

Algebra and Geometry Seminar

Speaker: Andrew P. Staal (University of Waterloo)

Title: Small Elementary Components of Hilbert Schemes of Points.

Abstract: Hilbert schemes of points are moduli spaces of fundamental importance in algebraic geometry, commutative algebra, and algebraic combinatorics. Since their construction by Grothendieck, they have seen broad-ranging applications from the McKay correspondence to Haiman's proof of the Macdonald positivity conjecture.

I will present some recent progress in the study of Hilbert schemes $\text{Hilb}^d(\mathbb{A}^n)$ of d points in affine space, and the related (local) punctual Hilbert schemes $\text{Hilb}^d(\mathcal{O}_{\mathbb{A}^n, p})$ at fixed $p \in \mathbb{A}^n$. Specifically, I will discuss some results on *elementary* components of Hilbert schemes of points and tie these to a question posed by Iarrobino in the 80's: does there exist an irreducible component of the punctual Hilbert scheme $\text{Hilb}^d(\mathcal{O}_{\mathbb{A}^n, p})$ of dimension less than $(n-1)(d-1)$? I will answer this question by describing a new infinite family of irreducible components satisfying this bound, when $n = 4$. A secondary family of elementary components also arises, providing further new examples of elementary components of Hilbert schemes of points, and improving our knowledge surrounding a folklore question on the existence of certain Gorenstein local Artinian rings.

This is joint work with Matt Satriano (U Waterloo).