Title : Groups, growth, hyperbolicity and beyond.

Abstract : Growth functions of finitely-generated groups count the number of elements that can be spelled as words in a generating alphabet, as a function of spelling length. One of the most striking results due to Gromov states that: [A group is virtually nilpotent if and only if its growth function is *bounded above* by a polynomial]. One can still wonder, however, whether the growth function is precisely polynomial. This turns out to be a bit too much to ask for nilpotent groups. Virtually abelian groups, for instance, have a slightly more general property called *rational growth*: no matter what finite generating set is chosen, the power series associated to the growth function represents a rational function. In 1984, Cannon gave an elegant proof showing that all *hyperbolic groups* (such as free groups, virtually free groups, surface groups and random groups) have rational growth; in this talk, I will define all the above objects, give ideas for Cannon's proof and present recent work generalizing Cannon's work to "all hyperbolic-like" elements in any finitely generated group.