Problems 08

Due: Friday, 6 March 2026 before 23:59 ET

P8.1. Prove that

$$\left|\mathrm{Gal}(\mathbb{Q}(\sqrt{p_1}, \sqrt{p_2}, \ldots, \sqrt{p_n}) \,/\, \mathbb{Q})\right| = 2^n$$

where $p_1, p_2, \ldots, p_n$ are the first n primes in the ring $\mathbb{Z}$ of integers.

P8.2. Find a subgroup of the symmetric group $\mathfrak{S}_4$ that is isomorphic to the Galois group

$$\mathrm{Gal}\Big(\mathbb{Q}(\mathrm{i}, \sqrt[4]{2}) \,/\, \mathbb{Q}\Big).$$

Can you name this group?

P8.3. Let L be the splitting field of $f := 2x^5 - 10x + 5 \in \mathbb{Q}[x]$. Prove that $\mathrm{Gal}(L/\mathbb{Q}) \cong \mathfrak{S}_5$.