

## ALGEBRA AND GEOMETRY SEMINAR

**Speaker:** Zachary Selk (Queen's University)

**Title:** The Algebra of Iterated Integrals

**Abstract:** The fundamental observation of rough paths theory, introduced in the 90s by Terry Lyons is that solving differential equations driven by irregular functions  $X : [0, T] \rightarrow \mathbb{R}^d$  can be reduced to constructing the collection of iterated integrals  $(\int \int dXdX, \int \int \int dXdXdX, \dots)$ . The collection of all iterated integrals, known as the “signature” of a path takes values in the tensor algebra  $T(\mathbb{R}^d)$ , or equivalently, in its topological dual. The tensor algebra can be turned into a Hopf algebra under the shuffle product. One good class of rough paths are “geometric” rough paths which satisfy “the usual rules of calculus” where the “usual rules of calculus” can be stated in terms of the shuffle product.

However, a generalization of “geometric” rough paths are “branched” rough paths introduced by Gubinelli in 2010. In analogy with Butcher series, a branched rough path takes values in the topological dual of the Connes-Kreimer Hopf algebra of rooted trees. The “geometric” condition of respecting the shuffle product is replaced by a condition of respecting the product in the Connes-Kreimer Hopf algebra.

In this talk, I will introduce basic definition and recall some achievements of branched rough paths.