Solutions 01

P1.1. Carefully read both pages of the course website. Give a brief review of the site: what did you like, what would you improve, what information is missing, list all errors you found, etc. Add an explicit acknowledgement that you understand the policies and procedures of this course.

Solution. I am excited to be part of the Magentaverse. I particularly like the Wikipedia links incorporated into the images. I did not see any ways to improve the website. Having created the policies and procedures, their necessity and details are painfully clear to me. □

P1.2. Provide a short explanation of why you are enrolled in this course. Why you are taking this course? How does it fits within your educational goals? What skills expect you anticipate developing? Is there anything the instructor should know about you?

Solution. I requested this teaching assignment. The equivalent course during second year of my undergraduate studies at Queen's literally changed the my life. I look forward to sharing my enthusiasm for algebra. Will this course have a significant impact on anyone? □

P1.3. For any nonnegative integer n, give two different proofs for the equation

$$\sum_{k=0}^{n} \frac{1}{(k+1)(k+2)} = \frac{n+1}{n+2}\,.$$

i. Verify this equation via induction on n.
ii. Derive this equation using partial fractions.

Solution.
i. We proceed by induction on n. When $n = 0$, we see that

$$\sum_{k=0}^{0} \frac{1}{(k+1)(k+2)} = \frac{1}{(1)(2)} = \frac{1}{2} = \frac{0+1}{0+2}\,,$$

so the base case holds. Assuming that $\sum_{k=0}^{n} \frac{1}{(k+1)(k+2)} = \frac{n+1}{n+2}$, it follows that

$$\begin{aligned}\sum_{k=0}^{n+1} \frac{1}{(k+1)(k+2)} &= \left(\sum_{k=0}^{n} \frac{1}{(k+1)(k+2)}\right) + \frac{1}{(n+2)(n+3)} \\ &= \frac{n+1}{n+2} + \frac{1}{(n+2)(n+3)} = \frac{(n+1)(n+3)+1}{(n+2)(n+3)} \\ &= \frac{n^2+4n+3+1}{(n+2)(n+3)} = \frac{(n+2)^2}{(n+2)(n+3)} = \frac{(n+1)+1}{(n+1)+2}\,,\end{aligned}$$

which completes the induction step.

ii. Since

$$\frac{1}{k+1} - \frac{1}{k+2} = \frac{(k+2)-(k+1)}{(k+1)(k+2)} = \frac{1}{(k+1)(k+2)},$$

we obtain the telescoping sum

$$\begin{aligned}
\sum_{k=0}^{n} \frac{1}{(k+1)(k+2)} &= \sum_{k=0}^{n} \left(\frac{1}{k+1} - \frac{1}{k+2}\right) \\
&= \frac{1}{0+1} + \sum_{k=1}^{n} \left(\frac{1}{k+1}\right) - \sum_{k=0}^{n-1} \left(\frac{1}{k+2}\right) - \frac{1}{n+2} \\
&= 1 + \sum_{k=1}^{n} \left(\frac{1}{k+1}\right) - \sum_{k=1}^{n} \left(\frac{1}{k+1}\right) - \frac{1}{n+2} \\
&= \frac{n+2}{n+2} - \frac{1}{n+2} = \frac{n+1}{n+2}.
\end{aligned}$$

□