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Title: Arithmetic dynamics on projective bundles over elliptic curves.

Abstract: Let $X$ be a smooth projective variety defined over a number field $K$. Suppose that $X$ is endowed with a surjective endomorphism $f : X \to X$. A numerical measure of the complexity of the morphism $f$ is its dynamical degree, which can be defined as the spectral radius of the pullback morphism $f^* : N^1(X) \to N^1(X)$, where $N^1(X)$ is the Neron-Severi group of $X$. On the other hand, given a point $P$ in $X$ defined over $K$, we have the following arithmetic measure of complexity of $f$ at $P$: The arithmetic degree of $P$ with respect to $f$ is defined to be the limit, as $n \to \infty$, $h(f^n(P))^{1/n}$ where $h(x)$ is the height of a point $x$ in $X$. The Kawaguchi-Silverman conjecture predicts that if the forward orbit of $P$, $\{P, f(P), f^2(P), \ldots\}$, is Zariski dense, then the arithmetic degree of $P$ with respect to $f$ equals the dynamical degree of $f$.

In this talk, we will discuss how to prove the Kawaguchi-Silverman conjecture when $X$ is the projectivization of certain vector bundles on an elliptic curve $C$. Specifically, Atiyah proved that for each integer $r > 0$, there is a unique indecomposable rank $r$ degree zero vector bundle $F_r$ on $C$ with a non-zero global section. We will discuss how one may prove the Kawaguchi-Silverman conjecture for the projectivizations of these bundles. Along the way, we will extend some results of Atiyah in the following way: Atiyah showed that the Iitaka dimension of the line bundle $O(1)$ on $P(F_2)$ is zero. We prove that the Iitaka dimension of the line bundle $O(1)$ on $P(F_r)$ is strictly positive whenever $r > 2$ and relate this to the Kawaguchi-Silverman conjecture.