N = 10^5$ source symbols are transmitted and the signal-to-distortion ratio (SDR) per source symbol is calculated. We ran each simulation 10 times and took average for ensuring consistent results.

A. Exploiting memory and soft-decision quantization

Table II depicts simulation results (in dB) for different sources over the NBNDC-QB model with several parameters of SNR, SQ codeword length n, noise correlation Cor, and soft-decision resolution q.

Memoryless sources

As can be seen from the table, the performance of a system with high noise correlation can be significantly better than a system working over a fully-interleaved (Cor $= 0$) channel. For example, more than 2.2 dB of SDR gain is obtained for memoryless Gaussian sources at $q = 2$, $n = 3$, SNR $= 10$. Furthermore, for $n = 1$ since the quantized codewords form a symmetric i.i.d. source, the results illustrate Theorem 1 of [17] and Theorem 1 of Section III (compare the results of Tables II and III for $n = 1$). Considerable gain (up to 2.25 dB) are also obtained by increasing the quantizer resolution to $q = 2$ (at $n = 3$, SNR $= 5$, Cor $= 0.9$ for Laplacian sources).

Gauss-Markov sources

For Gauss-Markov sources, we have up to 2.6 dB SDR gain (at $q = 2$, $n = 3$, SNR $= 2$), by exploiting the noise correlation instead of interleaving the channel. As can be seen, in general better performance is observed when channel is highly correlated.

At low rates, especially at $n = 1$, the SDR performance for the correlated channel is worse than that for the uncorrelated channel. This behavior was expected for $n = 1$ and $q = 1$ using Corollary 3 of [17]. According to this theorem and the numerical results, for the correlated channel, the source memory has a mismatch with the channel memory. As a result, increasing the channel noise correlation will also increase the mismatch between the source and channel memory information. This makes the SQ-MAP perform worse on correlated channels than over uncorrelated channels. However, this mismatch does not occur for higher rates ($n > 2$) and the SDR performance of the system significantly improves with increasing channel noise correlation.

It is also observed that system gains up to 2.8 dB (at $n = 3$, SNR $= 2$, Cor $= 0.9$ for Gauss-Markov sources) using only a 2-bit soft-decision quantizer in the receiver over a hard-decision quantizer ($q = 1$). More gain is obtained for a 3-bit quantizer.

B. Validating the NBNDC-QB model

We next assess how well the NBNDC-QB model can approximate the correlated Rayleigh DFC in terms of SDR performance of the SQ-MAP system.

For a given DFC (with fixed SNR and $f_D T$) and q, we choose the value of δ that maximizes the DFC's capacity. We also choose the parameters of the NBNDC-QB, $(\rho_0, \rho_1, \cdots, \rho_{2^q-1})$, M, ϵ and α, so that the two channel models are as close to each other as possible. We have used the values given in [19] in which the Kullback-Leibler (KL) divergence rate between the two channel (2^q-ary) noise processes is minimized over M, ε, α for $f_D T \in \{0.005, 0.01\}$, $\mathrm{SNR}_{(dB)} \in \{2.0, 5.0, 10.0, 15.0\}$, $q = 2$, $\rho_j = P^{(1)}_{DFC}(j)$ from (5), and the δ value which maximizes the DFC capacity.

To simulate the Rayleigh DFC, we generate the fading coefficients using the modified Clarke's method introduced in [20]. Simulation results (over the NBNDC-QB and Rayleigh DFC channels) in terms of SDR are shown in Table IV for memoryless i.i.d. Gaussian sources. Both systems use the same MAP decoder designed for the NBNDC-QB.

Comparing the performance of the system for the two channels, we observe that for lower rates (codeword lengths $n = 1$ and 2), there is a good conformity between the results for the two channel models. This agreement in SDR performance can be heuristically explained by noting that for low rates ($n = 1$ and 2), the SQ output sent to the channel input is nearly i.i.d. uniform. But the NBNDC-QB and DFC channels were matched by minimizing the divergence rate between their noise processes. Hence, when both channels are driven by the same capacity-achieving input (which must be i.i.d. uniform as both channels are symmetric), they will then have a similar probability of error performance in addition to nearly identical capacities. The same agreement in SDR performance is also observed for memoryless Laplacian and Gauss-Markov sources for $n = 1$. We finally note that for $n \geq 3$, some disagreement in SDR performance is observed between the two systems (in this case the SQ output is not i.i.d. uniform).

V. Conclusion

We designed a scalar quantizer MAP decoding system for the recently introduced channel model NBNDC-QB. We provided a necessary and sufficient condition for a specific case of the SQ-MAP over the NBNDC-QB, under which the MAP decoder can be replaced with an instantaneous symbol-by-symbol decoder. The system was tested for both correlated and uncorrelated source distributions and numerical results show that the proposed system can successfully use memory and soft-decision information over the NBNDC-QB channel model. Finally, the channel model was compared to the Rayleigh DFC, in terms of SDR, and it was shown to be a useful practical model for designing systems which are intended to be used over Rayleigh DFCs.

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TABLE I

SYMBOL ERROR RATE (IN%) FOR MAP DECODING AND INSTANTANEOUS MAPPING θ FOR SYMMETRIC BINARY MARKOV SOURCES WITH $p_{00} = 0.6, 0.7$. THE CHANNEL MODEL IS THE NBNDC-QB, WITH $M = 1$, $Cor = 0.0$, AND $q = 2, 3$. THE VALUES C ARE CALCULATED FROM (10). THE δ VALUES FOR SNRS (15,10,5,2) ARE (0.12,0.20,0.40,0.50) FOR $q = 2$ AND (0.06,0.11,0.18,0.25) FOR $q = 3$, RESPECTIVELY.

p_{00}	q	SNR (dB) 15		10		5		2	
		MAP	θ	MAP	θ	MAP	θ	MAP	θ
0.6	2	$C = 1.39 > 1$		$C = 1.25 > 1$		$C = 1.27 > 1$		$C = 1.03 > 1$	
		0.76	0.76	2.30	2.30	6.43	6.43	10.85	10.85
	3	$C = 0.80 < 1$		$C = 0.80 < 1$		$C = 0.73 < 1$		$C = 0.68 < 1$	
		0.73	0.76	2.22	2.30	6.21	6.43	10.51	10.85
0.7	2	$C = 0.57 < 1$		$C = 0.52 < 1$		$C = 0.52 < 1$		$C = 0.42 < 1$	
		0.64	0.76	1.91	2.30	5.55	6.43	9.50	10.85
	3	$C = 0.33 < 1$		$C = 0.33 < 1$		$C = 0.30 < 1$		$C = 0.28 < 1$	
		0.62	0.76	1.90	2.30	5.37	6.43	9.27	10.85

TABLE II

SQ-MAP TRAINING SDR RESULTS (IN DB) FOR MEMORYLESS NBNDC-QB AND HIGHLY CORRELATED NBNDC-QB WITH PARAMETERS $\alpha = 1$; G: MEMORYLESS GAUSSIAN SOURCE, L: MEMORYLESS LAPLACIAN SOURCE , GM: GAUSS-MARKOV SOURCE WITH $\phi = 0.9$.

Sou-rce	q	n	Memoryless (Cor=0) SNR (dB) 15	10	5	2	Cor=0.9 SNR (dB) 15	10	5	2
G	1	1	4.17	3.75	2.78	1.94	4.19	3.77	2.85	1.97
		2	8.15	6.49	3.85	2.14	8.37	6.89	4.47	2.84
		3	11.05	7.80	4.02	1.93	11.58	8.43	4.76	2.76
	2	1	4.17	3.75	2.78	1.94	4.19	3.77	2.85	1.97
		2	8.15	6.49	3.85	2.14	8.69	7.68	5.61	4.03
		3	11.10	7.94	4.33	2.53	12.61	10.15	6.64	4.51
L	1	1	2.87	2.62	2.00	1.44	2.88	2.63	2.05	1.45
		2	6.65	5.27	2.91	1.30	6.89	5.88	4.28	3.21
		3	9.59	6.49	2.72	0.58	10.14	7.64	4.88	3.34
	2	1	2.87	2.62	2.00	1.44	2.88	2.63	2.05	1.45
		2	6.69	5.42	3.32	2.01	7.26	6.72	5.53	4.47
		3	9.90	7.09	3.81	2.06	11.59	9.86	7.14	5.35
GM	1	1	4.21	3.78	3.74	3.22	4.23	3.81	2.89	2.01
		2	8.95	8.29	6.97	6.34	8.94	8.24	6.88	5.71
		3	13.38	11.84	9.46	7.52	13.89	12.69	10.43	8.69
	2	1	4.38	4.23	3.89	3.61	4.35	4.14	3.44	2.65
		2	9.16	8.81	8.00	6.98	9.24	8.97	8.26	7.36
		3	13.98	12.87	10.72	8.91	14.47	14.02	12.88	11.51

TABLE III

SQ WITH INSTANTANEOUS MAPPING- TRAINING SDR RESULTS (IN DB) FOR $n = 1$ AND THE MEMORYLESS NBNDC-QB AND THE HIGHLY CORRELATED NBNDC-QB WITH PARAMETERS $M = 1$, $\alpha = 1$; G: MEMORYLESS GAUSSIAN SOURCE, L: MEMORYLESS LAPLACIAN SOURCE, GM: GAUSS-MARKOV SOURCE WITH $\phi = 0.9$.

Source	q	Memoryless (Cor=0) SNR (dB) 15	10	5	2	Cor=0.9 SNR (dB) 15	10	5	2
G	1	4.17	3.75	2.78	1.94	4.19	3.77	2.85	1.97
	2	4.17	3.75	2.78	1.94	4.19	3.77	2.85	1.97
L	1	2.87	2.62	2.00	1.44	2.88	2.63	2.05	1.45
	2	2.87	2.62	2.00	1.44	2.88	2.63	2.05	1.45
GM	1	4.21	3.78	2.82	1.97	4.23	3.81	2.89	2.01
	2	4.21	3.78	2.82	1.97	4.23	3.81	2.89	2.01

TABLE IV

SQ-MAP SIMULATION SDR RESULTS (IN DB) FOR THE DFC-FITTED NBNDC-QB AND THE DFC; MEMORYLESS GAUSSIAN SOURCE, $q = 2$.

Channel model	$f_D T$	n	SNR (dB) 15 Cor=0.35	10 Cor=0.32	5 Cor=0.29	2 Cor=0.22
NBNDC-QB	0.005	1	4.18	3.76	2.77	1.94
		2	8.34	6.73	4.01	2.29
	0.01	1	4.17	3.75	2.80	1.94
		2	8.34	6.75	4.07	2.28
Rayliegh DFC	0.005	1	4.17	3.75	2.78	1.94
		2	8.15	6.52	3.91	2.18
	0.01	1	4.18	3.75	2.79	1.95
		2	8.17	6.51	3.88	2.18

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