Problem Set #9

Due: Thursday, 8 November 2012

Students registered in MATH 401 should submit solutions to three of the following problems. Students in MATH 801 should submit solutions to all five.

1. Prove that a tree T has a perfect matching if and only if $o(T - v) = 1$ for every $v \in V(T)$.

2. If G is a d-regular graph of even order that remains connected when any $d - 2$ edges are deleted, then prove that G has perfect matching.

3. Let G be a k-connected graph of even order having no $K_{1,k+1}$ as an induced subgraph. Prove that G has a perfect matching.

4. Let G be a graph whose odd cycles are pairwise intersecting, meaning that every two odd cycles in G have a common vertex. Prove that $\chi(G) \leq 5$.

5. Prove that every graph G has a vertex ordering relative to which the greedy algorithm uses $\chi(G)$ colours.