Algebra and Geometry Seminar

Speaker: Jacob Matherne (University of Bonn & Max Planck Institute)

Title: Singular Hodge theory for combinatorial geometries

Abstract: If you take a collection of planes in $\mathbb{R}^3$, then the number of lines you get by intersecting the planes is at least the number of planes. This is an example of a more general statement, called the "Top-Heavy Conjecture", that Dowling and Wilson conjectured in 1974. On the other hand, given a hyperplane arrangement, I will explain how to uniquely associate to it a certain polynomial, called its Kazhdan-Lusztig (KL) polynomial. I will spend some portion of the talk comparing and contrasting these KL polynomials with the classical ones in Lie theory.

The problems of proving the "Top-Heavy Conjecture" and the non-negativity of the coefficients of these KL polynomials are related, and they are controlled by the Hodge theory of a certain singular projective variety. The "Top-Heavy Conjecture" was proven for hyperplane arrangements by Huh and Wang in 2017, and the non-negativity was proven by Elias, Proudfoot, and Wakefield in 2016. I will discuss joint work with Tom Braden, June Huh, Nicholas Proudfoot, and Botong Wang which resolves these two problems for arbitrary matroids.