Soft-Decision COVQ for Turbo-Coded AWGN and Rayleigh Fading Channels

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***Abstract*—A robust soft-decision channel-optimized vector quantization (COVQ) scheme for Turbo-coded additive white Gaussian noise (AWGN) and Rayleigh fading channels is proposed. The log likelihood ratio (LLR) generated by the Turbo decoder is exploited via the use of a q-bit scalar soft-decision demodulator. The concatenation of the Turbo encoder, modulator, AWGN channel or Rayleigh fading channel, Turbo decoder, and q-bit soft-decision demodulator is modeled as an expanded discrete memoryless channel (DMC). A COVQ scheme for this expanded discrete channel is designed. Numerical results indicate substantial performance improvements over traditional tandem coding systems, COVQ schemes designed for hard-decision demodulated Turbo-coded channels ($q = 1$), as well as performance gains over a recent soft decoding COVQ scheme by Ho.**

***Index Terms*—AWGN channels, channel-optimized vector quantization, joint source-channel coding, Rayleigh fading channels, soft-decision demodulator, turbo codes.**

I. Introduction

Based on Shannon's separation principle [10], source and channel coding are often treated separately (resulting in what we call a *tandem* coding system). This separation of source and channel coding results in no loss of optimality provided unlimited coding delay and system complexity are allowed [10]. During the past few decades, significant improvements have been achieved in these two separate areas. One of the most noticeable techniques in fixed-rate source coding is source-optimized vector quantization (LBG-VQ) [8], while in channel coding, Turbo codes [5] have been widely recognized as the most exciting breakthrough due to their extraordinary performance. However, in practice, with constraints on delay and complexity, joint source-channel coding can significantly outperform traditional tandem coding systems (e.g., [1]–[4], [6], [7], [9], [11], [12]).

In this work, we design and implement a *robust soft-decision* channel-optimized vector quantization (COVQ) scheme for Turbo-coded AWGN channels and Rayleigh fading channels with known side information. More specifically, we employ the methods recently introduced in [1], [9] to design a COVQ system that improves the end-to-end performance of a Turbo-coded system by exploiting the log-likelihood ratio (LLR) generated by the Turbo decoder. This is achieved via the use of a q-bit scalar soft-decision demodulator at the output of the Turbo decoder, and by designing a COVQ scheme for the resulting *expanded discrete channel* which consists of the concatenation of the Turbo encoded and decoded channel with the soft-decision demodulator. Alternative approaches for channel-optimized quantization using Turbo codes have been previously studied by Bakus and Khandani for scalar quantization [3], [4], and by Ho for vector quantization [7], where the *entire* (unquantized) soft-decision information provided by the LLR of the Turbo decoder is utilized. Our scheme offers better performance than Ho's system; furthermore, the decoding complexity of our system is lower but the storage requirement might be higher.

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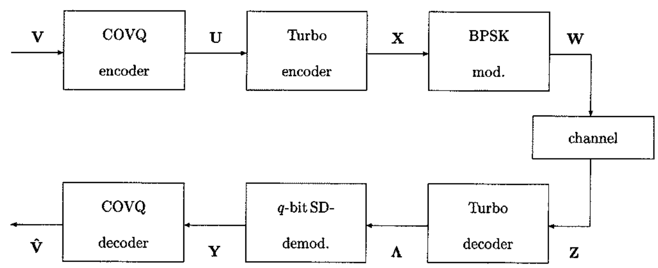

Fig. 1. Block diagram of the system.

II. System Design

The proposed system is as follows (see Fig. 1). The COVQ encoder takes a k-dimensional real vector $\mathbf{V}$ as its input, operates at a rate of R_s bits/source symbol, and generates kR_s bits as the output $\mathbf{U} \in \{0, 1\}^{kR_s}$. This output is then fed into a rate-R_c Turbo encoder, whose output $\mathbf{X}$ is binary phase-shift keying (BPSK) modulated as $\mathbf{W} \in \{-1, +1\}^{kR_s/R_c}$ (assuming kR_s/R_c is an integer) and then transmitted through an AWGN channel or a Rayleigh fading channel described by

$$Z_l = A_l W_l + N_l, \qquad l = 1, 2, 3, \ldots$$

where $W_l \in \{-1, +1\}$ is the BPSK signal of unit energy and $\{N_l\}$ is an i.i.d. Gaussian noise sequence with zero mean and variance $N_0/2$. For the AWGN channel, $A_l = 1$, $l = 1, 2, 3, \ldots$, while for the Rayleigh fading channel, the amplitude fading process $\{A_l\}$ (also known as the channel side information) is assumed to be i.i.d. and Rayleigh distributed. We assume that $\{A_l\}$ is known at the decoder, and that A_l, W_l, and N_l are independent of each other. At the receiver end, Turbo decoding is applied on the received sequence $\mathbf{Z}$ to compute the LLR given by

$$\Lambda_l = \log \frac{\Pr\{U_l = 1|\mathbf{Z}\}}{\Pr\{U_l = 0|\mathbf{Z}\}}, \qquad l = 1, 2, 3, \ldots$$

which is then demodulated via a q-bit uniform scalar quantizer $\alpha(\cdot)$ with quantization step Δ. The quantizer is described by $\alpha(\Lambda) = j$ if $\Lambda \in (T_{j-1}, T_j)$, where $j = 0, 1, \ldots, 2^q - 1$.

The thresholds $\{T_j\}$ are uniformly spaced with quantization step Δ; they satisfy

$$T_j = \begin{cases} -\infty, & \text{if } j = -1 \\ (j + 1 - 2^{q-1})\Delta, & \text{if } j = 0, 1, \ldots, 2^q - 2 \\ +\infty, & \text{if } j = 2^q - 1. \end{cases}$$

Finally, these qkR_s bits are passed to the COVQ decoder, from which $\hat{\mathbf{V}}$, an estimation of $\mathbf{V}$, is produced.

III. Expanded DMC Model and COVQ Design

The concatenation of the Turbo encoder, BPSK modulator, AWGN or Rayleigh fading channel, Turbo decoder, and q-bit soft-decision demodulator is modeled as a 2^{kR_s}-input, 2^{qkR_s}-output discrete (block) memoryless channel (DMC). To design a COVQ for this expanded DMC model, the transition matrix Π is needed. However, with Turbo encoding and decoding, obtaining a closed form expression for Π becomes intractable. For our design, we estimate Π using a long training sequence in the concatenated system. Note that Π is a function of the quantization step Δ and the channel signal-to-noise ratio (CSNR), which is defined as $\text{CSNR} = 2E_s/N_0 = 2R_cE_b/N_0$, where E_s and E_b are the average symbol energy and average energy per information bit, respectively. For each value of CSNR, we numerically choose the optimal quantization step size Δ in the sense that the capacity of this expanded channel is maximized [9]. The channel capacity is calculated by using Blahut's algorithm.

We then design a COVQ for this 2^{kR_s}-input, 2^{qkR_s}-output DMC using the iterative algorithm described in [6]. The COVQ system consists of an encoder mapping and decoder mapping, which are described by a partition and a codebook, respectively. The partition and the codebook are iteratively optimized such that the average squared-error distortion per sample is minimized [1], [9]. The codebook can be pre-computed off-line. Therefore, the COVQ decoding is implemented simply by a table-lookup with no extra computation. However, the memory for storing the codebook is high for large values of qkR_s.

IV. Numerical Results

In this section, we present the performance in terms of the signal-to-distortion ratio (SDR) of our soft-decision COVQ (SD-COVQ) scheme for the compression and transmission of a Gauss–Markov source with correlation coefficient $\rho = 0.9$ over Turbo-coded AWGN and Rayleigh fading channels. 80 000 training source vectors are used. The Turbo code is a rate $R_c = 1/2$, 16-state code with generator $(37, 21)$. The block length N is 65 536 bits and a pseudorandom interleaver is used [5], [7]. The number of decoding iterations is 10.

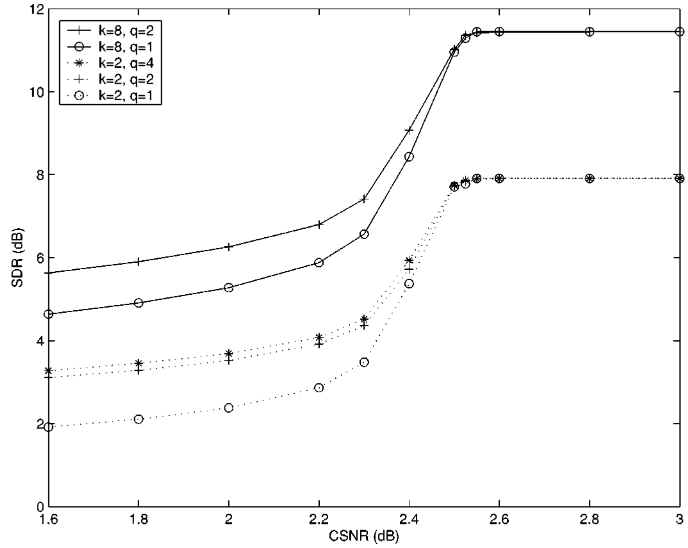

Fig. 2. SDR performance of SD-COVQ of a Gauss–Markov source (with $\rho = 0.9$) over a Turbo-coded Rayleigh fading channel, $R_s = 1$, $R_c = 1/2$.

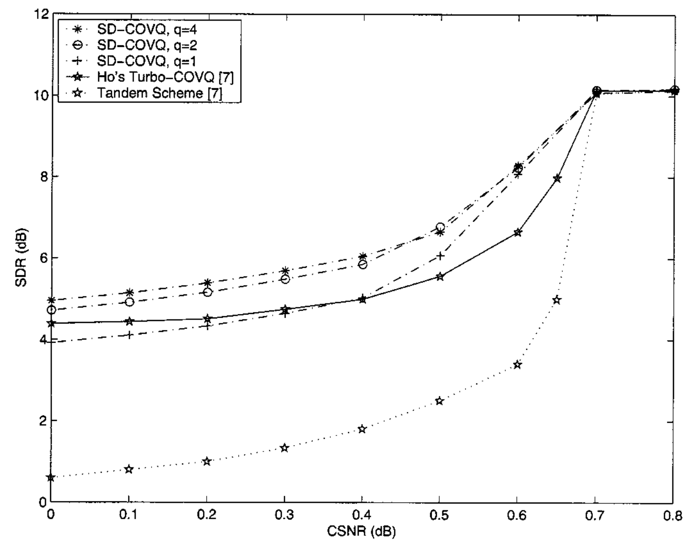

Fig. 3. SDR performance of SD-COVQ of a Gauss–Markov source (with $\rho = 0.9$) over a Turbo-coded AWGN channel, $k = 4$, $R_s = 1$, $R_c = 1/2$.

Fig. 2 shows the performance of our scheme over a Turbo-coded Rayleigh fading channel. Two sets of parameters are used for the source dimension: $k = 2$ and $k = 8$. The quantization rate is $R_s = 1$ bit/source symbol. When $\text{CSNR} > 2.3$ dB, where the BER performance curve of the Turbo code starts dropping down quickly, a slight increment of CSNR results in a drastic improvement of the SDR performance. When $\text{CSNR} \geq 2.55$ dB, the performance of our system reaches that of the COVQ designed for noiseless channels. For low CSNR's, the performance is improved when q increases, the most significant gain is achieved at $q = 2$. When $\text{CSNR} \geq 2.55$ dB, the use of hard-decision ($q = 1$) is sufficient. By using a high-rate Turbo code while maintaining the same overall rate, the performance can be further improved.

Fig. 3 shows the comparison of the performance generated by our scheme and other proposed schemes [7] over a Turbo-coded

AWGN channel. The source dimension is $k = 4$ and the quantization rate is $R_s = 1$. When $\mathrm{CSNR} = 0$ dB, our scheme offers a 4.3 dB gain over the traditional tandem scheme (which consists of a noiseless LBG-VQ followed by a regular Turbo code), and a 0.55 dB gain over Ho's system. When $\mathrm{CSNR} = 0.6$ dB, the gains over the two above schemes become 4.6 and 1.5 dB, respectively. The performance gain of our SD-COVQ scheme over Ho's scheme may be explained by the fact that in [7], the formulation of the LLR *a posteriori* probability is based on the LLR Λ instead of the channel output (compare [7, eqs. (1) and (2)]) and on the assumption of an equally likely binary source at the Turbo encoder input (see discussion following [7, eq. (2)]). Furthermore, in [7], the binary-input soft-output channel formed by the concatenation of the Turbo encoder, AWGN channel and Turbo decoder is modeled as a memoryless channel, while in our scheme we model it (in conjunction with a SD demodulator) as a block memoryless channel (expanded DMC).

V. Conclusion

In this letter, we design and implement a COVQ scheme for Turbo-coded AWGN and Rayleigh fading channels. The reliability information produced by the Turbo decoder is utilized via a q-bit scalar soft-decision demodulator. The concatenation of the Turbo encoder, BPSK modulator, AWGN channel or Rayleigh fading channel, Turbo decoder, and q-bit soft-decision demodulator is modeled as a 2^{kR_s}-input, 2^{qkR_s}-output expanded discrete block memoryless channel. The COVQ scheme is designed for this expanded channel model. Numerical results show significant gains over the traditional tandem schemes, COVQ systems designed for the hard-decision demodulated Turbo-coded channels, as well as Ho's recent soft decoding COVQ scheme [7].

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