Error Analysis of Space-Time Codes for Slow Rayleigh Fading Channels

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I. Introduction

We consider a communication system which employs L_T transmit and L_R receive antennas. The channel is assumed to be quasi-static Rayleigh flat fading. It is assumed that only the receiver has knowledge of the path gains. The additive noise at receiver j at symbol interval t, N_t^j, is assumed to be complex Gaussian with i.i.d. real and imaginary parts. Linear propagation is assumed, so that for a CSNR of γ_s at each receive antenna, the signal at receive antenna j can be written as $R_t^j = \sqrt{\frac{\gamma_s}{L_T}} \sum_{i=1}^{L_T} H_{ji} s_t^i + N_t^j$, where $\sqrt{\frac{\gamma_s}{L_T}} s_t^i$ is the signal sent from antenna i. $\boldsymbol{H}$ is the $L_R \times L_T$ path gains matrix. In the following, we consider the pairwise error probability (PEP) of space-time orthogonal block (STOB) codes, and then generalize the solution to space-time trellis (STT) codes, linear dispersion (LD) codes, and BLAST with ML decoding.

II. Exact PEP of STOB Codes

Using the properties of the Laplace transform, we derive the following expression for the exact PEP of ML decoded STOB codes:

$$P(\hat{c} = c_j | c = c_i) = \frac{1}{2}\left(1 - \frac{\delta_{ij}}{\sqrt{\delta_{ij}^2 + 2}} \sum_{k=0}^{n-1} \binom{2k}{k} \frac{1}{(2\delta_{ij}^2 + 4)^k}\right),$$

where $n = L_t L_R$ and $\delta_{ij} = \sqrt{\frac{g\gamma_s}{2L_T}} |c_i - c_j|$ (g is the inverse of the code rate). The above equation can also be derived using the error analysis results of MRC systems, as noticed in [1].

III. Exact PEP of STT Codes

Let $\mathbf{S}$ and $\hat{\mathbf{S}}$ be two paths on the trellis of a STT code. Also, define $d_{i,t} = s_t^i - \hat{s}_t^i$ and matrix $\mathbf{U}$ with elements $u_{k,i} = \sum_t d_{i,t} d_{k,t}^*$. Let us assume that $\mathbf{U}$ has K distinct non-zero eigenvalues λ_k each with multiplicity n_k. Setting $\delta_k^2 = \frac{1}{p_k} = \frac{\gamma_s \lambda_k}{2L_T}$ and denoting the residues of $\Phi_{\frac{1}{2}\Delta^2_{\mathbf{s},\hat{\mathbf{s}}}}(-s) = \prod_{k=1}^K \frac{1}{(1+\frac{s}{p_k})^{L_R n_k}}$ by $\alpha_{i,k}$, we get the pairwise error probability of STT codes as

$$P_{\mathbf{S}\to\hat{\mathbf{S}}} = \sum_{k=1}^{K} \sum_{i=1}^{L_R n_k} \frac{\beta_{i,k}}{2}\left(1 - \frac{\delta_k}{\sqrt{\delta_k^2 + 2}} \sum_{j=0}^{i-1} \binom{2j}{j} \frac{1}{(2\delta_k^2 + 4)^j}\right),$$

where $\beta_{i,k} = \delta_k^{2i} \alpha_{i,k}$. Since the channel is quasi-static, the same expression can be used for any other space-time codes (such as LD codes and V-BLAST). Considering error paths of length 2, a simple expression in the form of the union bound is used to approximate the system bit error rate (BER). This is plotted in Figure 1 for the first 4-state Q-PSK STT code.

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IV. Error Rate Bounds of STOB Codes

The exact PEP expression and the method used for its derivation can be used to obtain very tight Bonferonni-type upper and lower bounds [2] on the symbol error rate (SER) and BER for STOB codes. Numerical results, such as Figure 2, show that the bounds provide very good estimates on the system performance. In many cases, the upper and lower bounds coincide even at low channel signal to noise ratios and large constellations.

References

[1] G. Bauch, J. Hagenauer, and N. Seshadri, "Turbo processing in transmit antenna diversity systems," *Annals Telecommun.*, vol. 56, pp. 455-471, Aug. 2001.

[2] H. Kuai, F. Alajaji, and G. Takahara, "Tight error bounds for nonuniform signaling over AWGN channels," *IEEE Trans. Inform. Theory*, vol. 46, pp. 2712-2718, Nov. 2000.

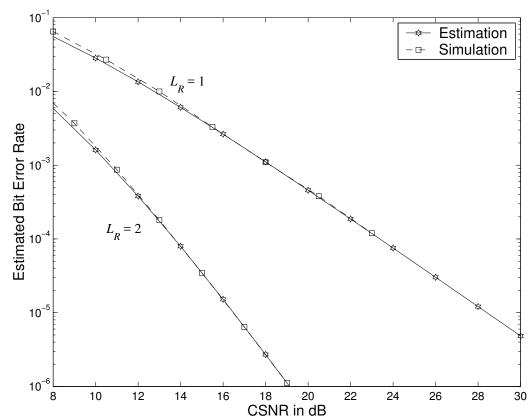

Figure 1: Results for the 4-state Q-PSK STTC.

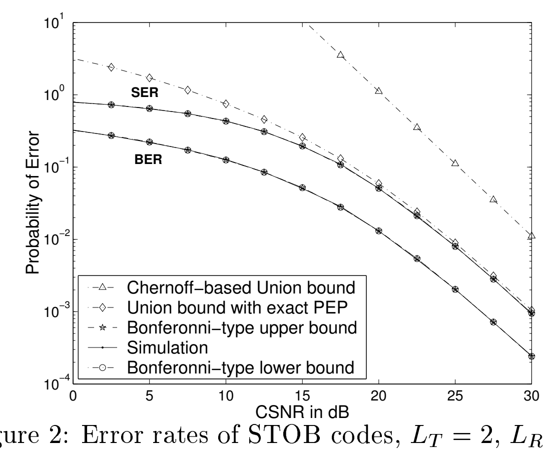

Figure 2: Error rates of STOB codes, $L_T = 2$, $L_R = 1$, and 16-PSK constellation.