ALGEBRA AND GEOMETRY SEMINAR

Speaker: Khoa Nguyen (Queen's University)

Title: A family of $\mathcal{U}(\mathfrak{h})$-free modules of rank 2 over $\mathfrak{sl}(2)$

Abstract: The study of simple $\mathfrak{sl}(2)$-modules can be divided into two categories. The first category consists of weight modules, namely, those that decompose into direct sums of their weight space with respect to a fixed Cartan subalgebra $\mathfrak{h}$. Such modules have been classified explicitly. The other category consists of non-weight modules. In particular, simple non-weight $\mathfrak{sl}(2)$-modules are $\mathcal{U}(\mathfrak{h})$-free modules of finite rank when restricted to $\mathcal{U}(\mathfrak{h})$. Block showed that there is a bijection between simple non-weight modules and irreducible elements in a non-commutative ring. However, an explicit realization of these modules has yet to be discovered.

In this talk, I will introduce a new family of $\mathcal{U}(\mathfrak{h})$-free modules of rank 2 over $\mathfrak{sl}(2)$. Nonetheless, we will also discuss other families of $\mathcal{U}(\mathfrak{h})$-free modules in the literature. This is based on a joint work with D. Grantcharov and K. Zhao.