Problems 05

Due: Friday, 6 February 2026 before 23:59 ET

P5.1. Let K be a field, and let $f, g \in K[x]$ be monic irreducible polynomials. Prove that, when f and g have a common root in some field extension $K \subseteq L$, we have $f = g$.

P5.2. Find the minimal polynomial of the 24th root of unity $\zeta_{24} := \exp(2\pi \mathrm{i}/24)$ as follows.

i. Factor $x^{24} - 1$ over $\mathbb{Q}$.

ii. Determine which of the factors is the minimal polynomial of ζ_{24}.

P5.3. Let K be a field

i. Demonstate that the polynomial $x^m - a \in K[x]$ is reducible whenever the positive integer m has a divisor d such that $d > 1$ and

$$a = \begin{cases} b^d & \text{if } b \in K, \\ -4c^4 & \text{if } d = 4 \text{ and } c \in K. \end{cases}$$

ii. Let $L := K(t)$ be the field of rational functions in t with coefficients in K. Consider $f := x^p - t \in L[x]$ where p is a positive prime integer. Prove that f is irreducible.