A Discrete Queue-Based Model for Soft-Decision Demodulated Correlated Fading Channels

Cecilio Pimentel*, Fady Alajaji†, Pedro Melo*

* Department of Electronics and Systems, Federal University of Pernambuco
Recife, PE, 50711-970, Brazil – Email: cecilio@ufpe.br, pmelo@hotmail.com

† Department of Mathematics and Statistics, Queen's University
Kingston, ON K7L 3N6, Canada – Email: fady@mast.queensu.ca

***Abstract*—A discrete fading channel (DFC) consisting of a binary modulated time-correlated Rayleigh fading channel used in conjunction with coherent soft-decision demodulation of resolution q is considered. The capacity of the binary input 2^q-ary output DFC, which can be explicitly expressed in terms of a non-binary noise discrete channel with stationary ergodic 2^q-ary noise, is evaluated in terms of q and the fading parameters. It is observed that considerable capacity gains can be achieved due to the channel's statistical memory and the use of as few as 2 bits for soft-decision over interleaving the channel (to render it memoryless) and hard-decision demodulation ($q = 1$). The DFC is next fitted by a recently introduced analytically tractable queue-based (QB) Markovian noise model. The QB parameters are estimated via an iterative procedure that minimizes the Kullback-Leibler divergence rate between the DFC and QB noise sources. Modeling results, measured in terms of both channel noise correlation function and capacity reveal a good agreement between the two channels for a broad range of fading conditions.**

I. Introduction

Binary finite-state Markov channels (FSMCs) have been extensively used to model the correlation structure of the error process of discrete time-correlated fading channels (from the input to the binary modulator to the output of the hard-quantized demodulator) [1]- [3]. An accurate FSMC provides an analytical description of the communication channel with memory that can be used to evaluate the performance of coded systems over such channels [4] as well as to design decoding strategies that exploit the channel statistical memory [5].

Motivated by well known results that soft-decision information can increase the capacity of several classes of channels [6], [7], a discrete communication channel with binary input and 2^q-ary output was recently introduced in [8] in order to capture both the statistical memory and the soft-decision information of binary phase-shift keying (BPSK) modulated time-correlated fading channels when they are coherently demodulated via a q-bit scalar quantizer. This channel is referred to as the non-binary noise discrete channel (NBNDC). It is shown in [8] that the output process of this channel can be expressed as an explicit simple function of the channel binary input process and a 2^q-ary noise process. A non-binary Markovian stationary ergodic queue-based (QB) noise source is also proposed and analyzed in [8] to model the noise process of the NBNDC. The resulting NBNDC with Markovian QB noise is an FSMC fully described by $2^q + 2$ independent parameters and generalizes the binary queue-based channel (QBC) proposed in [9]. This QB noise model inherits nice properties from the QBC such as a closed-form expression for its block transition probability and capacity.

In [10], a numerical modeling study demonstrates that the QBC provides a good approximation of binary input hard-decision demodulated time-correlated Rayleigh and Rician fading channels. The objective of this paper is to investigate the appropriateness of the NBNDC with non-binary QB noise in modeling a time-correlated discrete Rayleigh fading channel with soft decision demodulation.

We consider a discrete fading channel (DFC) composed of a binary BPSK modulator, a time-correlated flat Rayleigh fading channel and a q-bit soft-quantized coherent demodulator. We first evaluate numerically the effect of the quantizer resolution and channel correlation parameters on the capacity of the DFC. The capacity gains indicate that exploiting both the channel's memory and soft decision information is more worthwhile than ignoring either of them using channel interleaving or hard quantization. Next, we model the DFC via an NBNDC with QB noise. The 2^q+2 parameters of the NBNDC with QB noise are selected to minimize the Kullback-Leibler divergence rate between the DFC and the QB noise processes. The accuracy of the QB noise model is then measured in terms of the channel noise autocorrelation function and channel capacity. A good fit is obtained for a wide choice of fading conditions.

II. Discrete Fading Channel

A. DFC: Soft-Decision Demodulated Fading Channel

We define the input and output alphabets of the DFC by $\mathcal{X} = \{0, 1\}$, $\mathcal{Y} = \{0, 1, \cdots, 2^q - 1\}$, respectively. Let $\{X_k\}$, where $X_k \in \mathcal{X}$ for $k = 1, 2, \cdots$, be the DFC binary input process. The coherently demodulated received channel symbol at the kth signaling interval is given by

$$R_k = \sqrt{E_s} A_k S_k + N_k, \qquad k = 1, 2, \cdots,$$

where E_s is the energy of the transmitted signal, $S_k = 2X_k - 1$ is the BPSK input symbol taking values in $\{-1, +1\}$, and $\{N_k\}$ is the noise process consisting of a sequence of independent and identically distributed Gaussian random variables,

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each with zero-mean and variance $N_0/2$. Furthermore, $\{A_k\}$ is the channel's fading process with $A_k = |G_k|$, where $\{G_k\}$ is a time-correlated complex wide-sense stationary Gaussian process with zero-mean and autocorrelation function given by the Clarke's fading model [11] $R[k] = J_0(2\pi f_D T|k|)$, where $J_0(x)$ is the zero-order Bessel function of the first kind, and $f_D T$ is the maximum Doppler frequency normalized by the signaling rate $1/T$. As a result, each fading random variable A_k is Rayleigh distributed with unit second moment. The fading and noise processes are assumed to be independent of each other and of the input process.

Each received symbol R_k is next softly quantized via a q-bit scalar quantizer with normalized step-size δ yielding a DFC output $Y_k \in \mathcal{Y}$ according to the following operation

$$Y_k = j, \quad \text{if} \;\; R_k \in (T_{j-1}, T_j)$$

for $j \in \mathcal{Y}$ and the quantizer thresholds T_l satisfy [8]

$$T_l = \begin{cases} -\infty, & \text{if } l = -1 \\ (l + 1 - 2^{q-1})\delta, & \text{if } l = 0, 1, \cdots, 2^q - 2 \\ \infty, & \text{if } l = 2^q - 1. \end{cases}$$

Let $q_{i,j}(a_k) \triangleq \Pr(Y_k = j \mid X_k = i, A_k = a_k)$. Due to the symmetry of the BPSK constellation and the quantizer thresholds, we have that

$$q_{i,j}(a_k) = q_{0, \frac{j-(2^q-1)i}{(-1)^i}}(a_k).$$

The DFC with binary input process $\{X_k\}$ and 2^q-ary output process $\{Y_k\}$ is thus specified by the following sequence of n-fold (block) conditional probabilities for $n \geq 1$:

$$\begin{aligned} P^{(n)}_{\text{DFC}}(y^n \mid x^n) &\triangleq \Pr(Y^n = y^n \mid X^n = x^n) \\ &= \mathbf{E}_{A_1 \dots A_n}\left[\prod_{k=1}^{n} q_{0, \frac{y_k-(2^q-1)x_k}{(-1)^{x_k}}}(A_k)\right] \quad (1) \end{aligned}$$

where $y^n = (y_1, \cdots, y_n)$, $x^n = (x_1, \cdots, x_n)$ and $\mathbf{E}_X[\cdot]$ denotes expectation with respect to the random variable X. For $n = 1$, a closed-form expression for $P^{(1)}_{\text{DFC}}(y|x)$, $y \in \mathcal{Y}$ and $x \in \mathcal{X}$, is given by [12]

$$P^{(1)}_{\text{DFC}}(y|x) = m(-T_{j-1}) - m(-T_j) \quad (2)$$

where $j = \frac{y-(2^q-1)x}{(-1)^x} \in \mathcal{Y}$,

$$m(T_j) = 1 - Q(T_j\sqrt{2\gamma}) - \frac{\left[1 - Q\left(\frac{T_j\sqrt{2\gamma}}{\sqrt{\frac{1}{\gamma}+1}}\right)\right] e^{-\frac{T_j^2}{\left(\frac{1}{\gamma}+1\right)}}}{\sqrt{\frac{1}{\gamma}+1}}$$

and $\gamma = E_s/N_0$ is the signal-to-noise ratio (SNR) and $Q(x) = 1/\sqrt{2\pi}\int_x^\infty \exp\{-t^2/2\}dt$ is the Gaussian Q-function. The expected value in (1) can only be calculated for small values of n since the joint probability density function of arbitrarily correlated Rayleigh random variables is only known for $n \leq 3$ (e.g., see [13]), however the overall conditional probability in (1) can be determined via simulations. Hence, it is important to provide an effective model for $P^{(n)}_{\text{DFC}}(\cdot \mid \cdot)$.

B. DFC as an Non-Binary Noise Discrete Channel

Consider a binary-input 2^q-ary output channel, which we refer to as the non-binary noise discrete channel (NBNDC), where the output $Y_k \in \mathcal{Y}$ is explicitly expressed in terms of the input $X_k \in \mathcal{X}$ and a noise $Z_k \in \mathcal{Y}$ via

$$Y_k = (2^q - 1)X_k + (-1)^{X_k} Z_k \quad (3)$$

for $k = 1, 2, \cdots$, where the noise process $\{Z_k\}$ is independent of the input process $\{X_k\}$ and is governed by the n-fold distribution $P^{(n)}_{\text{NBNDC}}(z^n) \triangleq P^{(n)}_{\text{NBNDC}}(Z_1 = z_1, \cdots, Z_n = z_n)$ for $z_t \in \mathcal{Y}$, $t = 1, \cdots, n$. The channel n-fold conditional probability is given for each $n \geq 1$ by

$$P^{(n)}_{\text{NBNDC}}(y^n \mid x^n) = P^{(n)}_{\text{NBNDC}}(z^n)$$

where

$$z_t = \frac{y_t - (2^q - 1)x_t}{(-1)^{x_t}}, \quad t = 1, \cdots, n. \quad (4)$$

Now given $x^n \in \mathcal{X}^n$ and $y^n \in \mathcal{Y}^n$, whenever $P^{(n)}_{\text{NBNDC}}(z^n)$ is set to equal to (1) for each $n \geq 1$ with each z_t as given by (4), we obtain that $P^{(n)}_{\text{DFC}}(y^n \mid x^n) = P^{(n)}_{\text{NBNDC}}(y^n \mid x^n)$ for each $n \geq 1$. Thus the NBNDC provides an alternative description of the DFC.[1] In Sections IV and V, we use this fact to fit the DFC given by (1) via an NBNDC whose noise process $\{Z_k\}$ is an Mth order Markov source generated via a non-binary queue of length M. In the next section, we conduct a numerical capacity study of the DFC to determine the optimal values for the channel q-bit quantizer step-size δ and illustrate the potential gains in capacity due to the DFC's statistical memory and the use of soft-decision output quantization.

III. DFC Capacity Numerical Study

We herein examine the behavior of the capacity of the DFC in terms of the quantizer parameters (q and δ), the SNR (γ) and the normalized Doppler frequency ($f_D T$). As shown above, the DFC is an NBNDC described by (3) with a stationary ergodic 2^q-ary noise $\{Z_k\}$ whose n-fold distribution is given by (1) for each n. An expression for the capacity of this channel is derived in [8] in terms of the entropy rates of the noise process $\{Z_k\}$ and a related process $\{W_k\}$ with alphabet $\mathcal{W} = \{0, 1, \cdots 2^{q-1} - 1\}$ defined by

$$W_k \triangleq \min\{Z_k, 2^q - 1 - Z_k\}, \quad k = 1, 2, \cdots, \quad (5)$$

with resulting n-fold distribution

$$\Pr(W^n = w^n) = \sum_{x^n \in \mathcal{X}^n} \Pr\left(Z^n = \frac{w^n - (2^q - 1)x^n}{(-1)^{x^n}}\right)$$

where $Z^n = (w^n - (2^q - 1)x^n)/(-1)^{x^n}$ denotes the n-tuple obtained from component-wise operations, i.e., $(Z_1 = (w_1 - (2^q - 1)x_1)/(-1)^{x_1}, \cdots, Z_n = w_n - (2^q - 1)x_n)/(-1)^{x_n})$. The capacity C of the NBNDC is given by [8]

$$C = \lim_{n \to \infty} C^{(n)} = \sup_n C^{(n)}$$

[1]Note that the resulting NBNDC noise $\{Z_k\}$ is stationary and ergodic since the underlying fading process $\{A_k\}$ is stationary ergodic.

where

$$C^{(n)} = 1 + \frac{1}{n}[H(W^n) - H(Z^n)] \quad (6)$$

where $H(\cdot)$ denotes entropy. Thus

$$C = 1 + \lim_{n\to\infty} \frac{1}{n}[H(W^n) - H(Z^n)] = 1 + \mathcal{H}(W) - \mathcal{H}(Z)$$

in bits/channel use, where $\mathcal{H}(W) \triangleq \lim_{n\to\infty}(1/n)H(W^n)$ and $\mathcal{H}(Z) \triangleq \lim_{n\to\infty}(1/n)H(Z^n)$ denote the entropy rates of $\{W_n\}$ and $\{Z_n\}$, respectively. Since $C^{(n)} \le C$ and $H(Z^n)/n$ is decreasing in n for a stationary process $\{Z_n\}$, we obtain the following upper and lower bounds on C

$$C^{(n)} \le C \le \min\left\{1, 1 + \frac{1}{n}H(W^n) - \mathcal{H}(Z)\right\}.$$

As (1) cannot be determined for $n > 3$, we generate a realization of the noise process $\{Z_k\}$ via computer simulations for fixed DFC parameters (γ, $f_D T$, q, δ) and calculate $\{W_k\}$ using (5). The correlated Rayleigh fading samples are generated according to the method proposed in [14]. We then evaluate $\Pr(Z^n)$ and $\Pr(W^n)$ numerically for several values of n and compute the lower bound on the capacity $C^{(n)}$ using (6).

Fig. 1 presents $C^{(n)}$ versus the quantization step δ for several values of n for a DFC with parameters $\gamma = 10$ dB, $f_D T = 0.005$ and $q = 2$. One objective is to determine the optimal value of δ (in the sense of maximizing the channel capacity). We observe that for $n \ge 5$, $C^{(n)}$ is maximized for approximately $\delta = 0.2$. The curve $C^{(1)}$ corresponds to the capacity of a memoryless DFC (the channel resulting when perfect interleaving is employed on the DFC). In this case, the optimal value of δ is 0.27. We may also obtain values of the capacity for channels with hard quantization ($q = 1$), as this channel is equivalent to a DFC with $\delta = 0$. For example, the capacity of this DFC with hard-quantized and perfectly interleaved (obtained from the curve $C^{(1)}$ with $\delta = 0$) is 0.846. We also remark that increasing n further than 7 does not improve accuracy. We denote this maximum value by $n^\star$. Table I summarizes the values of $C^{(n^\star)}$ for $q = 1, 2$ (obtained for the optimal δ shown in the table) for selected values of $n^\star$ and γ. Note that the optimal values of δ provided in the table for $q > 1$ are different from those calculated in [6] for the memoryless DFC. We finally observe capacity gains due to the channel's memory and soft-decision quantization ($q > 1$) relative to hard-quantization ($q = 1$) and ideal channel interleaving (i.e., ignoring the channel's memory). For example, for $\gamma = 5$ dB, the capacity gain of $C^{(10)}$ ($q = 2$) over $C^{(10)}$ ($q = 1$) is 13 %, whereas it is 19 % when compared to the hard-quantized memoryless channel (with $C = 0.656$). For $\gamma = 2$ dB, the gains are 18.5% and 25.5%, respectively (in this case the capacity of the memoryless channel is $C = 0.51$).

IV. Non-Binary Queue-Based Markovian Noise

We next describe a non-binary queue-based (QB) noise model for the NBNDC which is a generalization of the binary queue noise source proposed in [9]. We briefly describe the generation of the noise symbol Z_k (a detailed description

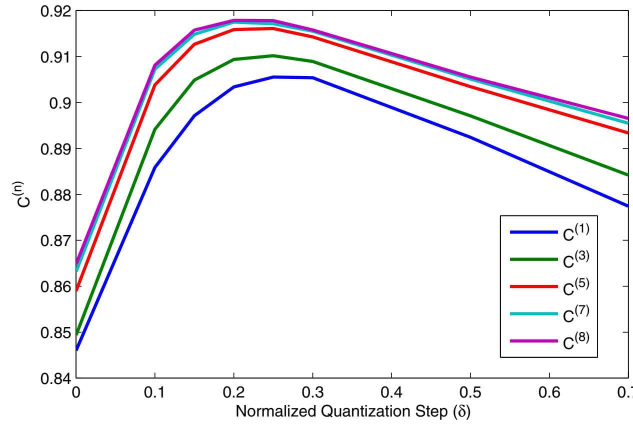

Fig. 1. $C^{(n)}$ versus the quantization step δ for different values of n; DFC with $q = 2$, $f_D T = 0.005$, $\gamma = 10$ dB.

TABLE I
Lower bound on channel capacity, $C^{(n)}$, in (bits/channel use) for DFCs with $f_D T = 0.005$.

γ	$n^\star$	$q = 1$	$q = 2$	
		$C^{(n^\star)}$	$C^{(n^\star)}$	optimal δ
2 dB	11	0.54	0.64	0.5
5 dB	10	0.689	0.78	0.4
10 dB	7	0.86	0.915	0.2
15 dB	3	0.939	0.969	0.12

is provided in [9]). First, one of two parcels (an urn and a queue of size M) is selected with probability distribution $\{\varepsilon, 1-\varepsilon\}$. The urn contains balls labeled with symbols in $\mathcal{Y}$ satisfying the probability distribution $\boldsymbol{\rho} = (\rho_0, \rho_1, \cdots, \rho_{2^q-1})$. If the urn is selected, a noise symbol $Z_k = i$ is selected with probability ρ_i, $i \in \mathcal{Y}$. If the queue is selected, a noise symbol is selected with a probability distribution that depends on M and a bias parameter α. The resulting QB noise process $\{Z_k\}_{k=1}^\infty$ is an Mth-order stationary ergodic Markov source with $2^q + 2$ independent parameters: the size of the queue, M, the probability distribution of the balls in the urn $\boldsymbol{\rho}$, and the parameters ε and α, where $0 \le \varepsilon < 1$, $\alpha \ge 0$. The state process $\{S_k\}_{k=1}^\infty$ of the QB noise, defined by $S_k \triangleq (Z_k, Z_{k-1}, \cdots, Z_{k-M+1})$, is a homogeneous first-order Markov process with state stationary distribution column vector $\boldsymbol{\Pi} = [\pi_{z^M}]$, $z^M \in \mathcal{Y}^M$, given by [8, Eq.(16)]. Several closed-form expressions for the statistical quantities of the QB noise model are herein summarized.

The QB noise n-fold distribution $P_{\text{QB}}^{(n)}(z^n) \triangleq \Pr(Z^n = z^n)$ is given by

$$P_{\text{QB}}^{(n)}(z^n) = \frac{\displaystyle\prod_{\ell=0}^{2^q-1} \prod_{m=0}^{\xi'_\ell - 1} \left((1-\varepsilon)\rho_\ell + m\frac{\varepsilon}{M-1+\alpha}\right)}{\displaystyle\prod_{k=0}^{n-1} \left((1-\varepsilon) + k\frac{\varepsilon}{M-1+\alpha}\right)} \quad (7)$$

for blocklength $n \le M$, where $\xi'_\ell = \sum_{k=1}^n \delta_{z_k,\ell}$ and $\delta_{i,j}$ is the Kronecker delta function. For blocklength $n \geq M+1$,

$$P_{\text{QB}}^{(n)}(z^n) = \prod_{i=M+1}^{n} \left[\left(\sum_{\ell=i-M+1}^{i-1} \delta_{z_i,z_\ell} + \alpha \delta_{z_i,z_{i-M}} \right) \times \frac{\varepsilon}{M-1+\alpha} + (1-\varepsilon)\rho_{z_i} \right] \pi_{(z_1,\cdots,z_M)} \quad (8)$$

where $\pi_{(z_1,\cdots,z_M)}$ is the stationary distribution given in [8, Eq.(16)]. We conclude from (7) that the one-dimensional distribution for Z_k is $\Pr(Z_k = z) = \rho_z$ for $z \in \mathcal{Y}$. The correlation coefficient for the QB noise is a non-negative quantity given by

$$\text{Cor}_{\text{QB}} = \frac{\mathbf{E}[Z_k Z_{k+1}] - \mathbf{E}[Z_k]^2}{\text{Var}(Z_k)} = \frac{\frac{\varepsilon}{M-1+\alpha}}{1 - \frac{(M-2+\alpha)\varepsilon}{M-1+\alpha}} \quad (9)$$

where $\text{Var}(Z_k)$ denotes the variance of Z_k. The autocorrelation function (ACF), defined as $R[m] = \mathbf{E}[Z_k Z_{k+m}]$, satisfies the formula at the top of the next page. Finally, the entropy rate of the QB noise is established in closed-form in [8].

V. DFC Modeling via the NBNDC with QB Noise

In the following, we fit the DFC using the NBNDC with QB noise. For this purpose, given a DFC with fixed parameters, we estimate the 2^q+2 independent parameters of the QB noise process such that the QB noise block probability approximates well the DFC channel block probability of (1). Specifically, we select the parameters of the QB noise that minimize the Kullback-Leibler divergence rate (KLDR) between the QB and the DFC noise processes defined as

$$D(P_{\text{DFC}}^{(n)} || P_{\text{QB}}^{(n)}) \triangleq \lim_{n\to\infty} \frac{1}{n} \sum_{z^n \in \mathcal{Y}^n} P_{\text{DFC}}^{(n)}(z^n) \log_2 \frac{P_{\text{DFC}}^{(n)}(z^n)}{P_{\text{QB}}^{(n)}(z^n)}$$

for identical one-dimensional probability distributions and noise correlation coefficients (we match the lower order statistics for both processes). Closed-form expressions for $P_{\text{QB}}^{(n)}(z^n)$ are given in (7) and (8), while $P_{\text{DFC}}(z^n)$ of (1) is calculated via computer simulations for $n > 1$. The minimization of the asymptotic KLDR quantity assures that both processes are statistically close for large blocklengths. Since the DFC noise process is stationary and the QB noise process is M'th order Markovian, the minimization of the KLDR over the QB noise parameters reduces to minimizing [10]

$$D_2^M \triangleq - \sum_{z^{M+1}} P_{\text{DFC}}^{(M+1)}(z^{M+1}) \log_2 P_{\text{QB}}(z_{M+1} \mid z^M) \quad (10)$$

where $P_{\text{QB}}(z_{M+1} \mid z^M)$ is the QB conditional probability of the noise symbol z_{M+1} given the previous M symbols, which is evaluated using the QB noise block probability (8) and is given by

$$P_{\text{QB}}(z_{M+1} \mid z^M) = \left(\alpha\, \delta_{z_{M+1},z_1} + \sum_{\ell=2}^{M} \delta_{Z_{M+1},z_\ell} \right) \frac{\varepsilon}{M-1+\alpha} + (1-\varepsilon)\rho_{z_{M+1}}. \quad (11)$$

We match the one-dimensional probability distribution by setting $\rho_j = P_{\text{DFC}}^{(1)}(j)$, $j = 0, \cdots, 2^q - 1$, where $P_{\text{DFC}}^{(1)}(j)$ is given by (2) in terms of the quantization parameters (δ and q) and γ. The remaining parameters (M, ε, α) are estimated as follows. We compute Cor_{DFC} and match the noise correlation coefficient $\text{Cor}_{\text{QB}} = \text{Cor}_{\text{DFC}}$. From (9) we can write the parameter α as

$$\alpha = \frac{\varepsilon + \text{Cor}_{\text{DFC}}(1-M) + \text{Cor}_{\text{DFC}}(M-2)\varepsilon}{\text{Cor}_{\text{DFC}}(1-\varepsilon)}. \quad (12)$$

For fixed DFC parameters, we substitute (12) into (11) and the result into (10) and find the value of ε that minimizes (10) for each value of M. For that purpose, we apply the Newton-Raphson's method to the derivative of (10), resulting in the following iterative procedure for estimating ε. Given an iteration point ε_n, the next iteration point is

$$\varepsilon_{n+1} = \varepsilon_n + \frac{\sum_{z^{M+1}} P_{\text{DFC}}(z^{M+1}) \frac{A_{z^{M+1}}}{A_{z^{M+1}}\varepsilon_n + B_{z^{M+1}}}}{\sum_{z^{M+1}} P_{\text{DFC}}(z^{M+1}) \frac{A_{z^{M+1}}^2}{(A_{z^{M+1}}\varepsilon_n + B_{z^{M+1}})^2}}$$

where

$$A_{z^{M+1}} = [1 + \text{Cor}_{\text{DFC}}(M-2)]\delta_{z_{M+1},z_1} - \left(\sum_{\ell=2}^{M} \delta_{z_{M+1},z_\ell} \right) \text{Cor}_{\text{DFC}} - (1 - \text{Cor}_{\text{DFC}})\rho_{z_{M+1}}$$

and

$$B_{z^{M+1}} = \left(\delta_{z_{M+1},z_1}(1-M) + \sum_{\ell=2}^{M} \delta_{z_{M+1},z_\ell} \right) \text{Cor}_{\text{DFC}} + (1 - \text{Cor}_{\text{DFC}})\rho_{z_{M+1}}.$$

From the constraint that $\alpha \geq 0$, we have from (12) that

$$\frac{\text{Cor}_{\text{DFC}}(M-1)}{1 + \text{Cor}_{\text{DFC}}(M-2)} \leq \varepsilon < 1.$$

In the algorithm, we used an empirically established estimate for the initial point ε_0 within this interval and observed convergence for all considered DFC parameters. We repeated this procedure for increasing values of M and chose a triplet (M, ε, α) to represent a specific DFC whenever D_2^M converges. The optimization procedure was carried out for DFCs with $q = 2$, four values of γ, and two values of $f_D T$ for each γ. We did not observe an important variation in the optimal value of δ for the considered $f_D T$ values, so we used the values provided in Table I. The vectors $\boldsymbol{\rho}$ calculated from (2) are $\boldsymbol{\rho} = (0.7067, 0.2016, 0.0762, 0.0155)$ for $\gamma = 2$ dB and $\delta = 0.5$, $\boldsymbol{\rho} = (0.8027, 0.1537, 0.0398, 0.0038)$ for $\gamma = 5$ dB and $\delta = 0.4$, $\boldsymbol{\rho} = (0.9239, 0.0528, 0.0187, 0.0046)$ for $\gamma = 10$ dB and $\delta = 0.2$, and $\boldsymbol{\rho} = (0.9722, 0.02, 0.0064, 0.0014)$ for $\gamma = 15$ dB and $\delta = 0.12$. Table II provides the remaining parameters of the QB noise process that fits a specific DFC.

We next use channel noise ACF and channel capacity as metrics for measuring the accuracy of the NBNDC with QB noise models of Table II in approximating the DFC. Fig. 2 compares the ACFs of the DFC and the QB noise processes for several values of M, for a DFC with $q = 2$, $f_D T = 0.005$,

$$R_{\text{QB}}[m] = \begin{cases} \mathbf{E}[Z_k^2], & \text{if } m=0 \\ \frac{1}{1-\frac{(M-2+\alpha)\varepsilon}{M-1+\alpha}} \left[\frac{\varepsilon}{M-1+\alpha}\mathbf{E}[Z_k^2] + (1-\varepsilon)\mathbf{E}[Z_k]^2 \right], & \text{if } 1 \le m \le M-1 \\ (1-\varepsilon)\mathbf{E}[Z_k]^2 + \frac{\varepsilon}{M-1+\alpha} \left[\sum_{i=1}^{M-1} R_{\text{QB}}[m-1] + \alpha\, R_{\text{QB}}[m-M] \right], & \text{if } m \ge M. \end{cases}$$

TABLE II
QB PARAMETERS FOR FITTING THE RAYLEIGH DFC WITH $q = 2$.

γ	$f_D T = 0.005$	$f_D T = 0.01$
2 dB ($\delta = 0.5$)	$M = 11$ $\varepsilon = 0.7537$ $\alpha = 0.6362$	$M = 8$ $\varepsilon = 0.6846$ $\alpha = 0.5313$
5 dB ($\delta = 0.4$)	$M = 10$ $\varepsilon = 0.7967$ $\alpha = 0.6318$	$M = 7$ $\varepsilon = 0.7260$ $\alpha = 0.5286$
10 dB ($\delta = 0.2$)	$M = 7$ $\varepsilon = 0.7563$ $\alpha = 0.5932$	$M = 5$ $\varepsilon = 0.6765$ $\alpha = 0.4818$
15 dB ($\delta = 0.12$)	$M = 5$ $\varepsilon = 0.7076$ $\alpha = 0.5511$	$M = 4$ $\varepsilon = 0.6371$ $\alpha = 0.399$

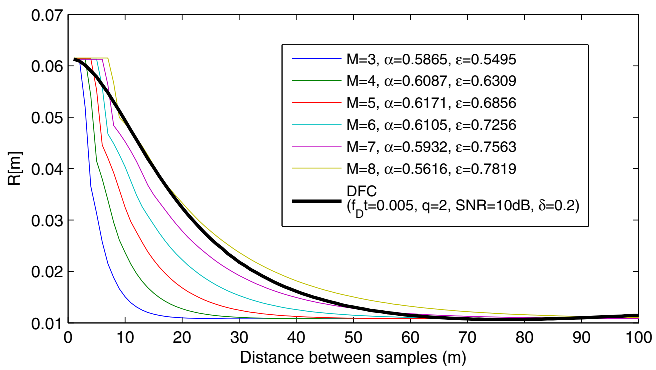

Fig. 2. Comparison of ACFs: DFC (with $f_D T = 0.005$, $q = 2$, $\gamma = 10$ dB and $\delta = 0.2$) vs NBNDC with QB noise.

$\gamma = 10$ dB, and $\delta = 0.2$. The figure shows a good agreement between the ACF of the DFC and that of the QB noise model with $M = 7$ described in Table II. A similar behavior is also observed for all QB models listed in this table (curves not shown), thus indicating that QB models satisfactorily approximate the ACF of the DFC process. Fig. 3 assesses the lower bound on the capacity of the DFC with that of the fitting QB noise model, where the DFC values of $n^\star$ and δ given in Table I for each SNR γ and the values of the QB noise are given in Table II. We observe a relatively close match in the capacity curves of both channels.

VI. CONCLUSION

The modeling results indicate that the DFC can be well approximated via the NBNDC with Markovian QB noise for a wide range of fading conditions. Given that the NBNDC with QB noise model is analytically tractable and offers closed-form expressions for its statistical and information-theoretic quantities, it provides an effective discrete model for which powerful coding schemes can be constructed to judiciously

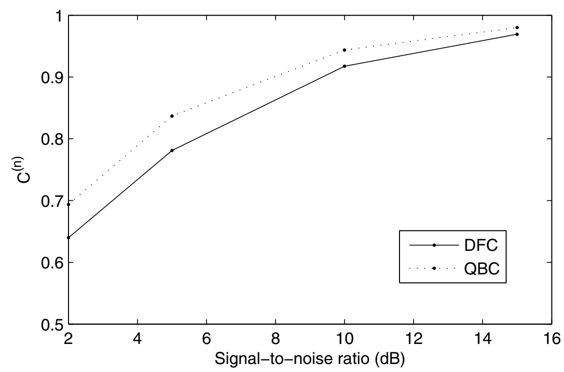

Fig. 3. Comparison of capacity lower bounds: DFC (with $f_D T = 0.005$ and $q = 2$) vs NBNDC with QB noise.

exploit channel memory and soft-decision information. This constitutes an interesting direction for future work.

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