Soft-Decision COVQ for Rayleigh-Fading Channels

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***Abstract*— A channel-optimized vector quantizer (COVQ) scheme that exploits the channel *soft-decision* information is proposed. The scheme is designed for stationary memoryless Gaussian and Gauss–Markov sources transmitted over BPSK-modulated Rayleigh-fading channels. It is demonstrated that substantial coding gains (2–3 dB in channel signal-to-noise ratio (SNR) and 1–1.5 dB in source signal-to-distortion ratio (SDR) can be achieved over COVQ systems designed for discrete (hard-decision demodulated) channels.**

***Index Terms*—Combined source-channel coding, COVQ, soft-decision decoding, Rayleigh-fading channels.**

I. Introduction

RECENT WORKS (e.g., [2], [3], [5]–[8]) on combined source-channel coding show that significant performance improvement can be realized for very noisy communication channels. Most of these works (with the exception of [8], [10]) deal with *discrete* channel models, i.e., channels used in conjunction with hard-decision demodulation.

In this letter, we incorporate the use of soft-decision information in the design of combined source–channel coding schemes. More specifically, we propose a channel-optimized vector quantizer (COVQ) [5], [3] for independent (fully interleaved) Rayleigh-fading channels with soft-decision binary phase-shift keying (BPSK) modulation. This scheme—which consists of a source code designed for noisy channels—is in many ways similar to channel-coding techniques that employ soft-decision coded modulation. Numerical results indicate that coding gains of up to 3 dB can be achieved over COVQ systems designed for hard-decision demodulated channels.

The main difference between this work and [8], [10] lies in the fact that we employ a soft-decision quantizer at the receiver. This results in an overall system with comparable performance [7] but significantly reduced decoder computational complexity; the memory requirement, however, is higher.

II. DMC Channel Model

The proposed system is illustrated in Fig. 1. The input source vector $\boldsymbol{V}$ is a real k-tuple, and the COVQ operates at a rate of r bits per source dimension. For each input vector, the encoder produces a binary vector $\boldsymbol{X} \in \{0,1\}^{kr}$ for transmission. Each of the kr bits of $\boldsymbol{X}$ is BPSK modulated, and the output $\boldsymbol{W} \in \{-1,+1\}^{kr}$ is transmitted over a Rayleigh-fading channel according to

$$Z_i = A_i W_i + N_i, \qquad i = 1, 2, \cdots, kr$$

where $W_i \in \{-1,+1\}$ is the BPSK signal of unit energy and $\{N_i\}$ is a sequence of independent and identically distributed (i.i.d.) zero-mean Gaussian random variables with variance σ^2. The amplitude fading process $\{A_i\}$ is assumed to be i.i.d. with probability density function (pdf)

$$f_A(a) = \begin{cases} 2ae^{-a^2}, & \text{if } a > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Note that $E[A_i^2] = 1$. We also assume that A_i, W_i, and N_i are independent of each other ($\forall i$), and that the values of the A_i's are not available at the receiver (no channel state information).

At the receiver, each vector $\boldsymbol{Z}$ is demodulated with q-bit soft decision (through the use of a uniform scalar quantizer) yielding $\boldsymbol{Y} \in \{0,1\}^{qkr}$. Thus for each k-dimensional source

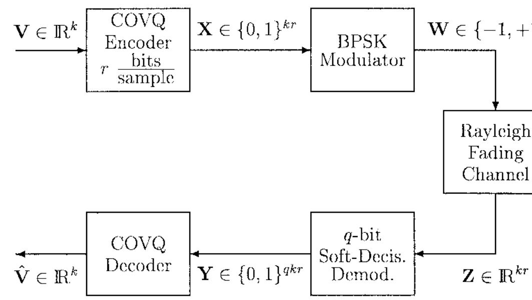

Fig. 1. Block diagram of the system.

TABLE I
Capacity (in Bits/Channel Use) of the DMC Derived from BPSK-Modulated Rayleigh-Fading Channel. Δ is the Step-Size of the Soft-Decision Demodulator Which Maximizes Capacity

Channel	$q=1$	$q=2$		$q=3$		$q=4$	
SNR (dB)	C	C	Δ	C	Δ	C	Δ
∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000
16.0	0.906	0.947	0.200	0.954	0.100	0.955	0.060
14.0	0.865	0.920	0.240	0.929	0.130	0.931	0.070
12.0	0.811	0.880	0.300	0.892	0.160	0.895	0.090
10.0	0.742	0.824	0.360	0.840	0.190	0.844	0.110
8.0	0.656	0.749	0.440	0.769	0.240	0.774	0.130
6.0	0.557	0.656	0.530	0.678	0.290	0.684	0.160
4.0	0.451	0.548	0.650	0.572	0.360	0.578	0.200
3.0	0.399	0.492	0.720	0.515	0.400	0.522	0.220
2.0	0.348	0.436	0.800	0.458	0.450	0.465	0.250
1.0	0.300	0.381	0.890	0.403	0.500	0.409	0.280
0.0	0.256	0.329	1.000	0.349	0.560	0.355	0.310
-1.0	0.216	0.281	1.110	0.299	0.630	0.305	0.350
-2.0	0.181	0.237	1.240	0.254	0.700	0.259	0.390
-3.0	0.150	0.199	1.390	0.213	0.790	0.217	0.440

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vector, qkr bits are produced at the demodulator output. These bits are then passed to the COVQ decoder.

We note that the concatenation of the modulator, channel, and demodulator constitutes indeed a 2^{kr}-input 2^{qkr}-output discrete memoryless channel (DMC). This channel is equivalent to a binary-input 2^q-output DMC used kr times. Its channel transition probability matrix can hence be computed from the channel noise variance and the complementary error function. More specifically, let the receiver's uniform scalar quantizer $\alpha(\cdot)$ with step-size Δ be defined as

$$\alpha(z) = j, \qquad \text{if } z \in (T_{j-1}, T_j)$$

where the thresholds $\{T_j\}$ satisfy

$$T_j = \begin{cases} -\infty, & \text{if } j = -1 \\ (j + 1 - 2^{q-1})\Delta, & \text{if } j = 0, 1, \cdots, 2^q - 2 \\ +\infty, & \text{if } j = 2^q - 1. \end{cases}$$

If $\mathcal{X} = \{0, 1\}$ and $\mathcal{Y} = \{0, 1, 2, \cdots, 2^q - 1\}$, then the transition probability matrix $\mathbf{\Pi}$ is given by

$$\mathbf{\Pi} = [\pi_{ij}], \qquad i \in \mathcal{X}, j \in \mathcal{Y}$$

where

$$\begin{aligned} \pi_{ij} &\triangleq \Pr\{Y = j | X = i\} \\ &= \Pr\{Z \in (T_{j-1}, T_j) | X = i\} \\ &= F_{Z|X}(T_j|i) - F_{Z|X}(T_{j-1}|i) \end{aligned}$$

where $F_{Z|X}(z|i) \triangleq \Pr\{Z \leq z | X = i\}$ is the conditional cumulative distribution function (cdf) of Z given X. For the Rayleigh-fading channel, we obtain that [9]

$$\begin{aligned} F_{Z|X}(z|1) &= 1 - F_{Z|X}(-z|0) \\ &= E_A[\Pr\{N \leq z - a\}] \\ &= 1 - \frac{1}{2}\operatorname{erfc}\left(\frac{z}{\sqrt{2\sigma^2}}\right) - \frac{1}{\sqrt{2\sigma^2 + 1}} \\ &\quad \times \left[1 - \frac{1}{2}\operatorname{erfc}\left(\frac{z}{\sqrt{2(2\sigma^2 + 1)\sigma^2}}\right)\right] \\ &\quad \cdot e^{-(z^2/(2\sigma^2 + 1))} \end{aligned}$$

where $\operatorname{erfc}(x) \triangleq (2/\sqrt{\pi}) \int_x^\infty e^{-t^2} \, dt$ is the complimentary error function. It can be observed that the above two-input 2^q-output DMC is "weakly" symmetric in the sense that its transition probability matrix $\mathbf{\Pi}$ can be partitioned (along its columns) into symmetric arrays—where a symmetric array is defined as an array having the property that all its rows are permutations of each other, and all its columns are permutations of each other [4], [1]. The symmetry property implies the fact that the capacity of this channel is achieved by a uniform input distribution [4]. Its capacity can, therefore, be easily computed by evaluating the mutual information between X and Y using a uniform distribution on X. In Table I, we display the channel capacity for different values of q and the channel signal-to-noise ratio (SNR)—$\text{SNR} = E[W^2]/E[N^2] = 1/\sigma^2$. For each channel SNR, we numerically select the value of the quantization step Δ which yields the maximum capacity of the binary-input 2^q-output DMC. Note that the capacity increases with q (as expected).[1] It is important to point out that the soft-decision information significantly increases the channel capacity during severe channel conditions; e.g., at a channel SNR of -3 dB, the capacity increases by 44.67% (from $q = 1$ to $q = 4$).

[1] Indeed, as $q \to \infty$, the capacity of the DMC monotonically converges to the capacity of the channel with unquantized output [9].

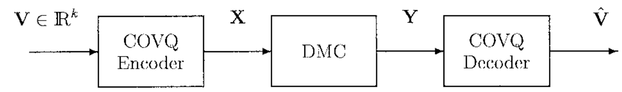

Fig. 2. Block diagram of a COVQ system.

III. COVQ DESIGN

Given the channel transition matrix $\mathbf{\Pi}$, we design the COVQ for the DMC using the algorithm proposed in [3]. The algorithm is an iterative algorithm which results in a locally optimal solution. We briefly describe it as follows.

Consider a real-valued i.i.d. source $\mathcal{V} = \{V_i\}_{i=1}^\infty$ with pdf $f(v)$. The source is to be encoded by a k-dimensional, kr-bit COVQ whose output is to be transmitted over the 2^{kr}-input 2^{qkr}-output DMC with transition probability distribution $P(\boldsymbol{y}|\boldsymbol{x}) = \prod_{l=1}^{kr} \pi_{x_l y_l}$, where $\boldsymbol{x} \in \mathcal{X}^{kr}$ and $\boldsymbol{y} \in \mathcal{Y}^{kr}$. The COVQ system, depicted in Fig. 2, consists of an encoder mapping γ and a decoder mapping, β. The encoder mapping γ: $\mathbb{R}^k \mapsto \mathcal{X}^{kr}$ is described in terms of a partition $\mathcal{P} = \{S_{\boldsymbol{x}} \subset \mathbb{R}^k$: $\boldsymbol{x} \in \mathcal{X}^{kr}\}$ of $\mathbb{R}^k$ according to $\gamma(\boldsymbol{v}) = \boldsymbol{x}$ if $\boldsymbol{v} \in S_{\boldsymbol{x}}, \boldsymbol{x} \in \mathcal{X}^{kr}$, where $\boldsymbol{v} = (v_1, v_2, \cdots, v_k)$ is a block of k successive source samples. The DMC takes an input sequence $\boldsymbol{x}$ and produces an output sequence $\boldsymbol{y}$. It is given in terms of the block channel transition matrix $P(\boldsymbol{y}|\boldsymbol{x})$. Finally, the decoder mapping β: $\mathcal{Y}^n \mapsto \mathbb{R}^k$ is described in terms of a codebook $\mathcal{C} = \{\boldsymbol{c_y} \in \mathbb{R}^k$:$\boldsymbol{y} \in \mathcal{Y}^{kr}\}$ according to $\beta(\boldsymbol{y}) = \boldsymbol{c_y}, \boldsymbol{y} \in \mathcal{Y}^{kr}$.

The encoding rate of the above system is r bits/sample and its average squared-error distortion per sample is given by [3]

$$D = \frac{1}{k} \sum_{\boldsymbol{x}} \int_{S_{\boldsymbol{x}}} f(\boldsymbol{v}) \left\{ \sum_{\boldsymbol{y}} P(\boldsymbol{y}|\boldsymbol{x}) ||\boldsymbol{v} - \boldsymbol{c_y}||^2 \right\} d\boldsymbol{v} \quad (1)$$

where $f(\boldsymbol{v}) = \prod_{i=1}^k f(v_i)$ is the k-dimensional source pdf. For a given source, channel k and kr, we wish to minimize D by proper choice of $\mathcal{P}$ and $\mathcal{C}$.

From (1), we see that for a fixed $\mathcal{C}$ the optimal partition $\mathcal{P}^* = \{S_{\boldsymbol{x}}^*\}$ is given by [3]

$$S_{\boldsymbol{x}}^* = \left\{ \boldsymbol{v}: \sum_{\boldsymbol{y}} P(\boldsymbol{y}|\boldsymbol{x}) ||\boldsymbol{v} - \boldsymbol{c_y}||^2 \right. \left. \leq \sum_{\boldsymbol{y}} P(\boldsymbol{y}|\tilde{\boldsymbol{x}}) ||\boldsymbol{v} - \boldsymbol{c_y}||^2 \quad \forall \tilde{\boldsymbol{x}} \in \mathcal{X}^{kr} \right\}$$

$\boldsymbol{x} \in \mathcal{X}^{kr}$. Similarly, the optimal codebook $\mathcal{C}^* = \{\boldsymbol{c}_{\boldsymbol{y}}^*\}$ for a given partition is [3]

$$\boldsymbol{c}_{\boldsymbol{y}}^* = \frac{\sum_{\boldsymbol{x}} P(\boldsymbol{y}|\boldsymbol{x}) \int_{S_{\boldsymbol{x}}} \boldsymbol{v} f(\boldsymbol{v}) \, d\boldsymbol{v}}{\sum_{\boldsymbol{x}} P(\boldsymbol{y}|\boldsymbol{x}) \int_{S_{\boldsymbol{x}}} f(\boldsymbol{v}) \, d\boldsymbol{v}}.$$

The codebook is precomputed offline. Hence there is no decoding computational requirements (as opposed to [8], [10]);

TABLE II
SOURCE SDR (IN DECIBELS) PERFORMANCES OF COVQ SYSTEM IN RAYLEIGH-FADING CHANNEL FOR DIFFERENT VALUES OF q (NUMBER OF SOFT-DECISION BITS); MEMORYLESS GAUSSIAN SOURCE; $r = 2$ BITS/SAMPLE; DIMENSION $k = 2$. NUMBERS IN BRACKETS INDICATE THE OPTIMAL PERFORMANCE THEORETICALLY ATTAINABLE (OPTA) FOR THE MEMORYLESS GAUSSIAN SOURCE AND DMC (DERIVED FROM THE RAYLEIGH-FADING CHANNEL)

Channel SNR (dB)	$q=1$	$q=2$	$q=3$	$q=4$
∞	9.60 [12.04]	9.60 [12.04]	9.60 [12.04]	9.60 [12.04]
16.0	7.90 [10.90]	8.14 [11.41]	8.24 [11.49]	8.26 [11.50]
14.0	7.25 [10.42]	7.56 [11.08]	7.67 [11.19]	7.70 [11.21]
12.0	6.47 [9.77]	6.86 [10.60]	6.98 [10.75]	7.02 [10.78]
10.0	5.73 [8.93]	6.41 [9.92]	6.56 [10.12]	6.59 [10.16]
8.0	4.93 [7.90]	5.63 [9.02]	5.79 [9.26]	5.83 [9.32]
6.0	4.08 [6.70]	4.75 [7.89]	4.91 [8.17]	4.95 [8.24]
4.0	3.23 [5.43]	3.84 [6.60]	3.99 [6.89]	4.03 [6.96]
3.0	2.83 [4.80]	3.41 [5.92]	3.55 [6.21]	3.59 [6.28]
2.0	2.46 [4.19]	3.00 [5.24]	3.13 [5.52]	3.16 [5.60]
1.0	2.13 [3.61]	2.62 [4.59]	2.74 [4.85]	2.77 [4.92]
0.0	1.85 [3.08]	2.30 [3.96]	2.42 [4.21]	2.45 [4.27]
-1.0	1.57 [2.60]	1.97 [3.38]	2.08 [3.60]	2.11 [3.67]
-2.0	1.32 [2.18]	1.68 [2.86]	1.77 [3.06]	1.80 [3.11]
-3.0	1.10 [1.80]	1.41 [2.39]	1.50 [2.57]	1.52 [2.62]

although the codebook size is $2^{(q-1)kr}$ times larger than the codebook in [8]. The above result can be easily generalized for sources with memory, e.g., a Gauss–Markov source [6].

IV. NUMERICAL RESULTS AND DISCUSSION

In Tables II and III, we present numerical results for our soft-decision COVQ scheme when the source is memoryless Gaussian and Gauss–Markov with parameter 0.9, respectively. The results are given in terms of the source signal-to-distortion ratio (SDR) for various values of the channel SNR. The numbers in brackets indicate the optimal performances theoretically attainable (OPTA) obtained by evaluating $D(rC)$, where $D(\cdot)$ is the distortion-rate function of the source (for the squared-error distortion measure), and C is the capacity of the DMC derived from the Rayleigh channel (and given in Table I). In both tables, the rate is $r = 2$ bits/sample and the dimension is $k = 2$. As many as 80 000 training vectors were employed in the COVQ design program. Note that the results for $q = 1$ correspond to hard-decision demodulation.

We observe from Tables II and III that the system performance improves as q increases. For both sources, the performance improvement in SDR due to the channel soft-decision information follows a similar pattern as the capacity gains reported in Table I. For the memoryless Gaussian source (Table II), at SNR = 8 dB, the SDR increases by 0.90 dB (from $q = 1$ to $q = 4$); while for the Gauss–Markov source (Table III), at $\mathrm{SNR} = 8$ dB, the improvement is by 1.5 dB.

In terms of coding gains in channel SNR, the best coding gains for both sources are around 2 dB when the channel is very noisy. Furthermore, for the Gauss–Markov source, at an SDR = 9.09 dB, the 16-bit soft-decision COVQ system achieves a coding gain of 3.4 dB over the hard-decision COVQ system.

TABLE III
SOURCE SDR (IN DECIBELS) PERFORMANCES OF COVQ SYSTEM IN RAYLEIGH-FADING CHANNEL FOR DIFFERENT VALUES OF q (NUMBER OF SOFT-DECISION BITS); GAUSS–MARKOV SOURCE WITH CORRELATION COEFFICIENT 0.9; $r = 2$ BITS/SAMPLE; DIMENSION $k = 2$. NUMBERS IN BRACKETS INDICATE THE OPTIMAL PERFORMANCE THEORETICALLY ATTAINABLE (OPTA) FOR THE GAUSS–MARKOV SOURCE ($\rho = 0.9$) AND DMC (DERIVED FROM RAYLEIGH-FADING CHANNEL)

Channel SNR (dB)	$q=1$	$q=2$	$q=3$	$q=4$
∞	13.52 [19.25]	13.52 [19.25]	13.52 [19.25]	13.52 [19.25]
16.0	9.93 [18.12]	10.86 [18.62]	11.00 [18.70]	11.04 [18.72]
14.0	9.09 [17.63]	10.31 [18.29]	10.49 [18.40]	10.50 [18.43]
12.0	8.29 [16.98]	9.50 [17.81]	9.74 [17.96]	9.79 [17.99]
10.0	7.36 [16.14]	8.45 [17.13]	8.72 [17.33]	8.79 [17.38]
8.0	6.60 [15.11]	7.31 [16.23]	7.54 [16.47]	7.59 [16.53]
6.0	5.55 [13.92]	6.30 [15.11]	6.55 [15.38]	6.62 [15.45]
4.0	4.45 [12.64]	5.43 [13.81]	5.69 [14.10]	5.76 [14.17]
3.0	3.92 [12.00]	4.85 [13.13]	5.10 [13.42]	5.17 [13.50]
2.0	3.41 [11.34]	4.28 [12.46]	4.51 [12.73]	4.58 [12.81]
1.0	2.94 [10.67]	3.72 [11.78]	3.94 [12.05]	3.99 [12.13]
0.0	2.51 [9.99]	3.21 [11.09]	3.40 [11.36]	3.45 [11.44]
-1.0	2.12 [9.30]	2.73 [10.39]	2.90 [10.66]	2.95 [10.74]
-2.0	1.78 [8.60]	2.31 [9.68]	2.46 [9.96]	2.50 [10.03]
-3.0	1.71 [7.91]	1.94 [8.97]	2.07 [9.24]	2.11 [9.32]

V. CONCLUSION

In this letter, we introduce a COVQ for binary-input continuous-output channels. It consists of a COVQ for a DMC derived from the q-bit quantized outputs of the original channel, thus incorporating the channel soft-decision information in the quantizer design. This technique is applied to Rayleigh-fading channels used in conjunction with BPSK-modulation. It is demonstrated that soft-decision demodulation always yields superior performance over hard-decision demodulation; coding gains of 2–3 dB in channel SNR are achieved.

REFERENCES

[1] R. E. Blahut, *Principles and Practice of Information Theory*. Reading, MA: Addison Wesley, 1988.
[2] J. Cheng and F. Alajaji, "Channel optimized quantization of images over bursty channels," in *Proc. Canadian Workshop on Information Theory*, Toronto, Canada, June 1997, pp. 49–52.
[3] N. Farvardin and V. Vaishampayan, "On the performance and complexity of channel-optimized vector quantizers," *IEEE Trans. Inform. Theory*, vol. 37, pp. 155–160, Jan. 1991.
[4] R. G. Gallager, *Information Theory and Reliable Communication*. New York: Wiley, 1968.
[5] H. Kumazawa, M. Kasahara, and T. Namekawa, "A construction of vector quantizers for noisy channels," *Electron. & Eng. Jpn.*, vol. 67-B, pp. 39–47, Jan. 1984.
[6] N. Phamdo, F. Alajaji, and N. Farvardin, "Quantization of memoryless and Gauss–Markov sources over binary Markov channels," *IEEE Trans. Commun.*, vol. 45, pp. 668–675, June 1997.
[7] N. Phamdo and F. Alajaji, "Performance of COVQ over AWGN/Rayleigh channels with soft-decision BPSK modulation," in *Proc. Conf. on Information Sciences and Systems*, Princeton, NJ, Mar. 1996, pp. 137–142.
[8] M. Skoglund, "On soft decoding and robust vector quantization," Ph.D. dissertation, Chalmers Univ. of Technol., Goteborg, Sweden, Tech. Rep. 302., Mar. 1997.
[9] G. Taricco, "On the capacity of the binary input Gaussian and Rayleigh fading channels," *Eur. Trans. Telecommun.*, vol. 7, no. 2, Mar.–Apr. 1996.
[10] H. Xiao and B. Vucetic, "Soft input source decoding in low-bit-rate speech tranmission," in *Proc. IEEE Int. Symp. on Information Theory*, Ulm, Germany, June–July 1997, p. 443.