Problems 04

Due: Friday, 3 February 2023 before 17:00 EST

P4.1. Define a binary relation on the set $\mathbb{R}$ of real numbers: for any two real numbers x and y, we have $x \sim y$ if there exists an integer k such that $x - y = 2\,k\,\pi$.

(i) Verify that this is an equivalence relation.

(ii) Describe a system of distinct representatives (also known as a transversal) for this equivalence relation.

(iii) Is addition well-defined on the quotient set $\mathbb{R}/\sim$?

(iv) Is multiplication well-defined on the quotient set $\mathbb{R}/\sim$?

P4.2. **(i)** Let m be an integer. Confirm that $m^2 \equiv 0$ or $1 \mod 3$.

(ii) Let p be a prime integer such that $p \geqslant 5$. Prove that $p^2 + 2$ is reducible.

P4.3. **(i)** Consider the integer $m = \sum_{j=0}^{k} d_j\, 10^j$ where k is a nonnegative integer and, for each j, the integer d_j satisfies $0 \leqslant d_j \leqslant 9$. Show that 9 divides m if and only if 9 divides $\sum_{j=0}^{k} d_j$.

(ii) Using part (i), determine if 9 divides 627 174.