# Combinatorial Algebra meets Algebraic Combinatorics — Schedule —

**Friday, 23 January 2015**; lectures in 110 Jeffery Hall

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| 13:00–13:50 | François Bergeron | Combinatorics and the Elliptic Hall Algebra |
| 14:00–14:20 | Hector Blandin | Generalized Polarization Modules |
| 14:30–15:30 | break | (5th floor of Jeffery) |
| 15:30–15:50 | Thomas Brüstle | On the non-leaving-face property for associahedra |
| 16:00–16:20 | Nathan Williams | Why the Fuss? |
| 16:30–16:50 | Amy Pang | Card-shuffling and convolutions |
| 18:00–20:00 | Kingston Brewing Company | Informal gathering |

**Saturday, 24 January 2015**; lectures in 234 Jeffery Hall

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| 9:00–9:30 | break | (234 Jeffery) |
| 9:30–10:20 | Cristian Lenart | Combinatorial aspects of Schubert calculus |
| 10:30–10:50 | Dominic Searles | Deformed cohomology of generalized flag varieties |
| 11:00 – 11:30 | break | |
| 11:30–11:50 | Anna Bertiger | The rim hook rule |
| 12:00–12:20 | Markus Schmidmeyer | Crossing and noncrossing partitions of the disk |
| 12:30–14:30 | lunch | (3rd floor in Jeffery) |
| 14:30–15:20 | Mats Boij | Cones of Betti tables and Hilbert functions |
| 15:30–15:50 | Li Li | Bases of cluster algebras |
| 16:00–16:30 | break | (2nd floor of Jeffery) |
| 16:30–17:20 | Eric Katz | Tropical Laplacians and curve arrangements |
| 19:30–21:00 | Milestones | Conference dinner |

**Sunday, 25 January 2015**; lectures in 234 Jeffery Hall

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| 9:00–9:30 | break | (234 Jeffery) |
| 9:30–10:20 | Patrica Hersh | Regular cell complexes in total positivity |
| 10:30–10:50 | Yannic Vargas | Packed words and generalized Bruhat orders |
| 11:00 – 11:30 | break | |
| 11:30–11:50 | Ali Alilooee | $j$-multiplicity of edge ideals |
| 12:00–12:20 | Will Traves | Ten points on a cubic curve |

# Abstracts

**Ali Alilooee:** *j-multiplicity of edge ideals*

It is a progress report of mt recent work with J. Validashti. j-multipilcity was introduced in a local ring in 1993 to extend the Hilbert-Samuel multiplicity for ideals which are not m-primary. Computing $j$-multiplicity in general is not easy, nevertheless many researches have investigated it for classes of ideals. In this talk I first define the Hilbert-Samuel multiplicity of $m$-primarly ideals in a local ring. Then I define j-multiplicity and I give some techniques for computing $j$-multiplicity of edge ideal of a graph.

**François Bergeron:** *Combinatorics and the elliptic Hall algebra*

Ties between an operator realization of the elliptic Hall algebra have an exciting impact in several fields: Knot Theory, Hilbert Schemes, Representation Theory, the Theory of Operators on Macdonald Polynomials, and Combinatorics. We will explain some of those that involve the combinatorics of generalized Dyck paths, and present very recent combinatorial developments that open up new questions regarding the generalization of this theory. Part of this work is in collaboration with E. Leven, A. Garsia and G. Xin; and the more recent portion is in collaboration with J.-C. Aval.

**Anna Bertiger:** *The rim hook rule: A story of two rings with a surprising and not so surprising relationship*

Joint work with Elizabeth Beazley (Haverford) and Kaisa Taipale (Minnesota). I will present the cohomology ring of the Grassmannian of k planes in complex n-space, roughly a ring with generators related to subvarieties of the Grassmannian in which the product encodes intersections of these subvarieties. It turns out this is a nice polynomial ring and module over the integers with many combinatorial properties. The same is true of the quantum cohomology ring of the Grassmannian, which has the same generators with a product encoding the "number" of curves with 3 marked points joining the subvarieties. Quantum cohomology of the Grassmannian is a Z[q]-module with the same basis as the integer basis for cohomology of the Grassmannian. These two rings, which seem very different when viewed from a geometric point of view are closely related as polynomial rings. They are also, surprisingly, related as modules with a product structure by a combinatorial result of Bertram, Ciocan-Fontanine and Fulton. I will explain this result, and also a new equivariant extension. I intend for the talk to be friendly, with all of the relevant geometric notions defined.

**Hector Blandin:** *Generalized polarization modules*

Inspired by M. Haiman's Operator Theorem, we study $\mathfrak{S}_n$-modules of polynomials in $\ell$ sets of $n$ variables, generated by a given homogeneous diagonally symmetric polynomial $f$. These modules are closed under taking partial derivatives, and generalized $\mathfrak{S}_n$-invariants polarization operators. We completely classify these modules (according to Frobenius transform) when they are generated by degree 2 and degree 3 homogeneous symmetric polynomials. For the classification of modules associated to homogeneous degree 3 symmetric polynomials we introduce the notion of $n$-exception and we give an interesting conjecture to characterize this notion. We compute general formulas for the vector-graded Frobenius transform of $\mathfrak{S}_n$-modules generated by degree 4 and degree 5 polynomials that seems to be universal.

**Mats Boij:** *Cones of Betti tables and Hilbert functions*

Studying the possible Betti tables of graded modules, it turned out to be useful to relax the question and only look at the Betti tables up to scaling, i.e., to study the cone spanned by the Betti tables in a suitable vector space. In the standard graded case, Macaulay's theorem gives us a complete classification of Hilbert functions of cyclic modules, but in other cases we are lacking such a classification and results on cones of Hilbert functions are useful. We can also combine the two questions and look at Hilbert functions of modules with some properties seen from the Betti table but not directly on the Hilbert function, as in the case of modules with bounded regularity.

I will give a survey on some of the work that has been done on cones of Betti tables and Hilbert functions over the last few years including some recent joint work with Gregory G. Smith.

**Thomas Brüstle:** *On the non-leaving-face property for associahedra of type ADE*

D. Sleator, R. Tarjan and W. Thurston showed in 1988 that the associahedron satisfies the non-leaving-face property, that is, every geodesic connecting two vertices stays in the minimal face containing both. Recently, C. Ceballos and V. Pilaud established the non-leaving face property for generalized associahedra of types B,C,D, and some exceptional types including $E_6$. The key ingredient in the proofs is a normalization, a sort of projection from the whole associahedron to a face. We use methods from cluster categories to define such a normalization, which allows us to establish the non-leaving face property at once for all finite cases that are modelled using cluster categories, namely the simply laced Dynkin diagrams. This talk reports on joint work with Jean-François Marceau.

**Patricia Hersh:** *Regular cell complexes in total positivity*

This talk will focus upon stratified spaces arising from total positivity theory and combinatorial representation theory, with background and motivations provided along the way. We prove that certain such stratified spaces having the Bruhat orders as their posets of closure relations are regular CW complexes homeomorphic to closed balls. A special case is the link of the identity in the space of upper triangular, totally nonnegative matrices with 1's on the diagonal, stratified according to which minors are positive and which are 0. This confirms a conjecture of Sergey Fomin and Michael Shapiro, completing the solution of a question of Joseph Bernstein. I will briefly discuss some ingredients that went into the proof, including the role of the 0-Hecke algebra of a finite Coxeter group, combinatorics of reduced and non-reduced words, and a new criterion for deciding if a finite CW complex is regular (with respect to a choice of characteristic maps) based on an interplay of combinatorics with topology. As time permits, I will also indicate ways in which this story could perhaps be generalized.

**Eric Katz:** *Tropical Laplacians and curve arrangements on surfaces*

Given a surface in space with a set of curves on it, one can ask which possible combinatorial arrangement of curves are possible. By studying a certain intersection matrix, the tropical Laplacian, we show that under natural conditions on the complement of the curve arrangement, the combinatorics of the arrangement are tightly constrained. We make connections to the realizability of homology classes in toric varieties and the Colin de Verdiere invariant of graphs. This is joint work with June Huh.

**Cristian Lenart:** *Combinatorial aspects of Schubert calculus in elliptic cohomology*

Modern Schubert calculus has been mostly concerned with the study of the cohomology and $K$-theory of flag manifolds. The basic results for other cohomology theories have only been obtained recently; additional complexity is due to the dependence of the geometrically defined classes on a reduced word for the corresponding Weyl group elements. After this main theory was developed, the next step is to derive explicit combinatorial formulas. I will describe my work with K. Zainoulline in this direction, which focuses on (torus equivariant) elliptic cohomology. We generalize the formulas of Billey (in ordinary cohomology) and Graham-Willems (in $K$-theory) for the equivariant Schubert classes. Another result is concerned with defining a Schubert basis (i.e., classes independent of a reduced word), using the Kazhdan-Lusztig basis of a certain Hecke algebra.

**Li Li:** *Bases of cluster algebras*
Cluster algebras were introduced by Fomin and Zelevinsky in trying to understand canonical bases in algebraic groups. A lot of activity in the theory of cluster algebras has been directed towards various constructions of natural bases in them. In this talk, I will discuss some recent advances in the study of cluster algebras, in particular, various natural bases of the cluster algebras and their connection to two classes of special varieties, namely the quiver Grassmannians and Nakajima's graded quiver varieties.

**Amy Pang:** *Card-shuffling and convolutions of projections on graded Hopf algebras*
Diaconis, Pitman and Fill studied many models of card-shuffling where a deck is cut into piles according to some distribution, then interleaved randomly. We observe that the transition probabilities of these shuffles can be expressed in terms of certain "convolutions of projections" operators on the shuffle algebra. I will explain how to mimic this connection to model the breaking-then-recombining of other combinatorial objects, such as trees and partitions. The structure of the underlying Hopf algebras gives a lot of information about these processes.

**Markus Schmidmeier:** *Crossing and noncrossing partitions of the disk*
According to a theorem by Green and Klein from 1968, there exists a short exact sequence

$$0 \to N_\alpha \to N_\beta \to N_\gamma \to 0$$

of nilpotent linear operators of Jordan types $\alpha$, $\beta$, $\gamma$, respectively, if and only if there is a Littlewood-Richardson tableau of shape $(\alpha, \beta, \gamma)$. Thus, the entries in a tableau provide us with isomorphism invariants for short exact sequences. In my talk I will discuss how the entries can be used to position objects within the category of all short exact sequences, and how they monitor the flow of homomorphisms in this category.

**Will Traves:** *Ten points on a cubic curve*
We ask a question with a simple answer, "When do 10 points lie on a cubic curve?" The condition can be described as the vanishing of a 10x10 determinant in the coordinates of the ten points, but more compact descriptions can be found using invariant theory. We describe one such compact description due to Reiss from 1867. We also describe a new ruler-and-compass construction producing the cubic constraint, derived using the Cayley-Bacharach Theorem. This is joint work with David Wehlau.

**Dominic Searles:** *Root-system combinatorics and deformed cohomology of generalized flag varieties*
In 2006, P. Belkale-S. Kumar introduced a new product on cohomology of generalized flag varieties. We present a new rule for the Belkale-Kumar product for flag varieties of type A, after the puzzle rule of A. Knutson–K. Purbhoo. Our rule uses the combinatorial model of root-theoretic Young diagrams.

Inspired by recent work of S. Evens-W. Graham, in joint work with O. Pechenik we also introduce a deformation of the cohomology of generalized flag varieties. A special case is the Belkale-Kumar deformation. This construction yields a new, short proof that the Belkale-Kumar product is well-defined. Another special case gives a different product structure, picking out triples of Schubert varieties that behave nicely under projections.

**Nathan Williams:** *Why the Fuss?*

We place the program of $m$-eralizing noncrossing Coxeter-Catalan combinatorics in the context of the corresponding positive Artin monoid. Both noncrossing partitions and cluster complexes have previously been successfully $m$-eralized, but no $m$-eralization of $c$-sortable elements has yet been given. We define $m$-$c$-sortable elements as certain elements of the corresponding Artin monoid, and relate these to the existing $m$-erlized Catalan objects. We define the $m$-eralized $c$-Cambrian lattice by naturally extending the construction of the $c$-Cambrian lattices as a restriction of the weak order to the $c$-sortable elements. We also construct these lattices using the $m$-eralized noncrossing partitions,using a construction that is new even for $m = 1$, and the cluster complexes. This is joint work with Christian Stump and Hugh Thomas.

**Yannic Vargas:** *Packed words and generalized Bruhat orders*

The right Permutohedron is the lattice of permutations ordered by the right weak Bruhat order. Taking the inverses of permutations, this lattice is order-isomorphic to the left Permutohedron. An analogue for packed words of the right Permutohedron was defined by Kenneth and Hoffman. Even though there is no inverse operation on packed words, we define an order in this setting which is an analogue of the left Permutohedron. We use these two lattices structures on packed words to define monomial bases for a Hopf algebra of packed words (isomorphic to NCQSYM), generalizing the monomial basis introduced by Aguiar and Sottile for the Hopf algebra of permutations FQSYM.