ALGEBRA AND GEOMETRY SEMINAR

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Title: 2-roots for simply laced Weyl groups

Abstract: We introduce and study "2-roots", which are symmetrized tensor products of orthogonal roots of Kac–Moody algebras. We concentrate on the case where W is the Weyl group of a simply laced Y-shaped Dynkin diagram with three branches of arbitrary finite lengths a, b and c; special cases of this include types D_n, E_n (for arbitrary $n \geq 6$), and affine E_6, E_7 and E_8.

We construct a natural codimension-1 submodule M of the symmetric square of the reflection representation of W, as well as a canonical basis $\mathcal{B}$ of M that consists of 2-roots. The 2-roots of W and the basis $\mathcal{B}$ share many similarities with the roots and the simple roots of W. In particular, we show that every 2-root is a linear combination of the basis $\mathcal{B}$ with coefficients of like sign. We also describe a natural partial order on the set of 2-roots and explicitly describe the "highest 2-roots", the maximal 2-roots with respect to this order. (This is joint work with Richard Green.)