

# Algebra and Geometry Seminar

**Speaker:** Brett Nasserden (University of Waterloo)

**Title:** Arithmetic dynamics on projective bundles over elliptic curves.

**Abstract:** Let  $X$  be a smooth projective variety defined over a number field  $K$ . Suppose that  $X$  is endowed with a surjective endomorphism  $f : X \rightarrow X$ . A numerical measure of the complexity of the morphism  $f$  is its dynamical degree, which can be defined as the spectral radius of the pullback morphism  $f^* : N^1(X) \rightarrow N^1(X)$ , where  $N^1(X)$  is the Neron-Severi group of  $X$ . On the other hand, given a point  $P$  in  $X$  defined over  $K$ , we have the following arithmetic measure of complexity of  $f$  at  $P$ : The arithmetic degree of  $P$  with respect to  $f$  is defined to be the limit, as  $n \rightarrow \infty$ ,  $h(f^n(P))^{1/n}$  where  $h(x)$  is the height of a point  $x$  in  $X$ . The Kawaguchi-Silverman conjecture predicts that if the forward orbit of  $P$ ,  $\{P, f(P), f^2(P), \dots\}$ , is Zariski dense, then the arithmetic degree of  $P$  with respect to  $f$  equals the dynamical degree of  $f$ .

In this talk, we will discuss how to prove the Kawaguchi-Silverman conjecture when  $X$  is the projectivization of certain vector bundles on an elliptic curve  $\mathcal{C}$ . Specifically, Atiyah proved that for each integer  $r > 0$ , there is a unique indecomposable rank  $r$  degree zero vector bundle  $F_r$  on  $\mathcal{C}$  with a non-zero global section. We will discuss how one may prove the Kawaguchi-Silverman conjecture for the projectivizations of these bundles. Along the way, we will extend some results of Atiyah in the following way: Atiyah showed that the Iitaka dimension of the line bundle  $\mathcal{O}(1)$  on  $P(F_2)$  is zero. We prove that the Iitaka dimension of the line bundle  $\mathcal{O}(1)$  on  $P(F_r)$  is strictly positive whenever  $r > 2$  and relate this to the Kawaguchi-Silverman conjecture.